University of Alabama
Department of Physics and Astronomy
Department of Electrical and Computer Engineering

PH 495/ECE 493 LeClair & Kung Spring 2011

Problem Set 1: "Review"

Instructions:

1. Answer all questions below. All questions have equal weight. Show your work for full credit.
2. All problems are due Thursday 20 January 2011 by 11:59pm.
3. You may collaborate, but everyone must turn in their own work.

1. *Hecht 2.17* The wavefunction of a transverse wave on a string is

$$\psi(x,t) = (30.0\,\text{cm}) \cos\left[(6.28\,\text{rad/m})\,x - (20.0\,\text{rad/s})\,t\right] \tag{1}$$

Compute the frequency, wavelength, period, amplitude, phase velocity, and direction of motion.

2. *Hecht 2.18* Show that

$$\psi(x,t) = A \sin k\,(x - vt) \tag{2}$$

is a solution of the differential wave equation.

3. *Hecht 2.31-2* Which of the following expressions correspond to traveling waves? For each of those, what is the speed of the wave? The quantities A, a, b, c are positive real constants.

$$\psi(x,t) = (ax - bt)^2 \tag{3}$$
$$\psi(x,t) = A \sin\left(ax^2 - bt^2\right) \tag{4}$$
$$\psi(x,t) = \frac{1}{ax^2 + b} \tag{5}$$
$$\psi(x,t) = A \sin 2\pi \left(\frac{x}{a} + \frac{t}{b}\right) \tag{6}$$

4. *Hecht 2.38* Show that the imaginary part of a complex number z is given by

$$\frac{z - z^*}{2i} \tag{7}$$

5. *Hecht 2.40* Show that the functions

$$\psi(x, y, z, t) = f(\alpha x + \beta y + \gamma z - \nu t) \tag{8}$$
$$\phi(x, y, z, t) = g(\alpha x + \beta y + \gamma z + \nu t) \tag{9}$$

which are plane waves of arbitrary form, satisfy the three-dimensional differential wave equation.

6. *Hecht 3.4* The time average of some function $f(t)$ taken over an interval T is given by

$$\langle f(t) \rangle = \frac{1}{T} \int_{t}^{T+t} f(t')\, dt' \tag{10}$$

where t' is just a dummy variable of integration. If $\tau = 2\pi/\omega$ is the period of a harmonic function, show that

$$\langle \sin^2 (kx - \omega t) \rangle = \frac{1}{2} \tag{11}$$
$$\langle \cos^2 (kx - \omega t) \rangle = \frac{1}{2} \tag{12}$$
$$\langle \sin (kx - \omega t) \cos (kx - \omega t) \rangle = 0 \tag{13}$$

when $T = \tau$ and when $T \gg \tau$.

7. *Hecht 3.5* An electromagnetic wave is specified (in SI units) by the following function:

$$\vec{E} = \left(-6\,\hat{\imath} + 3\sqrt{5}\,\hat{\jmath}\right) (10^4\ \mathrm{V/m})\, e^{i\left[\frac{1}{3}(\sqrt{5}x+2y)\pi\times 10^7 - 9.42\times 10^{15}\, t\right]} \tag{14}$$

Find **(a)** the direction along which the electric field oscillates, **(b)** the scalar value oft he amplitude of the electric field, **(c)** the direction of propagation of the wave, **(d)** the propagation number and wavelength, **(e)** the frequency and angular frequency, and **(f)** the speed.

8. The equation for a driven damped oscillator is

$$\frac{d^2x}{dt^2} + 2\gamma\omega_o \frac{dx}{dt} + \omega_o^2 x = \frac{q}{m} E(t) \tag{15}$$

(a) Explain the significance of each term.
(b) Let $E = E_o e^{i\omega t}$ and $x = x_o e^{i(\omega t - \alpha)}$ where E_o and x_o are real quantities. Substitute into the

above expression and show that

$$x_o = \frac{qE_o/m}{\sqrt{(\omega_o^2 - \omega^2)^2 + (2\gamma\omega\omega_o)^2}} \tag{16}$$

(c) Derive an expression for the phase lag α, and sketch it as a function of ω, indicating ω_o on the sketch.

9. Calculate the divergence ($\vec{\nabla}\cdot$) and curl ($\vec{\nabla}\times$) for the following vector functions $\vec{F}(x, y, z)$. Then verify that the divergence of the curl is zero for each, i.e., $\vec{\nabla} \cdot \left(\vec{\nabla} \times \vec{F}\right) = 0$.

$$x\hat{x} + (y+z)\,\hat{y} + (x+y+z)\,\hat{z}$$

$$f(x)\hat{x} + g(y)\hat{y} + h(z)\hat{z}$$

$$f(y,z)\hat{x} + g(z,x)\hat{y} + h(x,y)\hat{z}$$

$$(x+y+z)\,(x\hat{x} + y\hat{y} + z\hat{z})$$