UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

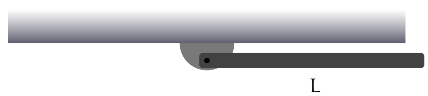
PH 101 LeClair Summer 2011

Exam 2 Practice Problems

1. A solid sphere of mass M and radius R starts from rest at the top of an inclined plane (height h, angle θ), and rolls down without slipping. What is the *linear* velocity of the center of mass at the bottom of the incline? For a solid sphere, $I = \frac{2}{5}MR^2$.

2. A star rotates with a period of 30 days about an axis through its center. After the star undergoes a supernova explosion, the stellar core, which had a radius of 1.0×10^4 km, collapses into a neutron star of radius 3.0 km. Determine the period of rotation of the neutron star. Note that $\omega = \frac{2\pi}{\text{period}}$

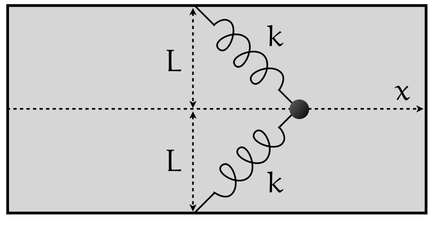
3. A pendulum is made from a rigid rod of length L and mass M hanging from a frictionless pivot point, as shown below. The rod is released from a horizontal position. How does the (tangential) acceleration of the end of the rod at the moment of release compare to g?



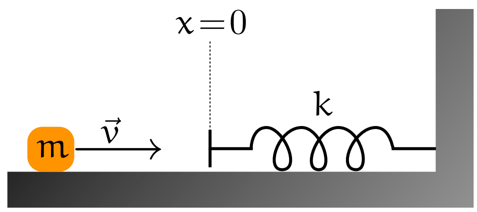
4. A wheel rotates with a constant angular acceleration $\alpha = 3.50\,\text{rad/s}^2$. If the angular speed is $\omega_i = 2.00\,\text{rad/s}$ at time $t_i = 0$, through what angular displacement does the wheel rotate in 2.00 s?

5. A pendulum consists of a sphere of mass m attached to a light cord of length L. The sphere is released from rest at an angle θ_i from the vertical. Find the speed of the mass at its lowest point.

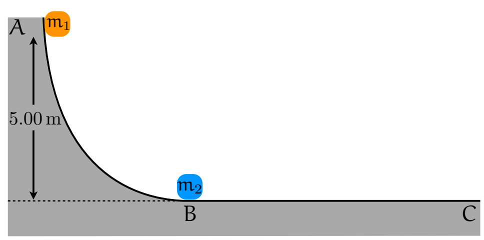
6. Consider the setup below with two springs connected to a mass on a *frictionless* table. Find an expression for the potential energy as a function of the displacement x. The springs are at their equilibrium length L when vertical. (Hint: consider the limiting cases $L \to 0$ and $x \to 0$ to check your solution.)



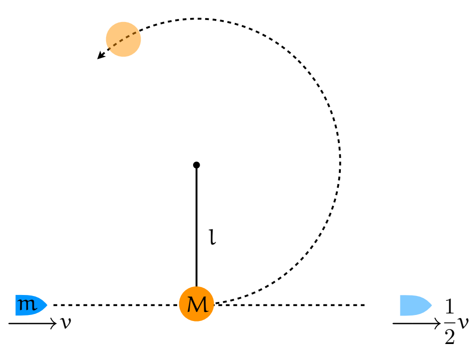
7. A block having a mass $m = 0.80$ kg is given an initial velocity of $v = 1.2$ m/s to the right, and it collides with a spring of negligible mass and force constant $k = 50$ N/m, as shown below. Assuming the surface to be frictionless, what is the maximum compression of the spring after the collision?



8. Two blocks are free to slide along a frictionless wooden track ABC as shown in below. The block of mass $m_1 = 4.92$ kg is released from A. Protruding from its front end is the north pole of a strong magnet, repelling the north pole of an identical magnet embedded in the back end of the block of mass $m_2 = 10.5$ kg, initially at rest. The two blocks never touch. Calculate the maximum height to which m_1 rises after the elastic collision.

