PH 102 Exam I Solution

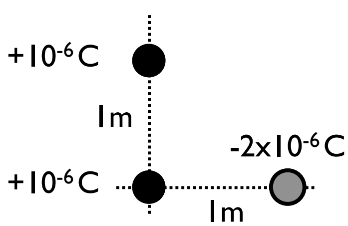
1. Two isolated identical conducting spheres have a charge of q and $-3q$, respectively. They are connected by a conducting wire, and after equilibrium is reached, the wire is removed (such that both spheres are again isolated). What is the charge on each sphere?

- ○ q, $-3q$
- ⊗ $-q$, $-q$
- ○ 0, $-2q$
- ○ $2q$, $-2q$

Percent answering correctly: 54.
Source: A variation on homework 2, problem 3.
Average score on HW2, #3: 78.1

The thing to remember is that any charge on a conductor spreads out evenly over its surface. When we have the conducting spheres isolated, they have q and $-3q$ respectively, and this charge is spread evenly over each sphere. When we connect them with a conducting wire, suddenly charges are free to move from one conductor, across the wire, into the other conductor. Its just the same as if we had one big conductor, and all the *total net charge of the two conductors combined* will spread out evenly over *both* spheres and the wire.

If the charge from each sphere is allowed to spread out evenly over both spheres, then the $-3q$ and $+q$ will both be spread out evenly everywhere. The $+q$ will cancel part of the $-3q$, leaving a total net charge of $-2q$ spread over evenly over both spheres, or $-q$ on each sphere. Once we disconnect the two spheres again, the charge remains equally distributed between the two.



2. Two charges of $+10^{-6}$ C are separated by 1 m along the vertical axis. What is the net **horizontal** force on a charge of -2×10^{-6} C placed one meter to the right of the lower charge?

- ○ 0.018 N
- ○ −0.031 N
- ⊗ −0.024 N
- ○ −0.051 N

Percent answering correctly: 33.
Similar to: homework 2, problem 5.
Average score on HW2, #5: 53.1

We are only interested in the x component of the force, which makes things easier. First, we are trying to find the force on a negative charge due to two positive charges. Both positive charges are to the left of the negative charge, and both forces will be attractive. We will adopt the usual convention that the positive horizontal direction is to the right and called $+x$, and the negative horizontal direction is to the left and called $-x$.

First, we will find the force on the negative charge due to the positive charge in the lower left, which we will call "1" to keep things straight. We will call the negative charge "2." This is easy, since the force is purely in the $-x$ direction:

$$
\begin{aligned}
F_{x,1} &= k_e \frac{q_1 q_2}{r_{12}^2} \\
&= \left(9\times10^9\,\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2\right)\frac{\left(10^{-6}\,\mathrm{C}\right)\cdot\left(-2\times10^{-6}\,\mathrm{C}\right)}{\left(1\,\mathrm{m}\right)^2} \\
&= \left(9\times10^9\,\mathrm{N}\cdot\cancel{\mathrm{m}^2}/\cancel{\mathrm{C}^2}\right)\frac{-2\times10^{-12}\,\cancel{\mathrm{C}^2}}{1\,\cancel{\mathrm{m}^2}} \\
&= -18\times10^{-3}
\end{aligned}
$$

So far so good, but now we have to include the force from the upper left-hand positive charge, which we'll call "3." We calculate the force in exactly the same way, with two little difference: the separation distance is slightly larger, and now the force has both a horizontal and vertical component. First, let's calculate the magnitude of the net force, we'll find the horizontal component after that.

Plane geometry tells us that the separation between charges 3 and 2 has to be $\sqrt{2}\cdot 1\,\mathrm{m}$, or $\sqrt{2}\,\mathrm{m}$ – connecting the charges with straight lines forms a 1-1-$\sqrt{2}$ right triangle, with 45° angles.

