University of Alabama
Department of Physics and Astronomy

PH 101 LeClair | Summer 2011

Exam 1 Solutions

1. A motorcycle is following a car that is traveling at constant speed on a straight highway. Initially, the car and the motorcycle are both traveling at the same speed of 18 m/s, and the distance between them is 92.0 m. After 2.50 s, the motorcycle starts to accelerate at a rate of 4.00 m/s^2. How long does it take from the moment the motorcycle starts to accelerate until it catches up to the car?

Solution: This was also problem 5b from homework 2 (HW average: 85.9%, exam average 83.7%). Since the car and motorcycle are moving at the same velocity until the moment the motorcycle accelerates, they will maintain a separation of $x_o = 92$ m until the moment the motorcycle accelerates. In this case, the time of 2.50 s is not relevant for solving the problem - it is only telling you when someone started measuring the motorcycle's and car's positions, and as such represents an arbitrary choice of origin. It is *not* a head start, like in previous problems similar to this one, since both vehicles are in motion at the start of the problem.

Let the motorcycle's position at the moment it accelerates be $x = 0$, and let time $t = 0$ be the moment when the motorcycle begins accelerating. We will call the initial velocity of both vehicles v_o, and the motorcycle's acceleration a_m. The motorcycle's position as a function of time is then

$$x_m(t) = v_o t + \frac{1}{2} a t^2 \tag{1}$$

which correctly captures the initial condition $x_m(0) = 0$. The car starts out a distance x_o ahead of the motorcycle, and continues at constant speed v_o, so its position is

$$x_c(t) = x_o + v_o t \tag{2}$$

which correctly captures the starting point $x_c(0) = x_o$. The motorcycle catches the car when their positions are equal:

$$x_c = x_m \tag{3}$$

$$x_o + v_o t = v_o t + \frac{1}{2} a t^2 \tag{4}$$

$$x_o = \frac{1}{2} a t^2 \tag{5}$$

$$t = \sqrt{\frac{2x_o}{a}} \approx 6.78\,\text{s} \tag{6}$$

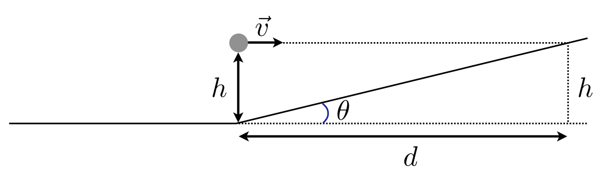
Note that the initial velocity v_o cancels out since both objects start out with the same initial velocity - only the *relative* velocity matters, which is entirely determined by the motorcycle's acceleration. An equivalent question would be how long does it take for the motorcycle to accelerate across the gap of distance x_o?

2. A pilot flies horizontally at 1300 km/h, at height $h=35$ m above initially level ground. However, at time $t=0$, the pilot begins to fly over ground sloping upward at angle $\theta=4.3°$. If the pilot does not change the airplane's heading, at what time t does the plane strike the ground?

Solution: (exam average 83.3%) This is a problem from a different textbook.
Given: The initial velocity and height of a plane flying toward an upward slope of angle θ.

Find: How long before the plane hits the slope? At time $t=0$, the plane is at the beginning of the slope, a height h above level ground. Assuming the plane continues at the same horizontal speed, we wish to find the time at which the plane hits the slope. Given the plane's velocity and height and the slope's angle, we can relate the horizontal distance to intercept the ramp to the plane's height.

Sketch: Assume a spherical plane (it doesn't matter). If the plane is at altitude h, it will hit the ramp after covering a horizontal distance d, where $\tan\theta = h/d$.



