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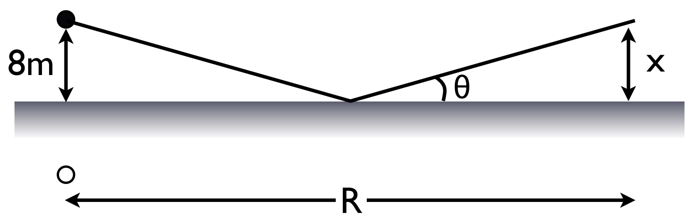
PH 495/ECE 493 LeClair & Kung   Spring 2011

Problem Set 5: Solutions

1. *Bekefi & Barrett 8.2; Hecht 9.24* A radar antenna operating on a wavelength of 0.10 m is located 8 m above the water line of a torpedo boat. Treat the reflected beam from the water as originating in a source 8 m below the water directly under the radar antenna. The dipole antenna is oriented perpendicular to the plane of the page.

is there a r



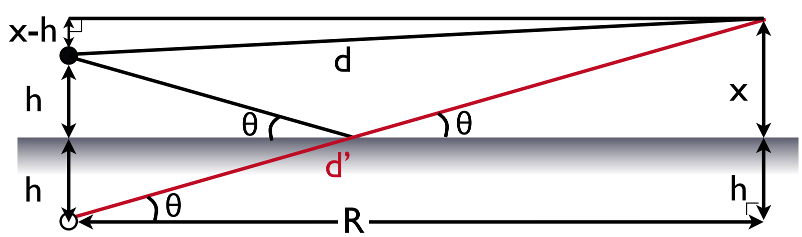
(a) What is the altitude x of an airplane 12 km from the boat if it is to be in the first interference minimum of the radar signal?

(b) What is the total number of minima one observes as one scans the sky in the vertical plane as a function of the angle θ, from $\theta = 0$ to $\theta = \pi$, keeping the distance R fixed?

Solution: From the geometry of the figure, referencing the crudely modified figure below, you should be able to convince yourself that

$$d^2 = (x-h)^2 + R^2 \tag{1}$$

$$d'^2 = (x+h)^2 + R^2 \tag{2}$$



The path difference between the direct ray (d) and the reflected ray d' is thus

$$d' - d = \sqrt{(x+h)^2 + R^2} - \sqrt{(x-h)^2 + R^2} \tag{3}$$

The phase difference δ between the two rays is this path difference multiplied by $k = 2\pi/\lambda$ *plus* an extra phase shift of π for the reflected ray:

$$\delta = \frac{2\pi}{\lambda}\left(d' - d\right) + \pi \tag{4}$$

The intensity is then easily found:

$$I = 4I_o \cos^2\left(\frac{\delta}{2}\right) = 4I_o \cos^2\left(\frac{\pi}{\lambda}\left(d' - d\right) + \pi\right) = 4I_o \sin^2\left(\frac{\pi}{\lambda}\left(d' - d\right)\right) \tag{5}$$

The minima in intensity occur when

$$\frac{\delta}{2} = \frac{\pi}{\lambda}\left(d' - d\right) = n\pi \tag{6}$$

$$n\lambda = d' - d = \sqrt{(x+h)^2 + R^2} - \sqrt{(x-h)^2 + R^2} \tag{7}$$

The problem is then the "simple" matter of solving the above for x. It can be done, with some tedium. First square both sides and isolate the remaining radical.

$$\begin{aligned} n\lambda &= \sqrt{(x+h)^2 + R^2} - \sqrt{(x-h)^2 + R^2} \\ n^2\lambda^2 &= (x+h)^2 + R^2 + (x-h)^2 + R^2 - 2\sqrt{(x+h)^2(x-h)^2 + R^4 + R^2\left((x+h)^2 + (x-h)^2\right)} \end{aligned}$$

Square both sides and collect terms ...

