University of Alabama
Department of Physics and Astronomy
Department of Electrical and Computer Engineering

PH 495/ECE 493 LeClair & Kung | Spring 2011

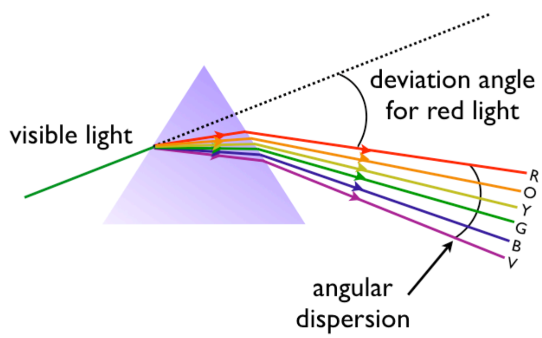
Problem Set 2

Instructions:

1. Answer all questions below. All questions have equal weight. Show your work for full credit.
2. All problems are due Thursday 27 January 2011 by 11:59pm.
3. You may collaborate, but everyone must turn in their own work.

1. The index of refraction for **violet** light in silica flint glass is $n_{\text{violet}} = 1.66$, and for **red** light it is $n_{\text{red}} = 1.62$. In air, $n = 1$ for both colors of light.

What is the **angular dispersion** of visible light (the angle between red and violet) passing through an equilateral triangle prism of silica flint glass, if the angle of incidence is 50°? The angle of incidence is that between the ray and a line *perpendicular* to the surface of the prism. Recall that all angles in an equilateral triangle are 60°.



2. *Hecht 3.48* Show that for substances of low density, such as gases, which have a single resonant frequency ω_o, the index of refraction is given by

$$n \approx 1 + \frac{N q_e^2}{2\epsilon_o m_e \left(\omega_o^2 - \omega^2\right)} \tag{1}$$

3. What is the apparent depth of a swimming pool in which there is water of depth 3 m, **(a)** When viewed from normal incidence? **(b)** When viewed at an angle of 60° with respect to the surface? The refractive index of water is 1.33.

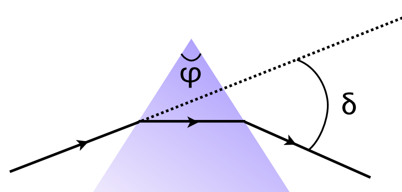
4. As light from the Sun enters the atmosphere, it refracts due to the small difference between the speeds of light in air and in vacuum. The optical length of the day is defined as the time interval between the instant when the top of the Sun is just visibly observed above the horizon, to the instant at which the top of the Sun just disappears below the horizon. The geometric length of the day is defined as the time interval between the instant when a geometric straight line drawn from the observer to the top of the Sun just clears the horizon, to the instant at which this line just dips below the horizon. The day's optical length is slightly larger than its geometric length.

By how much does the duration of an optical day exceed that of a geometric day? Model the Earth's atmosphere as uniform, with index of refraction $n = 1.000293$, a sharply defined upper surface, and depth 8767 m. Assume that the observer is at the Earth's equator so that the apparent path of the rising and setting Sun is perpendicular to the horizon. You may take the radius of the earth to be 6.378×10^6 m. Express your answer to the nearest hundredth of a second.

5. Light is deviated by a glass prism of index n as shown in the figure below. The ray in the prism is parallel to the base. Show that the refractive index is related to the deviation angle δ and the prism angle φ by the equation

$$n \sin \frac{\varphi}{2} = \sin \left(\frac{\varphi + \delta}{2} \right)$$

for this angle of incidence (*i.e.*,, the angle of incidence such that the ray in the prism is parallel to the base). The deviation angle δ is a minimum for this angle of incidence, and is known as the angle of minimum deviation. *Hint: You can solve this and the first problem together, if you keep things as general as possible - this is just a special case of the first problem.*



6. *Hecht 3.57* In 1871 Sellmeier derived the equation

$$n^2 = 1 + \sum_j \frac{A_j \lambda^2}{\lambda^2 - \lambda_{0j}^2} \tag{2}$$

where the A_j terms are constants and each λ_{0j} is the vacuum wavelength associated with a natural frequency ν_{0j}, such that $\lambda_{0j}\nu_{0j} = c$. This formulation is a considerable practical improvement over the Cauchy equation. Show that where $\lambda \gg \lambda_{0j}$, Cauchy's equation is an approximation of

Sellmeier's. *Hint:* write the above expression with only the first term in the sum; expand it by the binomial theorem; take the square root of n^2 and expand again.

7. *Hecht 4.5* Imagine that we have a non-absorbing glass plate of index n and thickness Δy, which stands between a source S and observer P.

(a) If the unobstructed wave (without the plate present) is $E_u = E_o \exp i\omega\,(t - y/c)$, show that with the plate in pace the observer sees a wave

$$E_p = E_o \exp i\omega\,[t - (n-1)\,\Delta y/c - y/c] \tag{3}$$

(b) Show that if either $n \approx 1$ or Δy is very small, then

$$E_p = E_u + \frac{\omega\,(n-1)\,\Delta y}{c} E_u e^{-i\pi/2} \tag{4}$$

The second term on the right may be envisioned as the field arising from the oscillators in the plate.

8. *Hecht 4.29* Starting with Snell's law, prove that the vector refraction equation has the form

$$n_t \hat{k}_t - n_i \hat{k} = (n_t \cos\theta_t - n_i \cos\theta_i)\,\hat{u}_n \tag{5}$$