Relevant equations: We can relate the horizontal distance to intersect the ramp to the plane's altitude using the known slope of ground:

$$\tan\theta = \frac{h}{d} \tag{7}$$

We can determine how long the horizontal distance d will be covered given the plane's constant horizontal speed v:

$$d = vt \tag{8}$$

Symbolic solution: Combining our equations above, the time t it takes for the plane to hit the slope is

$$t = \frac{d}{v} = \frac{h}{v \tan\theta} \tag{9}$$

Numeric solution: Using the numbers given, and converting units,

$$t = \frac{h}{v \tan\theta} = \frac{35\,\mathrm{m}}{1300\,\mathrm{km/h}\,(1000\,\mathrm{m/km})\,(1\,\mathrm{h}/3600\,\mathrm{s})\,(\tan 4.3°)} \approx 1.3\,\mathrm{s} \tag{10}$$

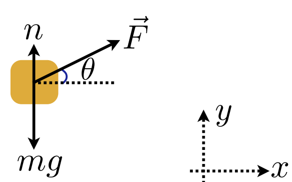
3. A block of mass $m = 5.00\,\mathrm{kg}$ is pulled along a horizontal frictionless floor by a cord that exerts a force of magnitude $F = 12.0\,\mathrm{N}$ at an angle of 65° with respect to horizontal. **(a)** What is the magnitude of the block's acceleration? **(b)** The force magnitude F is slowly increased. What is its value just before the block is lifted off the floor?

Solution: (exam average 51.9%) This is a problem from a different textbook.

Given: A block pulled along a frictionless floor by a force making an angle θ with the horizontal.

Find: The block's acceleration, the maximum force before the block leaves the floor, and the block's acceleration at that point.

Sketch: Let the x and y axes be horizontal and vertical, respectively. We have only the block's weight, the normal force, and the applied force.



Relevant equations: We need only Newton's second law and geometry.

Symbolic solution: Along the vertical direction, a force balance must give zero for the block to remain on the floor. This immediately yields the normal force.

$$\sum F_y = F_n - mg + F\sin\theta = 0 \qquad \Longrightarrow \qquad F_n = mg - F\sin\theta \tag{11}$$

A horizontal force balance gives us the acceleration:

$$\sum F_x = F\cos\theta = ma_x \qquad \Longrightarrow \qquad a_x = \frac{F}{m}\cos\theta \tag{12}$$

If the magnitude of the force is increased, the block will leave the floor as the normal force becomes zero:

$$F_n = mg - F\sin\theta = 0 \qquad \Longrightarrow \qquad F = \frac{mg}{\sin\theta} \tag{13}$$

At that point, its acceleration will be

$$a_x = \frac{F}{m}\cos\theta = \frac{g}{\tan\theta} \tag{14}$$

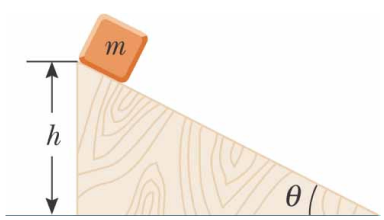
Numeric solution: Using the numbers given, the initial acceleration is

$$a_x = \frac{F}{m}\cos\theta = \frac{12.0\,\mathrm{N}}{5.00\,\mathrm{kg}}\cos 65° \approx 1.01\,\mathrm{m/s^2} \tag{15}$$

At the point the block is about to leave the floor, the required force is

$$F = \frac{mg}{\sin\theta} = \frac{(5.00\,\mathrm{kg})\left(9.81\,\mathrm{m/s^2}\right)}{\sin 65°} \approx 54\,\mathrm{N} \tag{16}$$

4. Consider the figure below. Let $h = 1\,\mathrm{m}$, $\theta = 15°$, and $m = 0.5\,\mathrm{kg}$. There is a coefficient of kinetic friction $\mu_k = 0.2$ between the mass and the inclined plane, and the mass m starts out at the very top of the incline with a velocity of $v_i = 0.1\,\mathrm{m/s}$. What is the speed of the mass at the bottom of the ramp?



Solution: (exam average 84.7%) This is a problem from the PH105 textbook.