$$
\begin{aligned}
F_{net,3} &= k_e \frac{q_2 q_3}{r_{23}^2} \\
&= \left(9\times10^9\,\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2\right)\frac{\left(10^{-6}\,\mathrm{C}\right)\cdot\left(-2\times10^{-6}\,\mathrm{C}\right)}{\left(\sqrt{2}\,\mathrm{m}\right)^2} \\
&= \left(9\times10^9\,\mathrm{N}\cdot\cancel{\mathrm{m}^2}/\cancel{\mathrm{C}^2}\right)\frac{-2\times10^{-12}\,\cancel{\mathrm{C}^2}}{2\,\cancel{\mathrm{m}^2}} \\
&= -9\times10^{-3}\,\mathrm{N}
\end{aligned}
$$

So the *net* force from the upper left charge is just half as much, since it is a factor $\sqrt{2}$ farther away. We only want the horizontal component though! Since we are dealing with a 45-45-90 triangle here, the horizontal component is just the net force times $\cos 45°$:

$$
\begin{aligned}
F_{x,3} &= F_{net,3}\cos 45° \\
&= -9\times10^{-3}\cdot\frac{\sqrt{2}}{2}\,\mathrm{N} = -9\times10^{-3}\cdot 0.707\,\mathrm{N} \\
&\approx -6.4\times10^{-3}\,\mathrm{N}
\end{aligned}
$$

The total horizontal force is just the sum of the horizontal forces from the two positive charges:

$$
\begin{aligned}
F_{x,\mathrm{total}} &= F_{x,1} + F_{x,3} \\
&= \left(-18\times10^{-3}\right) + \left(-6.4\times10^{-3}\right)\,\mathrm{N} \\
&= -24.4\times10^{-3}\,\mathrm{N} = -0.0244\,\mathrm{N}
\end{aligned}
$$

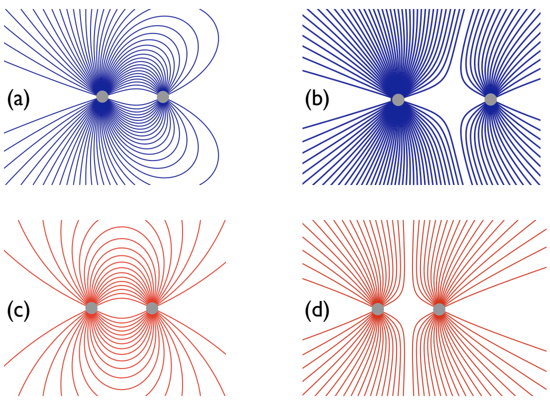
The multiple choice answers have only 2 significant digits, but clearly the answer is $-0.024\,\mathrm{N}$.

3. Which of the following is true for the electric force and **not true** for the gravitational force?

⊗ The force can be both attractive and repulsive between two particles.
◯ The force obeys the superposition principle.
◯ The force between two particles is inversely proportional to their separation distance squared.
◯ The force is conservative.

Percent answering correctly: 98%
Similar to: Quiz 2, #4.
Percent answering correctly on Quiz 2, #4: 86%

I think this one should be clear ... but just in case: both gravity and the electric force obey superposition, both are inverse square laws, and both are conservative. The gravitational force between two bodies cannot be repulsive, however, it is always attractive.



4. Which set of electric field lines could represent the electric field near two charges of the *same sign*, but *different magnitudes*?

◯ a
⊗ b
◯ c
◯ d

Percent answering correctly: 88%
Similar to: figures taken directly from notes.

This one is probably easiest to do by elimination and figure out which ones are clearly *not* correct.

If the charges are of the *opposite* sign, then the field lines would have to run from one charge directly to the other. Field lines start on a positive charge and end on a negative one, and there should be many lines which run from one charge to the other. Since opposite charges attract, the field between them is extremely strong, the lines should be densest right between the charges. This is the case in (a) and (b), so they are not the right ones.

By the same token, for charges of the *same* sign, the force is repulsive, and the electric field midway between them cancels. The field lines should "push away" from each other, and no field line from a given charge should reach the other charge – field lines cannot start and end on the same sign charge. This means that only (b) and (d) could possibly correspond to two charges of the same sign.

Next, the field lines leaving or entering a charge has to be proportional to the magnitude of the charge. In (d) there are the same number of lines entering and leaving each charge, so the charges are of the same magnitude. One can also see this from the fact that the lines are symmetric about a vertical line drawn midway between the charges. In (b) there are clearly many more lines near the left-most charge.