$$\begin{aligned} 2\left(x^2 + h^2 + R^2\right) - n^2\lambda^2 &= 2\sqrt{\left(x^2 - h^2\right)^2 + R^4 + 2R^2\left(x^2 + h^2\right)} \\ 4\left(x^2 + h^2 + R^2\right)^2 + n^4\lambda^4 - 4n^2\lambda^2\left(x^2 + h^2 + R^2\right) &= 4\left(x^4 + h^4 - 2x^2h^2 + R^4 + 2R^2\left(x^2 + h^2\right)\right) \\ 4x^4 + 4h^4 + 4R^4 - 8x^2h^2 + 8x^2R^2 + 8h^2R^2 &= 4x^4 + 4h^4 + 4R^4 + 8x^2h^2 + 8x^2R^2 \\ &\quad + 2h^2R^2 + n^4\lambda^4 - 4n^2\lambda^2\left(x^2 + h^2 + R^2\right) \\ 16x^2h^2 - 4n^2\lambda^2x^2 &= 4n^2\lambda^2\left(h^2 + R^2\right) - n^4\lambda^4 = n^2\lambda^2\left(4h^2 + 4R^2 - n^2\lambda^2\right) \\ x^2 &= \frac{n^2\lambda^2\left(4h^2 + 4R^2 - n^2\lambda^2\right)}{16h^2 - 4n^2\lambda^2} \\ x &= n\lambda\sqrt{\frac{4R^2 + 4h^2 - \lambda^2n^2}{16h^2 - 4\lambda^2n^2}} \approx 75.0\,\text{m} \end{aligned} \tag{8}$$

If we first use the very reasonable approximation $\lambda \ll \{R, h\}$ and then the nearly as reasonable $h \ll R$, we can come to something a bit simpler:

$$x = n\lambda\sqrt{\frac{4R^2+4h^2-\lambda^2n^2}{16h^2-4\lambda^2n^2}} \approx n\lambda\frac{\sqrt{R^2+h^2}}{2h} \approx n\lambda\frac{R}{2h} \tag{9}$$

These latter two forms are readily derived by starting from Eq. 7, factoring out $\sqrt{R^2+x^2+h^2}$ from both terms, and approximating the resulting radicals for $2xh \ll R^2+x^2+h^2$ using a binomial expansion (since x is at minimum h, this is ok). Using the numbers given, one finds the first minimum at $x \approx 75.0\,\mathrm{m}$.

How many minima must there be over the interval $[0,\pi]$? We have solved the problem of finding the minima only over the interval $[0,\pi/2]$. However, owing to the symmetry of the problem, we know that the interval $[\pi/2,\pi]$ has the same number of minima.

Unfortunately, we have no obvious restriction on magnitude x from the information given, and everything else is fixed. We do know that at $\theta=0$, $x=h$, but as θ approaches $\pi/2$, x increases without bound. What to do? The exact expression above does have an interesting condition built in, however: x must be real. For this to be true, the numerator and denominator in Eq. 8 must both be positive. That is,

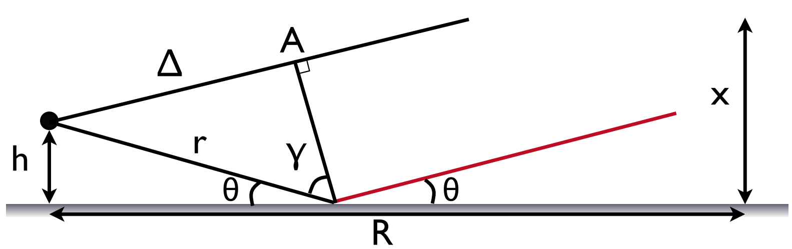
$$4R^2+4h^2-\lambda^2n^2>0 \qquad \Longrightarrow \qquad n_{\max}=\frac{2}{\lambda}\sqrt{R^2+h^2} \tag{10}$$

$$16h^2-4\lambda^2n^2>0 \qquad \Longrightarrow \qquad n_{\max}=\frac{2h}{\lambda} \tag{11}$$

Given that $R \gg h$, it is clear that the second expression is more restrictive. Using the numbers given, one finds $n=160$. This is for the interval $[0,\pi/2]$, the problem we have solved. The interval $(\pi/2,\pi]$ thus contains 159 minima (not double counting the minima at $\pi/2$, so in total over $\theta \in [0,\pi]$ we have $n=319$ minima.

