UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 101 LeClair & Kung Summer 2011

Exam 1 practice problems

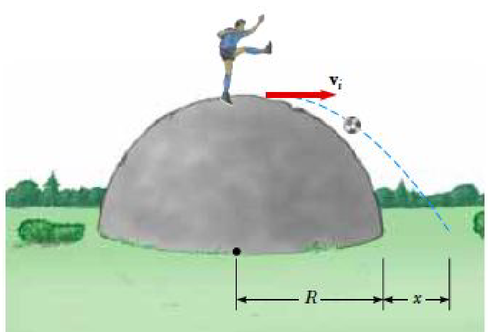
1. A car is traveling at a constant velocity of 18 m/s and passes a police cruiser. Exactly 2 seconds after passing, the cruiser begins pursuit, with a constant acceleration of $2.5\,\mathrm{m/s^2}$. How long does it take for the cruiser to overtake the car (from the moment the cop car starts)?

2. A rubber ball was thrown at a brick wall with an initial velocity of 10 m/s, and rebounded with a velocity of 8.5 m/s. The rebound was found to take 3.5×10^{-3} sec. What was the acceleration experienced by the ball during the rebound?

3. A projectile will be launched with an initial velocity of 750 m/s, and needs to hit a target 23 km away. What should the launch angle be? You can ignore air resistance.

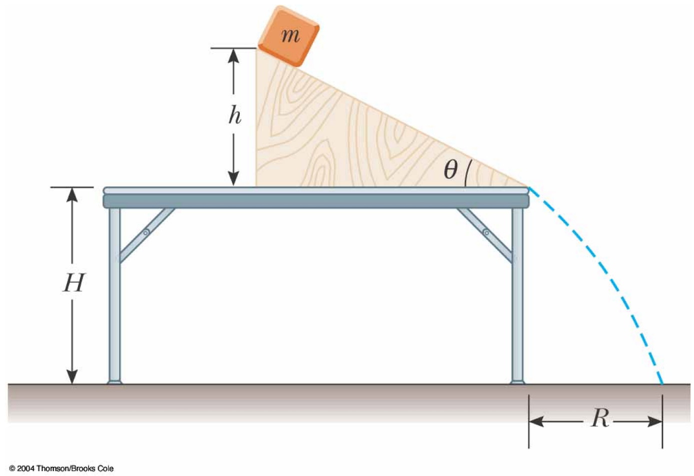
4. Joe fires his .270 Winchester, which has a muzzle velocity of 957 m/s using a 130 grain load, into the air at a 17° angle. Ignoring air resistance, how far away from Joe will the bullet land?

5. A person standing at the top of a hemispherical rock of radius R kicks a ball (initially at rest on the top of the rock) to give it horizontal velocity $\vec{v}_i$ as shown below. What must be its minimum initial speed if the ball is never to hit the rock after it is kicked? *Harder problem ...*

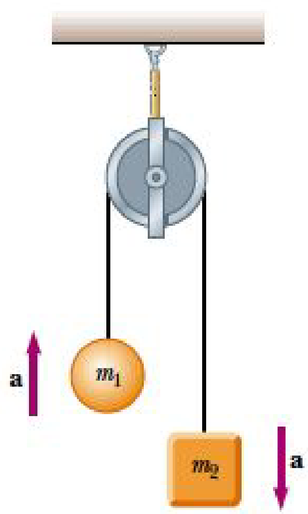


6. After being struck by a hockey stick, a hockey puck slides across the ice with an initial velocity of 7.0 m/s. If the coefficient of kinetic friction μ_k between the ice and the puck is 0.15, what is the velocity of the puck when it reaches the goal 10 m down the ice?

7. Consider the figure below. Let $h = 1\,\mathrm{m}$, $\theta = 37°$, $H = 2\,\mathrm{m}$, and $m = 1.1\,\mathrm{kg}$. There is a coefficient of kinetic friction $\mu_k = 0.2$ between the mass and the inclined plane, and the mass m starts out at the very top of the incline with a velocity of 0.1 m/s. Ignoring air resistance, what is R? *Much harder problem ...*



8. Consider the so-called "Atwood's machine" below. What is the acceleration of the two masses, if one ignores friction and the mass of the pulley and rope?



Solutions

1. Basically, we just want to write down the equations describing the position as a function of time for both the car and the cop, and set them equal to each other. We start with our basic equation for 1-D motion and fill in the terms we know: $x(t) = x_i + v_{ix}t + \frac{1}{2}a_x t^2$.