Or, right off the bat, you could notice that only (a) and (b) are asymmetric, and only (b) and (d) look like two like charges. No sense in over-thinking this one.

5. A single point charge $+q$ is placed exactly at the center of a hollow conducting sphere of radius R. Before placing the point charge, the conducting sphere had zero net charge. What is the magnitude of the electric field *outside* the conducting sphere at a distance r from the center of the conducting sphere? *I.e., the electric field for* $r > R$.

- ◯ $|\vec{\mathbf{E}}| = -\frac{k_e q}{r^2}$
- ◯ $|\vec{\mathbf{E}}| = \frac{k_e q}{(R+r)^2}$
- ◯ $|\vec{\mathbf{E}}| = \frac{k_e q}{R^2}$
- ⊗ $|\vec{\mathbf{E}}| = \frac{k_e q}{r^2}$

Percent answering correctly: 85%
Similar to: Homework 2, #10.
Average score on HW2, #10: 89.6%

The easiest way out of this one is Gauss' law. First, Gauss' law told us that any spherically symmetric charge distribution behaves as a point charge. Second, Gauss' law tells us that the electric flux out of some surface depends only on the enclosed charge. If we draw a spherical surface of radius r and area A around the shell and point charge, centered on the center of the conducting sphere, Gauss' law gives:

$$\begin{aligned}
\Phi_E &= \frac{q_{encl}}{\epsilon_0} = 4\pi k_e q_{encl} \\
EA &= 4\pi k_e q_{encl} \\
E &= \frac{4\pi k_e q_{encl}}{A}
\end{aligned}$$

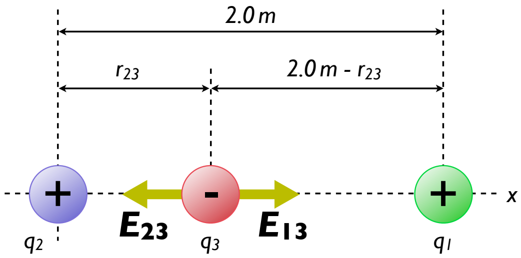
The surface area of a sphere is $A = 4\pi r^2$. In this case, the enclosed charge is just q, since the hollow conducting sphere itself has no charge of its own. Gauss' law only cares about the *total net charge* inside the surface of interest. This gives us:

$$E = \frac{4\pi k_e q}{4\pi r^2} = \frac{\cancel{4\pi} k_e q}{\cancel{4\pi} r^2} = \frac{k_e q}{r^2}$$

There we have it, it is just the field of a point charge q at a distance r.

If we want to get formal, we should point out that the point charge q induces a negative charge $-q$ on the inner surface of the hollow conducting sphere. Since the sphere is overall neutral, the outer surface must therefore have a net positive charge $+q$ on it. This makes no difference in the result – the total *enclosed* charge, for radii larger than that of the hollow conducting sphere ($r > R$), is still just q. If we start with an uncharged conducting sphere, and keep it physically isolated, any induced charges have to cancel each other over all.

If this is still a bit confusing, go back and think about induction charging again. A charged rod was used to induce a positive charge on one side of a conductor, and a negative charge on the other. *Overall*, the 'induced charge' was just a rearrangement of existing charges, so if the conductor started out neutral, no amount of 'inducing' will change that. We only ended up with a *net charge* on the conductor when we used a ground connection to 'drain away' some of the induced charges. Or, if you like, when we used a charged rod to repel some of the conductor's charges through the ground connection, leaving it with a net imbalance.



6. Three point charges lie along the x axis, as shown at left. A positive charge $q_1 = 15\,\mu\text{C}$ is at $x = 2\,\text{m}$, and a positive charge of $q_2 = 6\,\mu\text{C}$ is at the origin. Where must a *negative* charge q_3 be placed on the x-axis **between the two positive charges** such that the resulting electric force on it is zero?