Alternate method

The above is an *exact* method, though far from obvious. Given that both λ and h are tiny compared to R, we need not solve the problem exactly. We can use the same dipole approximation we employed previously to make things far simpler. The gist of the idea is that the source is very distant compared to the spacing of the real and virtual sources ($h << R$), and the geometry looks more like this:



We assume the source is sufficiently distant that the rays approaching the source can be approximated as parallel. Construct a line running from the intersection of the reflected ray with the water to the direct ray, such that this line meets the direct ray at a right angle (point A in the figure). The parallel ray approximation means that from point A rightward, both direct and reflected rays traverse the same distance. The only geometrical phase difference is thus due to the path difference: the reflected ray travels a distance r, while the direct ray travels a distance Δ. If we can find the difference $r-\Delta$, we are nearly done.

From the figure above, the surface of the water is made up from angles θ, γ, and 90, such that

$$\gamma = 180 - 90 - 2\theta = 90 - 2\theta \tag{12}$$

You can convince yourself that

$$r = \frac{h}{\sin\theta} \tag{13}$$

$$\Delta = r\sin\gamma \tag{14}$$

The path difference is then

$$r - \Delta = \frac{h}{\sin\theta} - r\sin\gamma = \frac{h}{\sin\theta} - \left(\frac{h}{\sin\theta}\right)\sin(90-2\theta) = \frac{h}{\sin\theta}(1-\cos 2\theta) \tag{15}$$

Accounting for the phase shift on reflection, our total phase shift is then

$$\delta = \frac{2\pi}{\lambda}(r-\Delta) + \pi = \frac{2\pi}{\lambda}\frac{h}{\sin\theta}(1-\cos 2\theta) + \pi = \frac{2\pi}{\lambda}\frac{h}{\sin\theta}\left(2\sin^2\theta\right) + \pi = \frac{4\pi h}{\lambda}\sin\theta + \pi \tag{16}$$

The intensity is then

$$I = 4I_o\cos^2\left(\frac{\delta}{2}\right) = 4I_o\cos^2\left(\frac{2\pi h}{\lambda}\sin\theta + \frac{\pi}{2}\right) = 4I_o\sin^2\left(\frac{2\pi h}{\lambda}\sin\theta\right) \tag{17}$$

The condition for a minimum is

$$\frac{2\pi h}{\lambda}\sin\theta = n\pi \qquad \text{or} \qquad \sin\theta = \frac{n\lambda}{2h} \tag{18}$$

For a very distant source, the separation of the parallel rays is small compared to x, and we may approximate $x \approx R\tan\theta \approx R\sin\theta = Rn\lambda/2h$, which gives $x \approx 75\,\mathrm{m}$ in agreement with our previous result.

As for the number of minima, we need only note from the above expression that $\sin\theta$ returns values only in the interval $[0,1]$ over $\theta \in [0, \pi/2]$. That is, since $\sin\theta$ is at most 1,

$$\frac{n\lambda}{2h} = \sin\theta \leqslant 1 \qquad \text{or} \qquad n \leqslant \frac{2h}{\lambda} = 160 \tag{19}$$

This is precisely the condition we derived earlier; accounting for the angles $[\pi/2, \pi]$, we end up with a total of $2 \times 160 - 1 = 319$ minima.

Whether you prefer the first or second method is really a matter of taste. I preferred the first since it gives an exact answer, though such precision is of limited utility - it is probably not worth the extra difficulty. The advantage of the second method is that it is *the same problem* we've solved many times already (it is just the double slit again, or Lloyd's mirror) with a couple of small twists.

2. *Bekefi & Barrett 8.3* Two dipole radiators (e.g., the oscillating current segments we discussed in class) are separated by a distance $\lambda/2$ along the x axis (half-wave dipole antenna). The dipoles are oriented along z, as in the problem we worked in class. Assume the distance to the observation point r satisfies $r \gg \lambda$.

(a) Plot the intensity of radiation in the x–y plane. Note the values of intensity at $\theta = 0, \pi/3, \pi/2, \pi$ if the oscillators are in phase.[i]

(b) Repeat (a) if the oscillators are 180° out of phase.

(c) The oscillators are now spaced by a distance $\lambda/4$ and are 90° out of phase. Repeat (a). Note that this configuration would be very useful for a broadcast station in a coastal city, for example ...