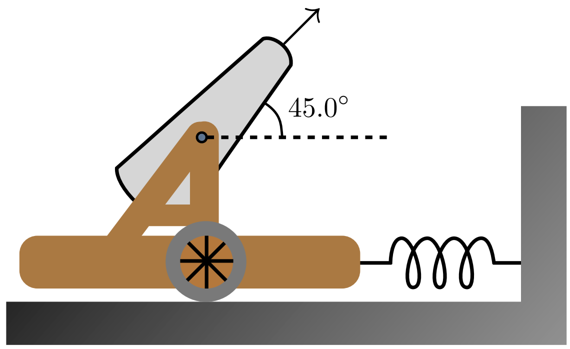


9. As shown below, a bullet of mass m and speed v passes completely through a pendulum bob of mass M. The bullet emerges with a speed of $v/2$. The pendulum bob is suspended by a stiff rod of length l and negligible mass. What is the minimum value of v such that the pendulum bob will barely swing through a complete vertical circle?



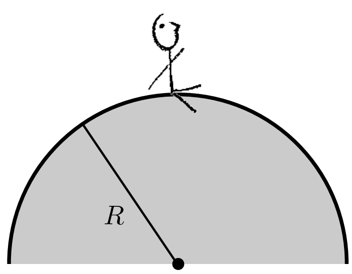
10. A cannon is rigidly attached to a carriage, which can move along horizontal rails but is connected to a post by a large spring, initially un-stretched and with force constant $k = 1.90 \times 10^4$ N/m, as shown below. The cannon fires a 200 kg projectile at a velocity of 125 m/s directed 45.0° above the horizontal.

If the mass of the cannon and its carriage is 4780 kg, find the maximum extension of the spring.

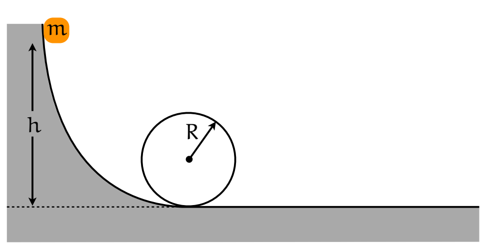


11. Determine the acceleration of the center of mass of a uniform solid disk rolling down an incline making angle θ with the horizontal.

12. A boy is initially seated on the top of a hemispherical ice mound of radius R. He begins to slide down the ice, with a negligible initial speed. Approximate the ice as being frictionless. At what height does the boy loose contact with the ice?



13. A solid sphere of mass m and radius r rolls without slipping along the track shown below. It starts from rest with the lowest point of the sphere at a height h above the bottom of the loop of radius R, much larger than r. What is the minimum value of h (in terms of R) such that the sphere completes the loop? The moment of inertia for a solid sphere is $I = \frac{2}{5}mr^2$.



Solutions

Solution 1: The simplest approach here is probably conservation of energy. At the top of the ramp, we have only potential energy, while at the bottom we have rotational kinetic energy and translational kinetic energy due to the overall motion of the center of mass (cm).

The vertical height of the ramp is h, and we will presume that the sphere starts out at the very top such that its change in center of mass height is h, rather than $h-R$. Conservation of energy gives:

$$Mgh = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\omega^2 \tag{1}$$

For pure rolling motion, the center of mass velocity must be the same as the velocity of the surface of the sphere, $R\omega = v_{cm}$, or $\omega = v_{cm}/R$.

$$Mgh = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\left(\frac{v_{cm}^2}{R^2}\right) = \frac{1}{2}v_{cm}^2\left(M + \frac{I}{R^2}\right) \tag{2}$$

$$\frac{1}{2}v_{cm}^2 = \frac{Mgh}{M + I/R^2} = \frac{gh}{1 + I/MR^2} \tag{3}$$

$$v_{cm}^2 = \frac{2gh}{1 + I/MR^2} \tag{4}$$

$$v_{cm} = \sqrt{\frac{2gh}{1 + I/MR^2}} \tag{5}$$

Using $I = \frac{2}{5}MR^2$, we find $v_{cm} = \sqrt{\frac{10}{7}gh}$, slower than what we would expect for pure sliding motion without friction ($v = \sqrt{2gh}$).