- ⊗ $x = +0.77\,\text{m}$
- ○ $x = -3.44\,\text{m}$
- ○ $x = +1.34\,\text{m}$
- ○ $x = -1.44\,\text{m}$

Percent answering correctly: 94%
Similar to: Homework 2, #7; practice exam.
Average score on HW2, #7: 81.2%

We have one negative charge (q_3) sitting between two positive charges (q_2 and q_1). The force from each positive charge will act in the opposite direction, and we want to find the position r_{23} such that both forces are equal in magnitude. All charges are on the x axis, so the problem is one-dimensional and does not require vectors.

Let F_{32} be the force on q_3 due to q_2, and F_{31} be the force on q_3 due to q_1, and we will take the positive x direction to be to the right. Since both forces are repulsive, F_{32} acts in the $-x$ direction *and must therefore be negative*, while F_{31} acts in the $+x$ direction and is positive. We are not told about any other forces acting, so our force balance is this:

$$-F_{32} + F_{31} = 0 \qquad \Longrightarrow \qquad F_{32} = F_{31}$$

It didn't really matter which one we called negative and which one we called positive, just that they have different signs. The separation between q_2 and q_3 is r_{23}, and the separation between q_1 and q_3 is then $2 - r_{23}$. Now we just need to down the electric forces. We will keep everything perfectly general, and plug in actual numbers at the end ... this is always safer.

$$\begin{aligned}
F_{32} &= F_{31} \\
\frac{k_e q_3 q_2}{r_{23}^2} &= \frac{k_3 q_3 q_2}{(2 - r_{23})^2} \\
\frac{\cancel{k_e}\cancel{q_3} q_2}{r_{23}^2} &= \frac{\cancel{k_3}\cancel{q_3} q_1}{(2 - r_{23})^2} \\
\frac{q_2}{r_{23}^2} &= \frac{q_1}{(2 - r_{23})^2}
\end{aligned}$$

Note how this doesn't depend at all on the actual magnitude *or sign* of the charge in the middle! From here, there are two ways to proceed. We could cross-multiply, use the quadratic formula, and that would be that. On the other hand, since we know that q_3 is supposed to be between the other two charges, then r_{23} must be positive, and less than 2. That means that we can just take the square root of both sides of the equation above without problem, since neither side would be negative afterward.[i] Using this approach first:

$$\begin{aligned} \frac{q_2}{r_{23}^2} &= \frac{q_1}{(2-r_{23})^2} \\ \Longrightarrow \frac{\sqrt{q_2}}{r_{23}} &= \frac{\sqrt{q_1}}{2-r_{23}} \end{aligned}$$

Now we can cross-multiply, and solve the resulting linear equation:

$$\begin{aligned} \sqrt{q_2}\,(2-r_{23}) &= \sqrt{q_1}r_{23} \\ 2\sqrt{q_2}-\sqrt{q_2}r_{23} &= \sqrt{q_1}r_{23} \\ 2\sqrt{q_2} &= \left(\sqrt{q_2}+\sqrt{q_1}\right)r_{23} \\ r_{23} &= \frac{2\sqrt{q_2}}{\sqrt{q_2}+\sqrt{q_1}} \end{aligned}$$

Plugging in the numbers we were given (and noting that all the units cancel):

$$r_{23} = \frac{2\sqrt{q_2}}{\sqrt{q_2}+\sqrt{q_1}} = \frac{2\sqrt{6\,\mu\mathrm{C}}}{\sqrt{6\,\mu\mathrm{C}}+\sqrt{15\,\mu\mathrm{C}}} = \frac{2\sqrt{6}}{\sqrt{6}+\sqrt{15}} = \frac{2\sqrt{2}}{\sqrt{2}+\sqrt{5}} \approx 0.77\,\mathrm{m}$$

For that very last step, we factored out $\sqrt{3}$ from the top and the bottom. An unnecessary step if you are using a calculator anyway, but we prefer to stay in practice.