Solution: The intensity is

$$I = 4I_o \cos^2\left[\frac{\pi d}{\lambda}\sin\psi - \frac{1}{2}(\varphi_1 - \varphi_2)\right] = 4I_o \cos^2\left[\frac{\pi}{2}\sin\psi - \frac{1}{2}(\varphi_1 - \varphi_2)\right] \tag{20}$$

with $d = \frac{\lambda}{2}$. For (a) and (b), we need only make polar plots with I/I_o as the radial coordinate and ψ as the angular coordinate. For (a) we have $\varphi_1 - \varphi_2 = 0$, and for (b) we have $\varphi_1 - \varphi_2 = \pi$. For (c), the spacing is now $\lambda/4$ and $\varphi_1 - \varphi_2 = \pi/4$. Can you see now why this would be useful for a transmitter in a coastal city?

[i] You want to make a polar plot with intensity as the radial distance and reference the angle from the midpoint between the two sources. Wolfram alpha is handy for this, `http://wolframalpha.com`.Try a query like "plot of r = 4cos^2(-(pi/4)*sin(theta) + pi/4)"

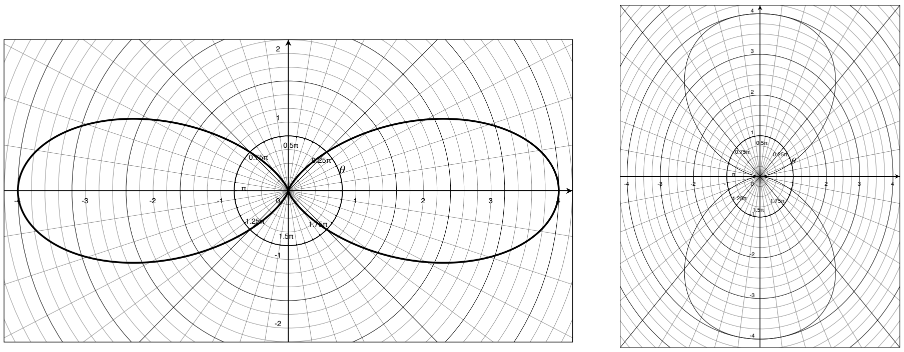


Figure 1: **(left)** *Intensity pattern for two oscillators with zero phase offset, spaced at* $d = \lambda/2$. *The oscillators are at* $y = \pm\lambda/4$. **(right)** *The same oscillators with a phase difference of* $\varphi_1 - \varphi_2 = \pi$. *The radiation pattern is rotated by* 90°.

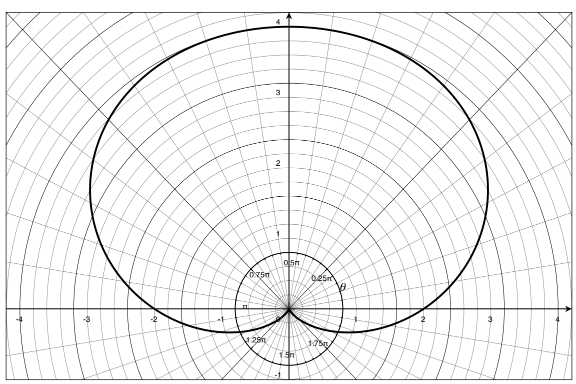


Figure 2: *Intensity pattern for two oscillators with phase offset* $\varphi_1 - \varphi_2 = \pi/2$, *spaced at* $d = \lambda/4$. *The oscillators are at* $y = \pm\lambda/8$.

3. *Bekefi & Barrett 8.5* We desire to superpose the oscillations of several simple harmonic oscillators having the same frequency ω and amplitude A, but differing from one another by constant phase increments α; that is,

$$E(t) = A\cos\omega t + A\cos(\omega t + \alpha) + A\cos(\omega t + 2\alpha) + A\cos(\omega t + 3\alpha) + \cdots \tag{21}$$

(a) Using graphical phasor addition, find $E(t)$; that is, writing $E(t) = A_o\cos(\omega t + \varphi)$, find A_o and φ for the case when there are five oscillators with $A = 3$ units and $\alpha = \pi/9$ radians.

(b) Study the polygon you obtained in part (a) and, using purely geometrical considerations, show that for N oscillators

$$E(t) = (NA)\,\frac{\sin N\alpha/2}{N\sin\alpha/2}\cos\left[\omega t + \left(\frac{N-1}{2}\right)\alpha\right] \tag{22}$$

(c) Sketch the amplitude of $E(t)$ as a function of α.

The above calculation is the basis of finding radiation from antenna arrays and diffraction gratings.