Solution 2: Presumably, the star's mass is essentially constant. We can use the initial period of the star T_i to find its original rate of rotation ω_i. WIth that in hand, conservation of angular momentum gives use the angular velocity after the explosion, and thus the new period.

$$\omega_i = \frac{2\pi}{T_i} \tag{6}$$

Noting that $v = R\omega$, conservation of angular momentum is straightforward:

$$L_i = mv_iR_i = L_f = mv_fR_f \tag{7}$$

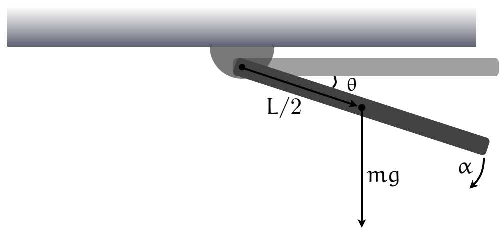
$$m\omega_iR_i^2 = m\omega_fR_f^2 \tag{8}$$

$$\omega_f = \frac{R_i^2}{R_f^2}\omega_i \tag{9}$$

Finally, the post-explosion period T_f:

$$T_f = \frac{2\pi}{\omega_f} = \frac{2\pi R_f^2}{\omega_i R_i^2} = T_i \frac{R_f^2}{R_i^2} \approx 0.23\,\text{s} \tag{10}$$

Solution 3: If we can get the angular acceleration, we can get the linear acceleration at any point, including the end of the rod. We can get that by finding the net torque on the system. First we should make a quick sketch of the situation:



The weight of the rod mg acts downward from its center of mass. Since the center of mass is a distance L/s from the pivot point, this will create a torque. The angle between the force mg and a line from the force to the pivot point is $90°$ when the rod starts to fall, so the magnitude of the torque is just

$$\tau_{\text{weight}} = -mg\left(\frac{L}{2}\right) \tag{11}$$

Here the minus sign indicates that the torque would cause a clockwise rotation, which is by convention negative. Since this is the only torque present, it must be equal to the moment of inertia times the angular acceleration. For a thin rod of mass M and length L, rotated about its end, the moment of inertia is $I = \frac{1}{3}ML^2$. Thus,

$$\sum \tau = I\alpha \tag{12}$$

$$-\frac{1}{2}mgL = \frac{1}{3}mL^2\alpha \tag{13}$$

$$\alpha = -\frac{3g}{2L} \tag{14}$$

The linear acceleration a can be found by noting $a = \alpha R$, where R is the distance from the point of rotation to the point of interest, in this case just L:

$$a = -\alpha L = -\frac{3}{2}g \tag{15}$$

The acceleration is half again as large as that due to gravity, meaning the end of the rod falls *faster* than if the rod were simply dropped! In fact, one can see that $a = g$ when $R = \frac{2}{3}L$, so points of the rod further than that from the pivot point fall faster than dropped objects.

Solution 4: We only need the equation for angle as a function of time under constant angular acceleration:

$$\theta(t) = \theta_o + \omega_i t + \frac{1}{2}\alpha t^2 \tag{16}$$

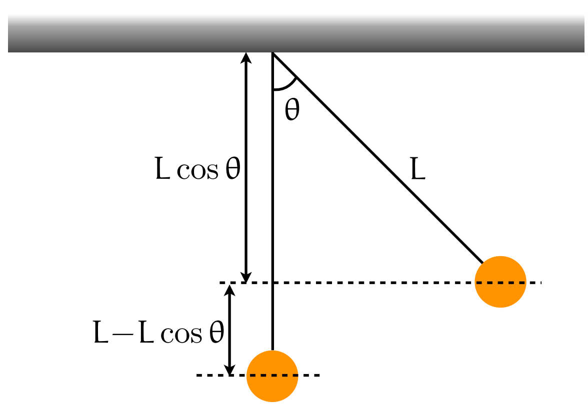
In this case, we only want the angular displacement $\Delta\theta(t) = \theta(t) - \theta_o$:

$$\Delta\theta(t) = \omega_i t + \frac{1}{2}\alpha t^2 \tag{17}$$

We are given $\omega_i = 2.00\,\text{rad/s}$ and $\alpha = 3.5\,\text{rad/s}^2$, so finding the angular displacement after $2.00\,\text{s}$ is just a matter of plugging in the numbers:

$$\Delta\theta(2\,\text{s}) = \left(2.00\,\frac{\text{rad}}{\text{s}}\right)(2\,\text{s}) + \frac{1}{2}\left(3.5\,\frac{\text{rad}}{\text{s}^2}\right)(2\,\text{s})^2 = 11.0\,\text{rad} \approx 1.75\,\text{rev} \tag{18}$$

Solution 5: This is a standard conservation of energy problem. Our initial state is the pendulum bob at an angle θ_i, the final state is that where the bob is hanging straight up and down.