The car is just traveling at constant velocity, with no acceleration. Call its starting position $x_i = 0$, and we can write:

$$x_{\text{car}} = 18t$$

The cop starts two seconds later, we have to be careful to use $t - 2$ wherever time appears. The initial velocity for the cop is zero, as is the initial position. We only need to account for the acceleration of $2.5\,\text{m/s}^2$.

$$x_{\text{cop}} = \frac{1}{2}(2.5)(t-2)^2 = 1.25t^2 - 5t + 5$$

At the moment when the cop overtakes the car, their positions are equal:

$$x_{\text{car}} = x_{\text{cop}} = 18t = 1.25t^2 - 5t + 5 \qquad \Rightarrow \qquad 1.25t^2 - 23t + 5 = 0$$

Now we just solve the quadratic equation, which gives us roots of $t = [0.22, 18.2]$. What we were looking for was the time *relative to when the cop started*, which means we want $t-2$. The negative root is not physical (it corresponds to a time before the cop started), so **16.2 s** is the answer.

Incidentally, we could have set $t = 0$ at the moment the cop started, and not have to worry about subtracting 2 seconds off of the final result. This would give us the following:

$$x_{\text{car}} = 18(t-2) = x_{\text{cop}} = \frac{1}{2}(2.5)\,t^2 \qquad \Rightarrow \qquad 1.25t^2 - 18t - 36 = 0$$

You can verify yourself that the positive root of this quadratic equation is 16.2 sec, the time we were looking for.

2. All we need to know for this one is the definition of average acceleration.

$$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{-8.5 - 10\,\text{m/s}}{3.5 \times 10^{-3}\,\text{s}} = -5290\,\text{m/s}^2$$

3. The easiest way to solve this one is to use the formula for the range of a projectile, which we derived in class (and which was given on the formula sheet). Of course it is only slightly more work just to write down $x(t)$ and $y(t)$ explicitly and solve ... Note the distance is given in *km* not m.

$$R = \frac{v_i^2 \sin 2\theta}{g} = \frac{750^2 \sin 2\theta}{g} = 23000 \qquad \Rightarrow \qquad \sin 2\theta = \frac{23000g}{750^2} \qquad \Rightarrow \qquad \theta = 11.8^\circ \approx 12^\circ$$

4. Again, using the range equation is most straightforward:

$$R = \frac{v_i^2 \sin 2\theta}{g} = \frac{957^2 \sin 34^\circ}{g} = 5.2 \times 10^4 \text{ m}$$

5. We know that the ball being kicked off of the rock will follow projectile motion, which describes a parabolic curve $y(x)$. The rock, being a hemisphere, can be described in the x-y plane by a circle, $y_{\text{rock}}^2 + x_{\text{rock}}^2 = R^2$. What we want to know, then, is for what minimum value of v_i does the parabola describing the ball's motion *not* intersect the quarter circle?

Measure heights relative to the ground and call the ball's starting position $x = 0$. The x position of the ball at any time t is just $x = v_i t$. The y position of the ball at any time t can be described by:

$$y_{\text{ball}} = y_i + v_{iy} t + \frac{1}{2} a_y t^2 = R + 0 - \frac{1}{2} g t^2 = R - \frac{g x^2}{2 v_i^2}$$

For the last step, we made use of $t = x/v_i$. Note that this is just the usual trajectory for projectile motion $y(x)$ with the starting height $y_i = R$ added in. In order for the ball not to hit the rock, the parabola above must everywhere lie above the circle describing the rock. In other words, $y_{\text{ball}} > y_{\text{rock}}$ at every x. We can write this as an inequality:

$$y_{\text{ball}}^2 + x_{\text{ball}}^2 > R^2$$

Now plug in what we know ...

$$\left(R - \frac{gx^2}{2v_i^2}\right)^2 + x^2 > R^2$$
$$R^2 - \frac{gx^2R}{v_i^2} + \frac{g^2x^4}{4v_i^4} + x^2 > R^2$$
$$\frac{g^2x^4}{4v_i^4} + x^2 > \frac{gx^2R}{v_i^2}$$
$$\frac{g^2x^2}{4v_i^4} + 1 > \frac{gR}{v_i^2}$$

Now we note that if this inequality is true at $x = 0$, it is true for all $x > 0$ (physically, this means that if the initial trajectory has a high enough radius of curvature, it clears the whole rock). This leads us to:

$$v_i > \sqrt{gR}$$

Note: this is too hard for an exam. If I asked anything like this, I would give you the initial velocity and radius, and ask you if the ball hits the rock or not.