Solution: You can find a quick solution using a phasor diagram here:

`http://ocw.mit.edu/courses/physics/8-03-physics-iii-vibrations-and-waves-fall-2004/assignments/soln10.pdf`

We will solve the N oscillator problem by more direct means. Each oscillator has a phase offset of α from its neighbor, so the N^{th} oscillator has a phase offset of $(N-1)\,\alpha$ from the first. Since the total field from all N oscillators is the sum of their individual electric fields, and thus[ii]

[ii] The symbol $\Re$ is the operator that takes the real part of an expression, and $\Im$ is the operator that takes the imaginary part. Also recall the sum of a finite geometric series $\sum_0^{n-1} ar^k = a(1-r^n)/(1-r)$.

$$E_{\text{tot}} = \sum_{n=0}^{N-1} A\cos(\omega t + n\alpha) = A\sum_{n=0}^{N-1}\left[\cos(\omega t)\cos(n\alpha) - \sin(\omega t)\sin(n\alpha)\right] \tag{23}$$

$$= A\cos(\omega t)\sum_{n=0}^{N-1}\cos(n\alpha) - A\sin(\omega t)\sum_{n=0}^{N-1}\sin(\omega t)\cos(n\alpha) \tag{24}$$

$$= A\cos\omega t\left[\frac{\cos\left(\frac{N-1}{2}\alpha\right)\sin\left(\frac{N\alpha}{2}\right)}{\sin\frac{\alpha}{2}}\right] - A\sin\omega t\left[\frac{\sin\left(\frac{N-1}{2}\alpha\right)\sin\left(\frac{N\alpha}{2}\right)}{\sin\frac{\alpha}{2}}\right] \tag{25}$$

$$= A\frac{\sin\left(\frac{N\alpha}{2}\right)}{\sin\frac{\alpha}{2}}\left[\cos(\omega t)\cos\left(\frac{(N-1)\alpha}{2}\right) - \sin(\omega t)\sin\left(\frac{(N-1)\alpha}{2}\right)\right] \tag{26}$$

$$= A\left(\frac{\sin\left(\frac{N\alpha}{2}\right)}{\sin\frac{\alpha}{2}}\right)\cos\left[\omega t + \frac{(N-1)\alpha}{2}\right] = (NA)\frac{\sin N\alpha/2}{N\sin\alpha/2}\cos\left[\omega t + \left(\frac{N-1}{2}\right)\alpha\right] \tag{27}$$

Thus, total field is that of a single oscillator of amplitude and phase

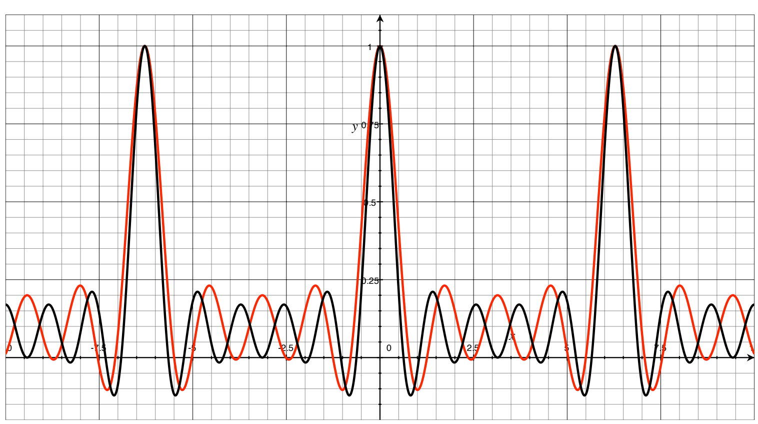
$$A_o = A\frac{\sin\left(\frac{N\alpha}{2}\right)}{\sin\frac{\alpha}{2}} \tag{28}$$

$$\varphi = \frac{(N-1)\alpha}{2} \tag{29}$$

One can also perform the sum using complex exponentials, which makes it a simple geometric series. Given $N=5$, $A=3$ and $\alpha=\pi/9$, one finds $A_o \approx 13.23$ and $\varphi = 2\pi/9 = 40°$. In order to sketch the amplitude as a function of α, we should recall that

$$\lim_{\beta\to 0}\frac{\sin N\beta}{\beta} = N \tag{30}$$

which means that the intensity has zeros at $N\alpha/2 = n\pi$ *unless* $\sin(\alpha/2)=0$, in which case we have a maximum. Below are plots of I/NA versus α for $N=5$ (red) and $N=6$ (black) for $\omega t=0$. Note the primary maxima at $\alpha=2n\pi$ and the $(N-2)$ secondary maxima in between.