The initial energy is purely gravitational potential energy. Geometry gives us the change in height after inclining the bob by θ_i. For convenience we choose the state of zero potential energy to be the final state, with the bob completely vertical. The final energy is purely kinetic. Setting the two equal and solving for v gives us our answer.

$$\begin{aligned}
K_i + U_i &= K_f + U_f \\
0 + mg(L - L\cos\theta_i) &= \frac{1}{2}mv^2 + 0 \\
\cancel{m}g(L - L\cos\theta_i) &= \frac{1}{2}\cancel{m}v^2 \\
2gL(1 - \cos\theta_i) &= v^2 \\
v &= \sqrt{2gL(1 - \cos\theta_i)}
\end{aligned}$$

Solution 6: Since potential energy is a scalar, we can just add the potential energies for each spring together. Since the two springs are equivalent, we can just figure out the potential energy of one of them and double it.

When $x = 0$, both springs have a length L. As soon as we pull on the mass and move it to some $x \neq 0$, we can find the new length of the spring from simple geometry as $\sqrt{x^2 + L^2}$. The Δx is the difference between these two lengths. Now we can easily write down $PE(x)$:

$$\begin{aligned}
PE(x) &= PE_{spring1} + PE_{spring2} \\
&= 2PE_{spring1} = 2\left(\frac{1}{2}k\,(\Delta x)^2\right) = k\,(\Delta x)^2 \\
&= k\left(\sqrt{x^2 + L^2} - L\right)^2 = kL^2 + kx^2 - 2kl\sqrt{x^2 + L^2} \\
&= kx^2 + 2kL\left(L - \sqrt{L^2 + x^2}\right)
\end{aligned}$$

Solution 7: Conservation of energy again. The initial kinetic energy of the block is converted into potential energy stored in the spring.

$$
\begin{aligned}
K_i + U_i &= K_f + U_f \\
\frac{1}{2} m v_A^2 + 0 &= 0 + \frac{1}{2} k (\Delta x)^2 \\
\cancel{\frac{1}{2}} m v_A^2 &= \cancel{\frac{1}{2}} k (\Delta x)^2 \\
(\Delta x)^2 &= \frac{m}{k} v_A^2 \\
\Delta x &= \sqrt{\frac{m}{k}} v_A = \sqrt{\frac{0.8\,\text{kg}}{50\,\text{N/m}}}\,(1.2\,\text{m/s}) \approx 0.15\,\text{m}
\end{aligned}
$$

Solution 8: This one has three basic steps. First, use conservation of energy to find the velocity of the first block just before the collision. Call the initial height $h = 5\,\text{m}$, and let the zero for potential energy be the vertical position of m_2:

$$
\begin{aligned}
K_i + U_i &= K_f + U_f \\
0 + mgh &= \frac{1}{2} m v_{1i}^2 \\
\cancel{m} gh &= \frac{1}{2} \cancel{m} v_{1i}^2
\end{aligned}
$$

$$
v_{1i} = \sqrt{2gh}
$$

Next, we handle the collision itself. It is an elastic collision, so both kinetic energy and momentum are conserved. We have already derived the equation for v_{1f} for a 1D elastic collision with one of the objects at rest, and only state the result below.

$$
v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) \sqrt{2gh}
$$

The last step is to use conservation of energy again to find the new height, which we'll call h'.

$$
\begin{aligned}
\frac{1}{2}\cancel{m_1}v_{1f}^2 &= \cancel{m_1}gh' \\
h' &= \frac{v_{1f}^2}{2g} = h\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 \\
h' &\approx 0.131h \approx 0.655\,\mathrm{m}
\end{aligned}
$$

Solution 9: This one is very much like the ballistic pendulum problem. The initial collision is inelastic, so we can't use conservation of energy, but we *can* use conservation of momentum. This is as follows:

$$
\begin{aligned}
\vec{p}_i &= \vec{p}_f \\
mv &= m\left(\frac{v}{2}\right) + Mv_{\mathrm{bob}} \\
v &= \left(\frac{v}{2}\right) + \frac{M}{m}v_{\mathrm{bob}}
\end{aligned}
$$