First focus on the block and incline, and draw the free body diagram for those two. Let the x axis point down the incline, and the y axis up, normal (perpendicular) to the incline. In the y direction, we have the normal force, and part of the weight of the block:

$$\Sigma F_y = F_n - mg\cos\theta = ma_y = 0 \qquad \Rightarrow \qquad F_n = mg\cos\theta \tag{17}$$

In the x direction, we have the other component of the weight, and opposing that we have friction:

$$\Sigma F_x = mg\sin\theta - f_k = mg\sin\theta - \mu_k F_n = mg\sin\theta - \mu_k mg\cos\theta = ma_x \tag{18}$$

$$\Rightarrow \qquad a_x = g\left(\sin\theta - \mu_k\cos\theta\right) \tag{19}$$

Given the acceleration a_x, the initial velocity $v_i = 0.1\,\mathrm{m/s}$, and the *length* of the ramp being $h/\sin\theta$, we can readily calculate the speed at the bottom of the ramp, which we'll call v_f:

$$v_f^2 = v_i^2 + 2a_x\Delta x = v_i^2 + 2g\left(\sin\theta - \mu_k\cos\theta\right)\left(\frac{h}{\sin\theta}\right) = v_i^2 + 2gh\left(1 - \frac{\mu_k}{\tan\theta}\right) \tag{20}$$

$$v_f = \sqrt{v_i^2 + 2gh\left(1 - \frac{\mu_k}{\tan\theta}\right)} \approx 2.23\,\mathrm{m/s} \tag{21}$$

5. A baseball leaves the bat at $30.0°$ above the horizontal and is caught by an outfielder 375 ft from home plate at the same height from which it left. What is the initial speed of the ball?

Solution: (exam average 89.2%) We know the launch angle and range of the projectile over level ground. For this situation, we've already derived the range equation, so we may as well use it:

$$R = \frac{v_o^2\sin 2\theta}{g} \tag{22}$$

where $\theta = 30.0°$ is the launch angle, v_o the launch velocity, and $R = 375\,\mathrm{ft} \approx 114\,\mathrm{m}$ is the range. Solving for v_o,

$$v_o = \sqrt{\frac{gR}{\sin 2\theta}} \approx 36\,\mathrm{m/s} \approx 118\,\mathrm{ft/s} \approx 81\,\mathrm{mph} \tag{23}$$

6. An advertisement claims that a particular automobile can "stop on a dime." What net force would actually be necessary to stop a 850 kg automobile traveling initially at 45.0 km/h in a distance equal to the diameter of a dime, which is 1.8 cm. *Hint: watch the units!*

Solution: (exam average 76.4%) We know the initial velocity $v_o = 45\,\mathrm{km/h} = 12.5\,\mathrm{m/s}$, the final velocity $v_f = 0$, and the distance over which the car accelerated, $\Delta x = 0.018\,\mathrm{m}$. This is enough to get us the net acceleration, which is then enough to get us the net force using Newton's second law. First the acceleration:

$$v_f^2 = v_o^2 + 2a\Delta x \tag{24}$$

$$a = -\frac{v_o^2}{2\Delta x} \tag{25}$$

The minus sign reminds us that the acceleration is in the direction opposite Δx. If this is the net acceleration, the net force causing it must follow $F = ma$, so the net force is in magnitude (i.e., we don't care about the sign)

$$|F_{\text{net}}| = ma = \frac{mv_o^2}{2\Delta x} \approx 3.7 \times 10^6\,\text{N} \approx 415\,\text{tons} \tag{26}$$

A scary, unsurvivable amount of force, equivalent to pulling about 440 g's (where 50 g's generally means serious injury or death).

One could also solve this with the work-energy theorem: the force F stopping the vehicle through a displacement Δx does work $W = F\Delta x$, and this must be the same as the car's change in kinetic energy, which is $\frac{1}{2}mv_o^2$. Equating and solving for F, we arrive at the same result.