University of Alabama
Department of Physics and Astronomy

PH 101 LeClair Summer 2011

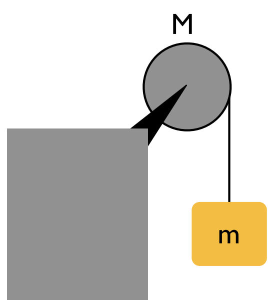
Exam 2

form d

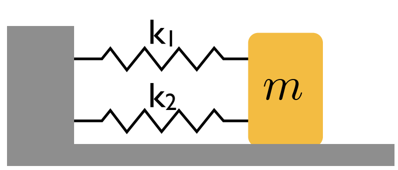
Instructions

1. Solve five of the six problems below.
2. All problems have equal weight. Do your work on separate sheets.
3. You are allowed 1 sheet of standard 8.5×11 in paper and a calculator.

☐ **1.** A uniform disk with mass $M=2.5$ kg and radius $R=20$ cm is mounted on a fixed horizontal axle, as shown below. A block of mass $m=1.2$ kg hangs from a massless cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension in the cord. *Note: the moment of inertia of a disk about its center of mass is* $I=\frac{1}{2}MR^2$.

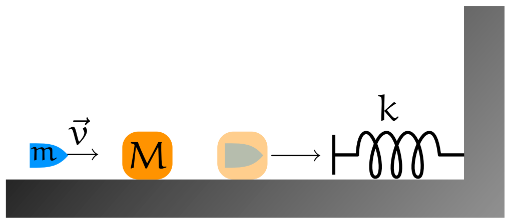


☐ **2.** A mass m is connected to two springs k_1 and k_2 as shown below. If the mass is pushed to the left such that the springs both compress by a distance x from equilibrium and released from rest, what is the velocity of the object as it passes through its original equilibrium position?

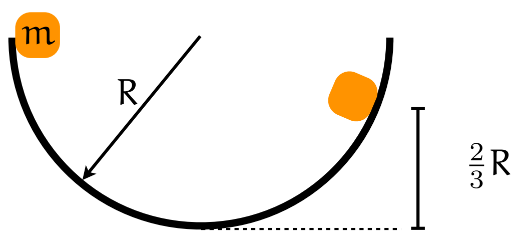


☐ **3.** A uniform cylinder of mass M and radius R rolls without slipping down a ramp at angle θ. The bottom edge of the cylinder starts at a vertical height h from the bottom of the ramp (i.e., such that the total vertical distance traveled by the center of the cylinder is h). What is the linear velocity of the cylinder when it reaches the bottom of the ramp? The moment of inertia for a solid cylinder is $I=\frac{1}{2}MR^2$.

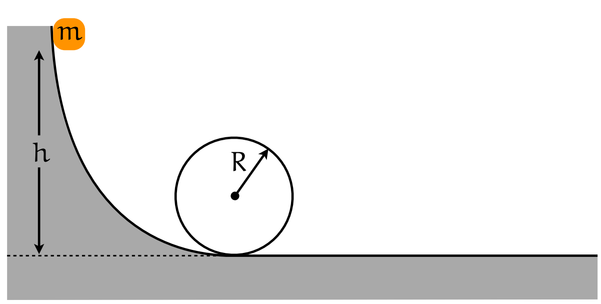
☐ **4.** A bullet of mass m and speed v is shot at a wooden block of mass M. The block is at rest before the shot is fired, and subsequently the bullet embeds itself in the block (i.e., perfectly inelastic collision). Both block and bullet then travel to the right over a frictionless surface and *elastically* strike a spring of force constant k. What is the maximum compression of the spring in terms of m, M, v, and k? *Hint: break the problem up into stages.*



☐ **5.** A particle of mass m is released from rest at the rim of a smooth bowl of radius R as shown below. It slides without friction down the bowl, and up the other side. When the particle is at a height $\frac{2}{3}R$ from the base of the bowl, what is its speed? Assume the particle is a point mass, and has no moment of inertia.



☐ **6.** A *point mass* m moves along the frictionless track shown below. It starts from rest at a height h above the bottom of the loop of radius R, where R is much larger than r. What is the minimum value of h (in terms of R) such that the object completes the loop?