Now we have the velocity of the bob after the collision. If the bob is going to make it all the way to the top of the arc, then its initial kinetic energy must at least match the gain in gravitational potential energy due to the change in height. This gives us a value for v_{bob}.

$$
\begin{aligned}
K_i + U_i &= K_f + U_f \\
\frac{1}{2}mv_{\mathrm{bob}}^2 + 0 &= 0 + mg\Delta y = mg(2l) \\
\cancel{m}g(2l) &= \frac{1}{2}\cancel{m}v_{\mathrm{bob}}^2 \\
v_{\mathrm{bob}} &= \sqrt{4gl} = 2\sqrt{gl}
\end{aligned}
$$

Now plug that value for v_{bob} into the first equation we got from momentum conservation, and we're done.

$$
\begin{aligned}
v &= \left(\frac{v}{2}\right) + \frac{M}{m} v_{\text{bob}} \\
v &= \left(\frac{v}{2}\right) + \frac{M}{m}\left(2\sqrt{gl}\right) \\
\frac{v}{2} &= 2\left(\frac{M}{m}\right)\sqrt{gl} \\
v &= \left(\frac{4M}{m}\right)\sqrt{gl}
\end{aligned}
$$

Solution 10: First, we want to find the recoil velocity of the cannon, from which we can use conservation of energy to get the maximal extension of the spring.

We can get the recoil velocity from conservation of momentum, but we have to be careful. The projectile's momentum has both x and y components, but the cannon will only move in the $-x$ direction. We have to write down conservation of momentum my components. In this case we only need the x components.

$$
\begin{aligned}
\vec{p}_i &= \vec{p}_f \\
p_{xi} &= p_{xf} \\
0 &= v_{\text{proj}} m_{\text{proj}} \cos 45^\circ + m_{\text{cannon}} v_{\text{cannon}} \\
v_{\text{cannon}} &= -\left(\frac{m_{\text{proj}}}{m_{\text{cannon}}}\right) v_{\text{proj}} \cos 45^\circ
\end{aligned}
$$

Now that we have the recoil velocity of the cannon, we can use conservation of energy to relate the cannon's kinetic energy to the energy stored in the spring.

$$
\begin{aligned}
\frac{1}{2}m_{\text{cannon}}v_{\text{cannon}}^2 &= \frac{1}{2}k\,(\Delta x)^2 \\
\Delta x &= \sqrt{\frac{m_{\text{cannon}}}{k}}v_{\text{cannon}} \\
\Delta x &= \sqrt{\frac{m_{\text{cannon}}}{k}}\left(\frac{m_{\text{proj}}}{m_{\text{cannon}}}\right)v_{\text{proj}}\cos 45^\circ \\
\Delta x &= \left[\frac{m_{\text{proj}}}{\sqrt{k m_{\text{cannon}}}}\right]v_{\text{proj}}\cos 45^\circ \\
\Delta x &= \left[\frac{200\,\text{kg}}{\sqrt{(1.90\times 10^4\,\text{N/m})\,(4780\,\text{kg})}}\right](125\,\text{m/s})\left(\frac{\sqrt{2}}{2}\right) \approx 1.85\,\text{m}
\end{aligned}
$$

Solution 11: There are two different ways to approach this one: using torque, and using energy conservation. First, we'll use energy conservation.

Using energy conservation to get the acceleration requires some insight. First, since gravity is the only force of interest here, the acceleration should be a constant. Second, if this is true, then we can use the velocity of the disc at any two distinct points on the ramp to get acceleration, *via*, $v_f^2 = v_i^2 + 2a\Delta x$. If the acceleration is constant, we can choose v_f and v_i to be any points on the ramp we want. So let us choose v_f as the speed at the bottom of the ramp, which we will calculate, and $v_i = 0$ at the top of the ramp.

