University of Alabama
Department of Physics and Astronomy
Department of Electrical and Computer Engineering

PH 495/ECE 493 LeClair & Kung — Spring 2011

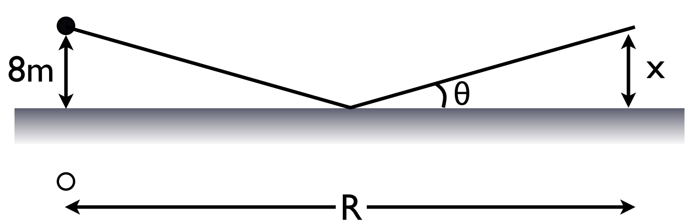
Problem Set 5

Instructions:

1. Answer all questions below. All questions have equal weight.
2. Show your work for full credit.
3. All problems are due Friday 25 March 2011 by 11:59pm.
4. You may collaborate, but everyone must turn in their own work.

Problems 9 and 10 involve a take-home experiment, for which you will need 2 glass slides and a plano-convex lens. These items will be provided in lecture on 10 March, or you may pick them up from Dr. LeClair by appointment after that time.

1. *Bekefi & Barrett 8.2; Hecht 9.24* A radar antenna operating on a wavelength of 0.10 m is located 8 m above the water line of a torpedo boat. Treat the reflected beam from the water as originating in a source 8 m below the water directly under the radar antenna. The dipole antenna is oriented perpendicular to the plane of the page.



(a) What is the altitude x of an airplane 12 km from the boat if it is to be in the first interference minimum of the radar signal?

(b) What is the total number of minima one observes as one scans the sky in the vertical plane as a function of the angle θ, from $\theta=0$ to $\theta=\pi$, keeping the distance R fixed?

2. *Bekefi & Barrett 8.3* Two dipole radiators (e.g., the oscillating current segments we discussed in class) are separated by a distance $\lambda/2$ along the x axis (half-wave dipole antenna). The dipoles are oriented along z, as in the problem we worked in class. Assume the distance to the observation point r satisfies $r \gg \lambda$.

(a) Plot the intensity of radiation in the x–y plane. Note the values of intensity at $\theta = 0, \pi/3, \pi/2, \pi$ if the oscillators are in phase.[i]

(b) Repeat (a) if the oscillators are 180° out of phase.

(c) The oscillators are now spaced by a distance $\lambda/4$ and are 90° out of phase. Repeat (a). Note that this configuration would be very useful for a broadcast station in a coastal city, for example ...

3. *Bekefi & Barrett 8.5* We desire to superpose the oscillations of several simple harmonic oscillators having the same frequency ω and amplitude A, but differing from one another by constant phase increments α; that is,

$$E(t) = A\cos\omega t + A\cos(\omega t + \alpha) + A\cos(\omega t + 2\alpha) + A\cos(\omega t + 3\alpha) + \cdots \tag{1}$$

(a) Using graphical phasor addition, find $E(t)$; that is, writing $E(t) = A_o \cos(\omega t + \varphi)$, find A_o and φ for the case when there are five oscillators with $A = 3$ units and $\alpha = \pi/9$ radians.

(b) Study the polygon you obtained in part (a) and, using purely geometrical considerations, show that for N oscillators

$$E(t) = (NA)\frac{\sin N\alpha/2}{N\sin\alpha/2}\cos\left[\omega t + \left(\frac{N-1}{2}\right)\alpha\right] \tag{2}$$

(c) Sketch the amplitude of $E(t)$ as a function of α.

The above calculation is the basis of finding radiation from antenna arrays and diffraction gratings.

4. *Hecht 9.10* White light falling on two narrow slits emerges and is observed on a distant screen. If red light ($\lambda_o = 780\,\text{nm}$) in the first-order fringe overlaps violet in the second-order fringe, what is the latter's wavelength?

5. *Hecht 9.26* A soap film surrounded by air has an index of refraction of 1.34. if a region of the film appears as bright red ($\lambda_o = 633\,\text{nm}$) in normally reflected light, what is its minimum thickness there?