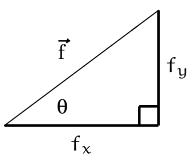


Formula sheet

$$g = 9.81\,\mathrm{m/s^2}$$

$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Generic vector $\vec{f}$:



$$\begin{aligned} f_y &= |\vec{f}| \sin\theta \\ f_x &= |\vec{f}| \cos\theta \\ \tan\theta &= f_y / f_x \\ |\vec{f}| &= \sqrt{f_x^2 + f_y^2} \end{aligned}$$

motion with constant acceleration:

$$\begin{aligned} x_f &= x_i + v_{xi} t + \frac{1}{2} a_x t^2 \\ v_f &= v_i + a_x t \\ v_{xf}^2 &= v_{xi}^2 + 2 a_x \Delta x \end{aligned}$$

Quantity	Unit	equivalent to
force	N	kg·m/s^2
energy	J	kg·m^2/s^2 = N·m
power	W	J/s=m^2·kg/s^3

Projectile motion:

$$\text{Range over level ground} = R = \frac{v_i^2 \sin 2\theta_i}{g}$$

$$\text{max height from origin} = H = \frac{v_i^2 \sin^2\theta_i}{2g}$$

$$y(x) = (\tan\theta_o)\,x - \frac{g x^2}{2 v_o^2 \cos^2\theta_o}$$

Isolated systems: $\vec{p}$, $E = K + PE$, L are all conserved.
Static equilibrium: $\sum F = 0$ and $\sum \tau = 0$ about any axis.
Elastic collision: KE and p are both conserved.
Inelastic collision: only p is conserved, not KE.

Force:

$$\Sigma\vec{F} = \vec{F}_{\text{net}} = m\vec{a} \qquad \Sigma F_x = m a_x \qquad \Sigma F_y = m a_y$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$F_{\text{gravity}} = mg = \text{weight} \qquad F_{\text{spring}} = -k\Delta x$$

$$\text{friction} \begin{cases} f_s & \leqslant \mu_s n \\ f_k & = \mu_k n \end{cases}$$

Work-Energy:

$$W = F_{||}\Delta x = F \Delta x \cos\theta \qquad P = F_{||} v = F v \cos\theta \quad \text{power}$$

$$K = \frac{1}{2} m v^2 = \frac{p^2}{2m}$$

$$\Delta K = K_f - K_i = W = -\Delta PE$$

$$PE_g(y) = mgy \qquad PE_s(x) = \frac{1}{2} k x^2$$

$$K_i + PE_i = K_f + PE_f + W_{\text{ext}}$$

Momentum, etc.:

$$\vec{p} = m\vec{v} \qquad \Delta p = p_f - p_i = F_{\text{avg}} \Delta t \qquad (\Delta p = 0 \text{ for isolated system})$$

$$x_{\text{com}} = \frac{1}{M_{\text{tot}}} \sum_{i=1}^{n} m_i x_i = \frac{m_1 x_1 + m_2 x_2 + \ldots m_n x_n}{m_1 + m_2 + \ldots m_n}$$

$$v_{\text{com}} = \frac{1}{M_{\text{tot}}} \sum_{i=1}^{n} m_i v_i = \frac{m_1 v_1 + m_2 v_2 + \ldots m_n v_n}{m_1 + m_2 + \ldots m_n}$$

$$\sum F_{\text{ext}} = M_{\text{tot}} a_{\text{com}} = \frac{\Delta p}{\Delta t}$$

$$p_{\text{tot}} = M_{\text{tot}} v_{\text{com}}$$

Rotation: we use radians

$$s = \Delta x = \theta r \qquad \omega = \frac{\Delta\theta}{\Delta t} = \frac{v}{r} \qquad \alpha = \frac{\Delta\omega}{\Delta t}$$

$$a_t = \alpha r \quad \text{tangential} \qquad a_r = a_{\text{centr}} = \frac{v^2}{r} = \omega^2 r \quad \text{radial}$$

$$x \to \theta \qquad v \to \omega \qquad a \to \alpha \qquad \text{convert linear to rotation eqns}$$

$$I = \sum_i m_i r_i^2 = k m r^2 \qquad \text{property of a given shape}$$

$$I_z = I_{com} + m d^2 \qquad \text{axis } z \text{ parallel, dist } d$$

$$\tau = r F \sin\theta_{rF} \qquad \sum \tau = I\alpha = \frac{\Delta L}{\Delta t}$$

$$L = r p \sin\theta = m v r \sin\theta = I\omega$$

$$K_{\text{rot}} = \frac{1}{2} I \omega^2 = \frac{L^2}{2I} \qquad K_{\text{tot}} = K_{\text{rot}} + K_{\text{transl}} = \frac{1}{2} M v_{\text{cm}}^2 + \frac{1}{2} I_{\text{cm}} \omega^2$$

$$\Delta K = W = \tau\Delta\theta \qquad P = \frac{\Delta W}{\Delta t} = \tau\omega$$

$$T = \frac{2\pi r}{v} \quad \text{period of motion}$$