6. First draw a free-body diagram for the hockey puck, let us call the x axis parallel to the ice and the y axis normal to it. In the y direction, we have only the normal force n and the puck's weight $-mg$.

$$\Sigma F_y = n - mg = ma_y = 0 \Rightarrow n = mg$$

In the x direction, we have only kinetic friction.

$$\Sigma F_x = -f_k = -\mu_k n = -\mu_k mg = ma_x \qquad \Rightarrow \qquad a_x = -\mu_k g$$

Given the acceleration, we can readily find the final velocity:

$$v_f^2 = v_i^2 + 2a_x\Delta x = v_i^2 - 2\mu_k g\Delta x = 7^2 - 2\,(0.15)\,(9.8)\,(10) \qquad \Rightarrow \qquad v_f = 4.42\,\mathrm{m/s} \approx 4.4\,\mathrm{m/s}$$

7. This is a problem that really has to be worked in stages. First focus on the block and incline, and draw the free body diagram for those two. Let the x axis point down the incline, and the y

axis point up normal to the incline. In the y direction, we have the normal force, and part of the weight of the block:

$$\Sigma F_y = n - mg\cos\theta = ma_y = 0 \qquad \Rightarrow \qquad n = mg\cos\theta$$

In the x direction, we have the other component of the weight, and opposing that we have friction:

$$\Sigma F_x = mg\sin\theta - f_k = mg\sin\theta - \mu_k n = mg\sin\theta - \mu_k mg\cos\theta = ma_x$$
$$\Rightarrow \qquad a_x = g\left(\sin\theta - \mu_k\cos\theta\right)$$

Given the acceleration a_x, the initial velocity $v_i = 0.1\,\mathrm{m/s}$, and the length of the ramp being $h/\sin\theta$, we can readily calculate the speed at the bottom of the ramp, which we'll call v_f:

$$v_f^2 = v_i^2 + 2a_x\Delta x = v_i^2 + 2g\left(\sin\theta - \mu_k\cos\theta\right)\left(\frac{h}{\sin\theta}\right) \qquad \Rightarrow \qquad v_f \approx 3.79\,\frac{\mathrm{m}}{\mathrm{s}}$$

Next, to find the position R we can just use the usual equations for projectile motion. In this case, the projectile starts off with a velocity of v_f, with $y_i = H$, and an angle of $-\theta$. We can easily write down the equations for $y(t)$ and $x(t)$, noting that $\cos(-\theta) = \cos\theta$ and $\sin(-\theta) = -\sin\theta$:

$$\begin{aligned} x(t) &= (v_f\cos\theta)\,t \\ y(t) &= H - (v_f\sin\theta)\,t - \frac{1}{2}gt^2 \end{aligned}$$

At the point where the block strikes the floor, $y = 0$. Setting $y(t) = 0$ and using the quadratic equation, the block must hit the floor at:

$$t_{\mathrm{hit}} = \frac{1}{g}\left(-v_f\sin\theta \pm \sqrt{v_f^2\sin^2\theta + 2gH}\right)$$

Only the positive root is physical, which gives $t_{\mathrm{hit}} = 0.449\,\mathrm{sec}$. Finally, the quantity R must be

$$x(t_{\mathrm{hit}}) = (v_f\cos\theta)\,t_{\mathrm{hit}} = 1.35 \approx 1.4\,\mathrm{m}$$

Definitely nothing like this on the exam - it is very long and difficult - but it is an example of a impossible-seeming problem that you have all the techniques to solve. If you can do this one, you are ready for the exam. If you can follow the solution easily, you're more than likely ready for the exam.

8. Pick out each mass separately, and draw the free body diagram. For each mass, there is only its weight and the tension in the string, which we'll call T. We note that since a is the same for both masses, this means the string does not go slack and T is the same on both sides of the pulley. First, we write the force balance for m_1, calling up the $+y$ direction.

$$\Sigma F = T - m_1 g = m_1 a$$

For m_1 the acceleration is positive, since it moves m_1 in the $+y$ direction. Now we write the force balance for m_2, noting that the acceleration is *negative* for m_2!

$$\Sigma F = T - m_2 g = -m_2 a$$

Now, subtract the second equation from the first, being very careful with signs:

$$-m_1 g + m_2 g = m_1 a + m_2 a \qquad \Rightarrow \qquad a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$