The more general solution is to go back before we took the square root of both sides of the equation and solve it completely:

$$\begin{aligned} \frac{q_2}{r_{23}^2} &= \frac{q_1}{(2-r_{23})^2} \\ q_2\,(2-r_{23})^2 &= q_1 r_{23}^2 \\ q_2\left(4-4r_{23}+r_{23}^2\right) &= q_1 r_{23}^2 \\ (q_2-q_1)\,r_{23}^2-4q_2r_{23}+4q_2 &= 0 \end{aligned}$$

Now we just have to solve the quadratic ...

$$\begin{aligned} r_{23} &= \frac{4q_2 \pm \sqrt{(-4q_2)^2-4\,(q_2-q_1)\cdot 4q_2}}{2\,(q_1-q_2)}\,\mathrm{m} \\ &= \frac{4\cdot 6\,\mu\mathrm{C} \pm \sqrt{(-4\cdot 6\,\mu\mathrm{C})^2-4\,(6\,\mu\mathrm{C}-15\,\mu\mathrm{C})\cdot 4\cdot 6\,\mu\mathrm{C}}}{2\,(6\,\mu\mathrm{C}-15\,\mu\mathrm{C})}\,\mathrm{m} \end{aligned}$$

We can cancel all of the μC ...

[i] This would not work if we wanted the point to the left of q_2.

$$\begin{aligned} r_{23} &= \frac{24 \pm \sqrt{24^2 - 4(-9)(4)(6)}}{2(-9)}\ \text{m} \\ &= \frac{24 \pm \sqrt{24^2 + 36(24)}}{-18}\ \text{m} \\ &= \frac{-24 \mp \sqrt{1440}}{18}\ \text{m} \\ &= (0.775, -3.44)\ \text{m} \end{aligned}$$

So there is one solution where q_3 is right between the two positive charges, at $r_{23} = 0.77\,\text{m}$, and one solution where q_3 is to the left of q_2 by 3.44 m. We were asked to find the point between the two charges where the force is zero, so we discard the negative solution.

7. A proton at rest is accelerated parallel to a uniform electric field of magnitude 8.36 V/m over a distance of 1.10 m. If the electric force is the only one acting on the proton, what is its velocity in km/s after it has been accelerated over 1.10 m? *The proton mass is given at the end of the exam.*

- ○ 30.0 km/s
- ○ 1800 km/s
- ⊗ 42.0 km/s
- ○ 21.0 km/s

Percent answering correctly: 75%
Similar to: Homework 3, #2.
Average score on HW3, #2: 96.5%

Of course, 42 is the answer to life, the universe, and everything.[ii]
The proton starts from rest, and hence has no kinetic energy. It is accelerated by an electric field, and thus gains kinetic energy. The kinetic energy gained must come from the electric field. A charge q moving parallel to a constant electric field E over a distance Δx changes its potential energy by:

$$\Delta PE = qE\Delta x$$

The charge on a proton is just $+e$, and E and Δx are given. The change in kinetic energy is just the final kinetic energy of the proton, since it started from rest. The gain in kinetic energy must equal the change in potential energy:

[ii] From *Hitchhiker's Guide to the Galaxy ... there are often nerd jokes on physics exams.*

$$\begin{aligned}
\Delta PE = PE_{\text{initial}} - PE_{\text{final}} &= -\Delta KE = -\left(KE_{\text{initial}} - KE_{\text{final}}\right) \\
eE\Delta x - 0 &= -\left(0 - \frac{1}{2}m_p v_{\text{final}}^2\right) \\
eE\Delta x &= \frac{1}{2}m_p v_{\text{final}}^2 \\
\implies v_{\text{final}}^2 &= \frac{2eE\Delta x}{m_p} \\
v_{\text{final}} &= \sqrt{\frac{2eE\Delta x}{m_p}}
\end{aligned}$$

Plugging in what we are given ...

$$\begin{aligned}
v_{\text{final}} &= \sqrt{\frac{2\,(1.6\times10^{-19}\,\text{C})\,(8.36\,\text{V/m})\,(1.10\,\text{m})}{1.67\times10^{-27}\,\text{kg}}} \\
&\approx 42000\,\sqrt{\text{C}\cdot\text{V/kg}} \\
&= 42000\,\sqrt{\text{J/kg}} \\
&= 42000\,\sqrt{\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2\cdot\text{kg}}} \\
&= 42\,\text{km/s}
\end{aligned}$$

Making absolutely sure that the units work out, one should note that Coulombs times Volts is Joules, or kg·m^2/s^2. If you always use proper SI units, it will work out though, and you won't have to remember lots of unit conversions.