4. *Hecht 9.10* White light falling on two narrow slits emerges and is observed on a distant screen. If red light ($\lambda_o = 780\,\text{nm}$) in the first-order fringe overlaps violet in the second-order fringe, what is the latter's wavelength?

Solution: The interference condition for a fringe to occur is

$$\sin\theta_m \approx \theta_m = \frac{m\lambda}{a} \tag{31}$$

where a is the slit spacing. If $\theta_{\text{red},m=1} = \theta_{\text{violet},m=2}$, this implies

$$\lambda_{\text{violet}} = \frac{1}{2}\lambda_{\text{red}} = 390\,\text{nm} \tag{32}$$

5. *Hecht 9.26* A soap film surrounded by air has an index of refraction of 1.34. if a region of the film appears as bright red ($\lambda_o = 633\,\text{nm}$) in normally reflected light, what is its minimum thickness there?

Solution: If the film appears red, it is because the thickness is such that interference creates a minimum in intensity for red light at wavelength λ_o in vacuum. These minima occur for a film of thickness d and index n at

$$d\cos\theta_t = (2m+1)\,\frac{\lambda_f}{4} \tag{33}$$

where the wavelength of light in the film λ_f relates to the vacuum wavelength via $\lambda_f = \lambda_o/n$. If we consider the intensity at normal incidence ($\theta_t = 0$) and the minimum thickness (corresponding to $m=0$),

$$d = \frac{\lambda_f}{4} = \frac{\lambda_o}{4n} \approx 118\,\text{nm} \tag{34}$$

6. *Hecht 9.36* One of the mirrors in a Michelson interferometer is moved, and 1000 fringe *pairs* shift past the hairline in a viewing telescope during the process. If the device is illuminated with 500 nm light, how far was the mirror moved?

Solution: A fringe pair will shift pass the hairline whenever one of the mirrors moves by $\lambda/2$. If N fringe pairs move past the hairline, one of the mirrors must have moved by a distance

$$\Delta d = \frac{1}{2}N\lambda \tag{35}$$

With $N = 1000$ and $\lambda_o = 500\,\text{nm}$, $\Delta d = 250\,\mu\text{m}$.

7. *Hecht 9.47* A glass camera lens with an index of refraction of 1.55 is to be coated with a cryolite film ($n \approx 1.30$) to decrease the reflection of normally incident green light ($\lambda_o = 500\,\text{nm}$). What thickness should be deposited on the lens?

Solution: Reflectivity minima occur for a film of thickness d and index n when

$$d \cos\theta_t = (2m+1)\frac{\lambda_f}{4} \tag{36}$$

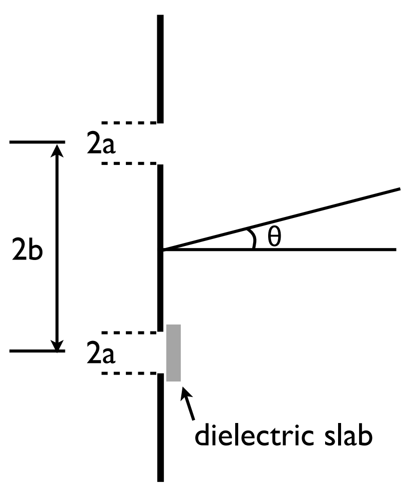
where the wavelength of light in the film λ_f relates to the vacuum wavelength λ_o via $\lambda_f = \lambda_o/n$. If we consider the intensity at normal incidence ($\theta_t = 0$), the minimum thickness must be

$$d = (2m+1)\frac{\lambda_f}{4} = \frac{\lambda_f}{4} = \frac{\lambda_o}{4n} \approx 96\,\text{nm} \tag{37}$$

8. *Bekefi & Barrett 8.9* A plane electromagnetic wave of wavelength λ_o is incident on two long, narrow slits, each having width $2a$ and separated by a distance $2b$, with $b \gg a$. **One** of the slits is covered by a thin dielectric plate of thickness d, and dielectric coefficient κ, with d chosen so that $(\sqrt{\kappa}-1)d/\lambda_o = 5/2$.