Releasing the disc from rest at the top of the ramp, whose height we'll call h, we can write conservation of energy in terms of the initial potential energy, and the final rotational plus linear kinetic energies:

$$
\begin{aligned}
mgh &= \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 \\
mgh &= \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\frac{v_{cm}^2}{R^2} \\
mgh &= \frac{1}{2}v_{cm}^2\left(m + \frac{I_{cm}}{R^2}\right) = \frac{1}{2}mv_{cm}^2\left(1 + \frac{I_{cm}}{mR^2}\right) \\
2gh &= v_{cm}^2\left(1 + \frac{I_{cm}}{mR^2}\right) \\
v_{cm}^2 &= \frac{2gh}{1 + I_{cm}/mR^2}
\end{aligned}
$$

Now we can relate the linear velocity at the bottom of the ramp v_{cm} to the acceleration, noting that the length of the ramp is $h/\sin\theta$

$$\begin{aligned}
v_{cm}^2 &= 2a\Delta x \\
\frac{2gh}{1 + I_{cm}/mR^2} &= 2a\left(\frac{h}{\sin\theta}\right) \\
a &= \frac{g\sin\theta}{1 + I_{cm}/mR^2}
\end{aligned}$$

For a disc, $I = \frac{1}{2}mR^2$, leading us to

$$a = \frac{2}{3}g\sin\theta$$

The other way to do this problem is to realize that the weight of the disc mg is actually a force applied at an angle to the incline, and represents a torque. The torque is the weight mg times the line of action, $R\sin\theta$. Equivalently, we could say it is the component of the disc's weight $mg\sin\theta$ along the ramp times the radial distance R. In either case:

$$\begin{aligned}
\Sigma\tau = I\alpha &= I\frac{a}{R} \\
\tau_{\text{weight}} = mgR\sin\theta &= I\frac{a}{R}
\end{aligned}$$

What is I in this case? Solving the problem with conservation of energy we used I_{cm}, but *only* because we had previously derived the equation for total kinetic energy of a rolling object in terms of I_{cm}. In the present case, we have to find I explicitly using the parallel axis theorem. We are rotating the disc about a point on its surface a distance R away, so this means:

$$I = I_{cm} + mR^2 = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2$$

For the last step, we have used $I_{\text{disc}} = \frac{1}{2}mR^2$. Now we can put this into our torque equation above:

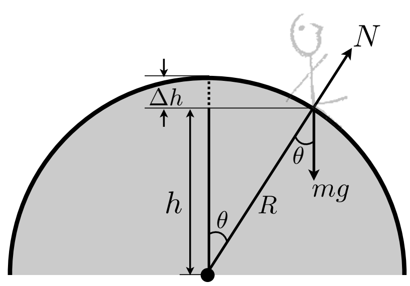
$$
\begin{aligned}
\tau_{\text{weight}} &= mgR\sin\theta = I\frac{a}{R} \\
\cancel{m}g\cancel{R}\sin\theta &= \frac{3}{2}\cancel{m}\cancel{R^2}\frac{a}{\cancel{R}} \\
a &= \frac{2}{3}g\sin\theta
\end{aligned}
$$

Personally, I find the energy approach more intuitive, but one has to be careful. It would not have worked if the acceleration was not constant, for instance.

Solution 12: **Find:** The point at which the boy will loose contact with the hemisphere. If we know the point on the sphere, we can easily find the height. Since the radius of the hemisphere is constant, we need only the boy's angular position. The point at which the boy will leave the sphere will be the point at which the normal force is zero – i.e., the point at which there is no longer a force constraining him to stay on the surface.

Given: The radius of the sphere and the fact that it is frictionless.

Sketch: The only thing we need to add to the given picture is a free-body diagram and a coordinate system. Clearly, (r, θ) polar coordinates with an origin at the center of the hemisphere will be convenient. Let $\theta = 0$, $r = R$ define the boy's initial position.



Once the boy is at a given angle θ, his height above the ground will be $h = R\cos\theta$, meaning he has moved downward from his starting position by an amount $\Delta h = R - h$. The boy's weight mg will be acting downward making the same angle θ with respect to a radial line at that point, while the normal force N will act in the positive radial direction.

Relevant equations: We need only energy conservation and centripetal force combined with Newton's second law. First, the force balance. Along the hemisphere's radial direction $(\hat{r})$, the net force must result in the centripetal force:

$$\sum F_r = \frac{-mv^2}{R}$$

We do not need the force balance along the angular ($\hat{\theta}$) direction. Additionally, we will need conservation of energy to find the velocity at any point. Since only conservative forces are present,

$$K_i + U_i = K_f + U_f$$

Symbolic solution: We first apply conservation of mechanical energy to find the boy's velocity when he has slid down the hemisphere through an angle θ. We choose the ground level to be our zero for gravitational potential energy. Between the boy's starting point (i) and a later position θ (f),

$$\begin{aligned}
K_i + U_i &= K_f + U_f \\
0 + mgR &= \frac{1}{2}mv^2 + mgR\cos\theta \\
\frac{v^2}{2} &= gR\left(1 - \cos\theta\right) \\
v^2 &= 2gR\left(1 - \cos\theta\right)
\end{aligned}$$