8. It takes 3×10^6 J of energy to fully recharge a 9 V battery. How many electrons must be moved across the 9 V potential difference to fully recharge the battery?

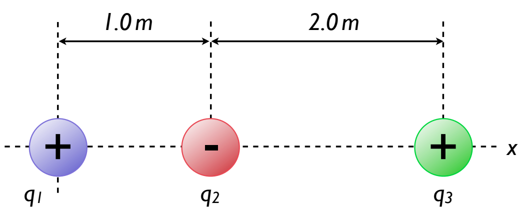
- ○ 1×10^{25} electrons
- ⊗ 2×10^{24} electrons
- ○ 4×10^{12} electrons
- ○ 8×10^{13} electrons

Percent answering correctly: 73%
Similar to: Homework 3, #3.
Average score on HW3, #3: 90.7%

The energy required to charge the battery is just the amount that the potential energy of all the charges changes by. Each electron is moved through 9 V, which means each electron changes its potential energy by $-e\cdot 9\,\text{V}$, where e is the charge on one electron. The total potential energy is the potential energy per electron times the number of electrons, n. Basically, this is conservation of energy: the total energy into the battery has to equal the amount of energy to move one electron across 9 V times the number of electrons.

$$
\begin{aligned}
\Delta E_{in} + \Delta PE &= 0 \\
3.6\times10^{6}\,\mathrm{J} + n(-e\cdot 9\,\mathrm{V}) &= 0 \\
ne\cdot 9\,\mathrm{V} &= 3.6\times10^{6}\,\mathrm{J} \\
n &= \frac{3.6\times10^{6}\,\mathrm{J}}{e\cdot 9\,\mathrm{V}} \\
&= \frac{3.6\times10^{6}\,\mathrm{J}}{(1.6\times10^{-19}\,\mathrm{C})\,(9\,\mathrm{V})} \\
&= \frac{3.6\times10^{6}}{(1.6\times10^{-19})\,(9)} \\
&\approx 2\times10^{24}
\end{aligned}
$$

Again, we make use of the fact that Coulombs times Volts is Joules. Again, if you just use proper SI units throughout, the units will work out on their own.



9. Three charges are positioned along the x axis, as shown at left. All three charges have the same *magnitude* of charge, $|q_1| = |q_2| = |q_3| = 10^{-9}\,\mathrm{C}$ (note that q_2 is negative though). What is the total **potential energy** of this system of charges? We define potential energy zero to be all charges infinitely far apart.

- ○ 2.3×10^{-9} J
- ○ -6.7×10^{-10} J
- ○ 1.8×10^{-9} J
- ⊗ -1.0×10^{-8} J

Percent answering correctly: 40%
Similar to: Homework 3, #4; examples in notes.
Average score on HW3, #4: 72.1%

The potential energy of a system of charges can be found by superposition, by adding together the potential energy of all *unique* pairs of charges. In this case, we have three distinct pairs of charges – (1,2), (1,3), and (2,3). The potential energy of the pair (1,2) is the electric potential that charge 2 feels due to charge 1, times charge 2:

$$
PE_{(1,2)} = k_e q_2 \frac{q_1}{r_{12}^2} = k_e \frac{q_1 q_2}{r_{12}^2}
$$

Here r_{12} is the separation between charges 1 and 2, or just 1.0 m in this case. We do the same for the other two pairs of charges, and add all three energies together (being very careful with signs):

$$
\begin{aligned}
PE_{\text{total}} &= PE_{(1,2)} + PE_{(1,3)} + PE_{(2,3)} \\
&= k_e \frac{q_1 q_2}{r_{12}} + k_e \frac{q_1 q_3}{r_{13}} + k_e \frac{q_2 q_3}{r_{23}} \\
&= k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) \\
&= \left(9 \times 10^9 \, \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}} \right) \left[\frac{\left(-10^{-9}\,\mathrm{C}\right)\left(10^{-9}\,\mathrm{C}\right)}{1\,\mathrm{m}} + \frac{\left(10^{-9}\,\mathrm{C}\right)\left(10^{-9}\,\mathrm{C}\right)}{3\,\mathrm{m}} + \frac{\left(-10^{-9}\,\mathrm{C}\right)\left(10^{-9}\,\mathrm{C}\right)}{2\,\mathrm{m}} \right] \\
&= \left(9 \times 10^9 \, \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}} \right) \left(\frac{10^{-18}\,\mathrm{C^2}}{\mathrm{m}} \right) \left[-1 + \frac{1}{3} - \frac{1}{2} \right] \\
&= \left(9 \times 10^{-9}\,\mathrm{N \cdot m} \right) \left[\frac{-7}{6} \right] \\
&\approx -1.1 \times 10^{-8}\,\mathrm{J}
\end{aligned}
$$