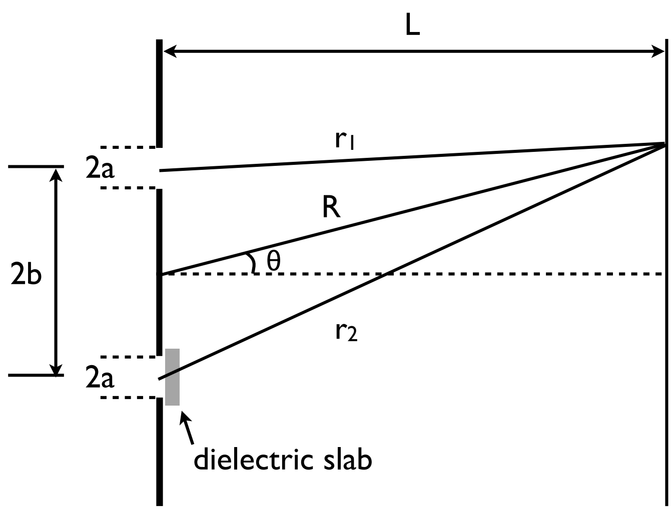
The interference pattern due to the slits is observed in a plane a distance L from the slits, where L is large enough so that the far field approximations may be used, that is, the pattern depends only on the angle θ from the normal to the slits, as shown.

(a) Consider effects due to interference only. What is the condition for a maximum in the pattern? Sketch the interference pattern.



(b) Now include effects due to both interference and diffraction. How is the intensity distribution modified from that obtained in (a)? Let $b/a = 10$, sketch the resulting interference-diffraction pattern. (Assume that all angles involved are small enough so that $\cos\theta \approx 1$, and hence that the optical path through the dielectric is independent of angle.)

Solution: Consider the revised figure below, which places a screen a distance $L \gg \{a, b\}$ from the slits. An observation point on the screen is then a distance r_1 from one slit and and r_2 from the other, as well as a distance R from the midpoint of the two slits.



The law of cosines allows us to find the distances r_1 and r_2 in terms of b, R and θ

$$r_2^2 = b^2 + R^2 - 2bR\cos(\theta + 90) = b^2 + R^2 + 2bR\sin(\theta) \tag{38}$$
$$r_1^2 = b^2 + R^2 - 2bR\sin(\theta) \tag{39}$$

The difference of the squared differences is then

$$r_2^2 - r_1^2 = 4bR\sin(\theta) = (r_1 + r_2)(r_2 - r_1) \tag{40}$$
$$r_2 - r_1 = \frac{4bR\sin(\theta)}{r_1 + r_2} \tag{41}$$

One can solve this analytically for r_2 in terms of r_1 and then compute $r_2 - r_1$ exactly.[iii] However, since we are explicitly encouraged to use the far field approximation, we may as well save a bit of tedium. If we make the approximation that θ is small, amounting to the condition $L \approx R$, then

[iii] This results in $r_2 - r_1 \approx \sqrt{2r_2^2 - 8bR\sin\theta}$, which does *not* depend only on θ as the far field approximation requires.

$r_1 \approx r_2$ and thus $r_1 + r_2 \approx 2R$. We *cannot* do the same for $r_2 - r_1$, however, since we are interested in difference between r_2 and r_1 on the scale of the incident wavelength. This leads us to

$$r_2 - r_1 \approx \frac{4bR\sin(\theta)}{2R} \tag{42}$$

This is the geometric path difference between the two beams, exactly as we have previously derived for the two slit problem. Ignoring the finite width of the slits for the moment, this would be sufficient to calculate the two slit interference pattern. In the present case, we must also take into account the propagation delay suffered by the second beam resulting from its transit through the dielectric. In crossing a distance d through the dielectric, the second beam requires a time

$$t_2 = \frac{d}{v} = \frac{dn}{c} \tag{43}$$

while the first transiting only vacuum through the same distance d requires a time

$$t_1 = \frac{d}{c} \tag{44}$$

The second beam therefore suffers a delay of

$$\delta t_2 = \frac{d}{c}(n-1) = \frac{d}{c}\left(\sqrt{\kappa} - 1\right) \tag{45}$$

where we have used the result $n = \sqrt{\kappa}$ in a medium where $\mu \approx \mu_o$. This time delay implies a phase delay for the second beam relative to the first of

$$\omega \delta t_2 = \frac{\omega d}{c}\left(\sqrt{\kappa} - 1\right) = \frac{d}{\lambda}\left(\sqrt{\kappa} - 1\right) = 5\pi \tag{46}$$