6. *Hecht 9.36* One of the mirrors in a Michelson interferometer is moved, and 1000 fringe *pairs*

[i] You want to make a polar plot with intensity as the radial distance and reference the angle from the midpoint between the two sources. Wolfram alpha is handy for this, `http://wolframalpha.com`.Try a query like "plot of r = 4cos^2(-(pi/4)*sin(theta) + pi/4)"

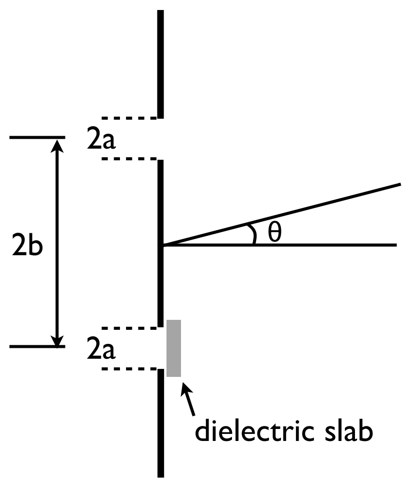
shift past the hairline in a viewing telescope during the process. If the device is illuminated with 500 nm light, how far was the mirror moved?

7. *Hecht 9.47* A glass camera lens with an index of refraction of 1.55 is to be coated with a cryolite film ($n \approx 1.30$) to decrease the reflection of normally incident green light ($\lambda_o = 500\,\text{nm}$). What thickness should be deposited on the lens?

8. *Bekefi & Barrett 8.9* A plane electromagnetic wave of wavelength λ_o is incident on two long, narrow slits, each having width $2a$ and separated by a distance $2b$, with $b \gg a$. **One** of the slits is covered by a thin dielectric plate of thickness d, and dielectric coefficient κ, with d chosen so that $(\sqrt{\kappa}-1)d/\lambda_o = 5/2$.

The interference pattern due to the slits is observed in a plane a distance L from the slits, where L is large enough so that the far field approximations may be used, that is, the pattern depends only on the angle θ from the normal to the slits, as shown.

(a) Consider effects due to interference only. What is the condition for a maximum in the pattern? Sketch the interference pattern.



(b) Now include effects due to both interference and diffraction. How is the intensity distribution modified from that obtained in (a)? Let $b/a = 10$, sketch the resulting interference-diffraction pattern. (Assume that all angles involved are small enough so that $\cos\theta \approx 1$, and hence that the optical path through the dielectric is independent of angle.)

For questions 9 and 10, download the procedure for the take-home experiment here:[ii]

[ii]These experiments have been adapted from Mavalvala, Nergis, Walter Lewin, and Wolfgang Ketterle, "8.03

http://ocw.mit.edu/courses/physics/8-03-physics-iii-vibrations-and-waves-fall-2004/assignments/take_home_exp6.pdf

In these experiments, you will investigate two interference effects. The phenomena are not hard to observe, and the mathematical derivations and descriptions (found in your text) are not difficult. However, until you see these effects yourself and explore them a bit, you will not have a true understanding of interference. Plus, the interference patterns can be quite striking and colorful!

9. Perform the interference experiment with two glass slides. Take care in making sure your slides are clean and dust-free. Observing the fringes may be easier using a desk lamp close to the slides, position yourself such that you see the mirror-like reflection from the glass surface. If you do not see the fringes clearly, try gently pressing the slides together with a pencil so they make better contact. For a color filter, any colored glass or plastic will do (perhaps a bottle). If you can't find anything, try using a colored light source, such as bright LED. An interesting variation of the fringe patterns can be seen if you press (carefully!) on the slides with a pencil to slightly bow the slides - you are essentially observing the strain field created in the glass.

Briefly, in a paragraph or two, discuss your findings qualitatively, and make an order-of-magnitude estimate of the amount by which your slides deviate from being perfectly flat.

10. Perform the experiment on Newton's rings using your plano-convex lens and one of the glass slides. Note that the rings are quite small and harder to observe than the fringes in the previous experiment – using a second lens or magnifying glass to more easily observe the rings may be a good idea.

Briefly discuss your findings qualitatively. Note in particular why the rings have the size that they do, and how you might make them bigger. Use the rings to *roughly* estimate the radius of curvature of your lens. From an earlier lab, you should know how to verify the radius of curvature from the focal distance of the lens.