Here we used the fact that a $1\,\mathrm{J} \equiv 1\,\mathrm{N \cdot m}$

10. A parallel plate capacitor is shrunk by a factor of two in every dimension – the separation between the plates, as well as the plates' length and width are all two times smaller. If the original capacitance is C_0, what is the capacitance after all dimensions are shrunk?

○ $2C_0$
⊗ $\frac{1}{2}C_0$
○ $4C_0$
○ $\frac{1}{4}C_0$

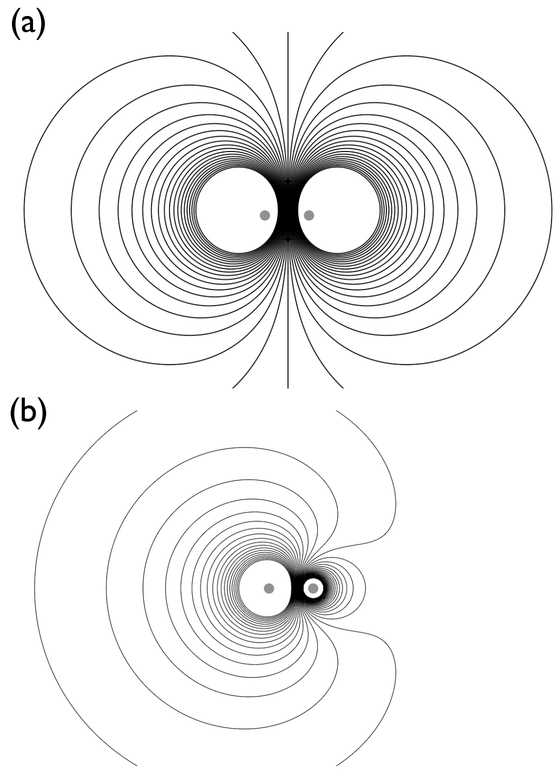
Percent answering correctly: 70.8%
Similar to: n/a.

The capacitance of a parallel plate capacitor whose plates have an area A and a separation d is $C = \frac{\epsilon_0 A}{d}$. If we imagine the plates to be rectangular of length l and width w, the area A is $A = lw$. Let the capacitance of the capacitor be $C_0 = \frac{\epsilon_0 l w}{d}$ *before* dimensions are shrunk. Once we reduce the length, width, and separation by two times, we have:

$$
C = \frac{\epsilon_0 \left(\frac{1}{2} l\right) \left(\frac{1}{2} w\right)}{\left(\frac{1}{2} d\right)} = \frac{\epsilon_0 \frac{1}{2} l w}{d} = \frac{1}{2} C_0
$$

It is easy to prove that if we chose, *e.g.*, circular plates, the answer would be the same – for any reasonable shape, the area goes down as the *square* of the dimensional decrease, while the separation just goes down as the factor itself.

11. The figure at right shows the **equipotential** lines for two different configurations of two charges (the charges are the solid grey circles). Which of the following is true?



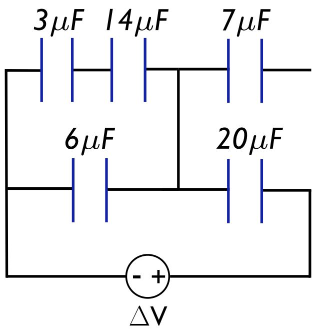
- ○ The charges in (a) are of the same sign and magnitude, the charges in (b) are of the same sign and different magnitude.
- ⊗ The charges in (a) are of opposite sign and of the same magnitude, the charges in (b) are of the opposite sign and different magnitude.
- ○ The charges in (a) are of the same sign and magnitude, the charges in (b) are of the opposite sign and the same magnitude.
- ○ The charges in (a) are of the opposite sign and different magnitude, the charges in (b) are of the same sign and different magnitude.