Here we have used $\lambda = c/f = 2\pi c/\omega$ and the given relationship $(\sqrt{\kappa} - 1)d/\lambda_o = 5/2$. We must add this phase delay to the geometric phase delay $k(r_2 - r_1)$ to find the total phase difference between the two beams:

$$\delta_{tot} = 2kb\sin\theta + 5\pi \tag{47}$$

This gives the intensity on the screen as[iv]

$$I = 4I_o\cos^2\left(\frac{\delta_{tot}}{2}\right) = 4I_o\cos^2\left(kb\sin\theta - \frac{5\pi}{2}\right) \tag{48}$$

We will have maxima when the argument of $\sin^2$ is an integer multiple of π:

$$kb\sin\theta - \frac{5\pi}{2} = m\pi \qquad \text{or} \qquad 2b\sin\theta = \frac{\lambda}{2}\left(m + \frac{5}{2}\right) \tag{49}$$

[iv]Note that whether we pick $+5\pi/2$ or $-5\pi/2$ makes no difference here, it physically only amounts to asking whether we are considering points above or below the midpoint of the screen, and ultimately makes no difference at all.

Since m is just an integer, what this formula is really telling us is that the path difference $2b\sin\theta$ must be a half-integer multiple of $\lambda/2$. We could just as well re-index the starting point of m and write

$$r_2 - r_1 \approx 2b\sin\theta = \frac{\lambda}{2}\left(m+\frac{1}{2}\right) \tag{50}$$

The gist of the matter is that the interference pattern now has a minimum at $\theta = 0$ as a result of the dielectric slab, rather than a maximum. Otherwise, it looks just like the usual two-slit problem.

The inclusion of diffraction essentially turns each slit into an array of point sources rather than a single point source. When we take into account the finite slit width, in the far-field regime we can imagine each point within a slit acts as a new source of spherical waves which will interfere with each other. We can model this as an array of point sources, as in problem 3, letting $N \to \infty$. Moreover, now the constant phase difference between neighboring oscillators makes sense - since each successive evenly-spaced source is slightly farther from the observation point, this phase offset is accounting for the propagation delay due to the spacing between oscillators.

If we have a slit of width $2a$ made up of N oscillators, then the spacing between oscillators is $d = 2a/N$, or $N = 2a/d$. Our slit is the case that $d \to 0$ with fixed $2a$. From problem 3, the total field of N oscillators in an array, each of intensity A, was

$$E = (NA)\frac{\sin N\alpha/2}{N\sin\alpha/2}\cos\left[\omega t + \left(\frac{N-1}{2}\right)\alpha\right] \tag{51}$$

The phase difference α for oscillators spaced a distance d from each other can be written (in analogy to our work above) $\alpha = kd\sin\psi$, where $\psi \approx \theta$ is the angle to the point of interest. Adding $2a = Nd$ and squaring to find the intensity (and noting $k = 2\pi/\lambda$),

$$I = A^2\frac{\sin^2\left(d\frac{2\pi a}{\lambda}\sin\psi\right)}{\sin^2\left(\frac{2\pi a}{\lambda N}\sin\psi\right)} \tag{52}$$

Taking the limit that $N \to \infty$, with $\psi \approx \theta$ and defining $I_o \equiv A^2$, the intensity of a slit of finite width $2a$ is

$$I = N^2 I_o\left(\frac{\sin\beta}{\beta}\right)^2 \qquad \text{with} \qquad \beta = \frac{2\pi a}{\lambda}\sin\theta \tag{53}$$

In the end, accounting for the finite width of the slits merely modulates the intensity by a factor $\sin^2(\beta)/\beta$. Thus, for the problem at hand,

$$I = 4I_o\left[\frac{\sin\left(\frac{2\pi a}{\lambda}\sin\theta\right)}{\frac{2\pi a}{\lambda}\sin\theta}\right]^2\cos^2\left(\frac{2\pi b}{\lambda}\sin\theta - \frac{5\pi}{2}\right) \tag{54}$$

The intensity plot is left as an exercise to the reader ...