Since the centripetal force expression we need also involves v^2, solving for v^2 should be sufficient. Next, we need to apply a radial force balance to find the normal force on the boy. When the normal force vanishes, the boy will leave the sphere. Using our sketch, we see that the radial component of the boy's weight is $mg\cos\theta$. The only other radial force is the normal force:

$$\begin{aligned}
\sum F_r = \frac{-mv^2}{R} &= N - mg\cos\theta \\
N &= mg\cos\theta - \frac{mv^2}{R} = mg\cos\theta - 2mg\left(1 - \cos\theta\right) = mg\left(3\cos\theta - 2\right) = 0 \\
\Longrightarrow \quad \cos\theta &= \frac{2}{3}
\end{aligned}$$

Thus, the boy leaves the hemisphere at $\theta = \cos^{-1}\left(\frac{2}{3}\right) \approx 48.2°$ His height above the ground at this point will be

$$h = R\cos\theta = \frac{2R}{3}$$

Numeric solution: Since we are not given a specific radius of the rock, this is as good as it gets.

Solution 13: Conservation of energy will allow us to find the velocity at the top of the loop, and we can find the minimum velocity required to stay on the track by considering the forces at the top of the loop.

Comparing the two will give us an expression for h in terms of R. First, conservation of energy.

Let the ground level be our zero point for potential energy. Before the mass is released, it has only potential energy based on its height h above the ground. At the top of the loop, it still has potential energy due to its height $2R$ above the ground, but now also has linear kinetic energy due to the motion of its center of mass at speed v_{cm} and rotational kinetic energy due to its rotating at angular velocity ω.

$$mgh = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2 + mg\left(2r\right) \tag{19}$$

$$mg\left(h - 2r\right) = \frac{1}{2}m\left(v_{cm}^2 + I\omega^2\right) \tag{20}$$

We can relate v_{cm} and ω by noting that the horizontal distance the sphere covers in rolling through θ radians is the arclength of the circle through the same angle, $r\theta$, and the angle θ at constant angular velocity is ωt. Since the horizontal distance covered is also $v_{cm}t$, we have $r\omega t = v_{cm}t$, or $\omega = v_{cm}/r$.

$$mg\left(h - 2r\right) = \frac{1}{2}m\left(v_{cm}^2 + I\frac{v^2}{r^2}\right) \tag{21}$$

$$2g\left(h - 2r\right) = v^2\left(1 + \frac{I}{mr^2}\right) \tag{22}$$

$$v^2 = \frac{2g\left(h - 2r\right)}{1 + I/mr^2} \tag{23}$$

This is the actual speed the sphere will have at the top of the loop. We must compare this to the minimum speed required by centripetal acceleration. At the top of the loop, the only two forces will be the sphere's weight mg and the normal force F_n, both pointing downward toward the center of the circle. These two forces must equal the centripetal force required to stay on the track, which also acts downward toward the center of the circle:

$$\sum F = mg + F_n = \frac{mv^2}{R} \tag{24}$$

$$F_n = \frac{mv^2}{R} - mg \tag{25}$$

The sphere will stay on the track so long as the normal force is positive, i.e., when

$$v^2 > Rg \tag{26}$$

The actual speed of the sphere must be larger than this. Using the speed we found, we can solve for h to find the minimum requisite height.

$$v^2 = \frac{2g\left(h - 2r\right)}{1 + I/mr^2} > Rg \tag{27}$$

$$2g\left(h - 2r\right) > Rg\left(1 + \frac{I}{mr^2}\right) \tag{28}$$

$$h - 2R > \frac{R}{2}\left(1 + \frac{I}{mr^2}\right) \tag{29}$$

$$h > R\left(2 + \frac{1}{2} + \frac{I}{2mr^2}\right) \tag{30}$$

Noting that $I = \frac{2}{5}mr^2$, $I/2mr^2 = \frac{1}{5}$, so

$$h > R\left(2 + \frac{1}{2} + \frac{1}{5}\right) = \frac{27}{10}R = 2.7R \tag{31}$$