Percent answering correctly: 66.7%
Similar to: Examples in notes.

This is probably another question most easily answered by elimination. In (a), the charges are clearly of the same magnitude, since the graph is perfectly symmetric, while in (b) the charges must be of different magnitude to explain the asymmetric graph. Therefore, the third answer cannot be correct.

In (a), the potential is constant along a vertical line separating the two charges (since there is a perfectly vertical line running halfway between the charges). This would only be true if they are of *opposite* signs. If the charges were of the same sign, there would be equipotential lines running horizontally from charge to charge. Similarly, the charges must also be of opposite sign in (b). This also rules out the first answer.

Based on similarity of (a) and (b), it must be that if (a) has charges of opposite magnitude, then so does (b). This also means that the fourth answer is out, which leaves only the second answer as a possibility. If you are still not clear on why the correct answer must be the second one, you may want to look carefully at the examples of equipotential lines in different situations presented in the textbook and course notes.



12. What is the equivalent capacitance for the five capacitors at left (approximately)?

⊗ $6.0\,\mu$F
○ $29\,\mu$F
○ $24\,\mu$F
○ $1.7\,\mu$F

Percent answering correctly: 85.4%
Similar to: Homework 3, #7; practice exam; notes.
Average score on HW3, #7: 89.7%

First of all, we should notice that the $7\,\mu$F capacitor has nothing connected to its right wire, so it can't possibly be doing anything in this circuit. We can safely ignore it. Next, the $3\,\mu$F and $14\,\mu$F capacitors are simply in series, so we can readily find their equivalent capacitor:

$$C_{\text{eff,3\&14}} = \frac{(3\,\mu\text{F})(14\,\mu\text{F})}{(3\,\mu\text{F}) + (14\,\mu\text{F})} \approx (2.65\,\mu\text{F})$$

This $2.65\,\mu$F effective capacitor is purely in parallel with the $6\,\mu$F capacitor. We can therefore just add the two capacitances together and come up with an equivalent capacitance for the 3, 14, and $6\,\mu$F capacitors:

$$C_{\text{eff,3,14,\&6}} = C_{eff,3\&14} + 6\,\mu\text{F} = 8.65\,\mu\text{F}$$

Finally, *that* equivalent capacitance is just in series with the $20\,\mu$F capacitor, so the overall equivalent capacitance is readily found:

$$C_{\text{eff, total}} = \frac{C_{\text{eff,3,14,\&6}} 20\,\mu\text{F}}{C_{\text{eff,3,14,\&6}} + 20\,\mu\text{F}} \approx 6\,\mu\text{F}$$

If this is still giving you trouble, try re-working the example in Sect. 4.6 of the course notes.

13. If you double the current through a resistor ...

⊗ The potential difference doubles.
○ The potential difference is half as much.
○ The potential difference is the same.
○ None of the above.

Percent answering correctly: 81.3%
Similar to: Quiz 4, #3.
Average score on Quiz 4, #3: 67%

This is a conceptual question, but one that is most easily answered with a bit of algebra. Recall the relation between potential difference, current, and resistance (Ohm's law):

$$R = \frac{\Delta V}{I}$$

If we double the current I to $2I$, and the resistance remains the same, it is easy to see that the ΔV must also double:

$$R = \frac{(?)\Delta V}{2I} \qquad \Longrightarrow \qquad (?) \text{ must equal } 2$$

14. If the number of carriers in a conductor n decreases by 100 times, but the carriers' drift velocity v_d *increases* by 5 times, by how much does its **resistance** change?

- ⊗ It increases by 20 times.
- ○ It decreases by 500 times.
- ○ It decreases by 20 times.
- ○ It increases by 500 times.

Percent answering correctly: 43.8%
Similar to: n/a.

Just like the last question, this is easily answered with some algebra. First, we recall the relation between *current* and drift velocity:

$$I = nqAv_d$$

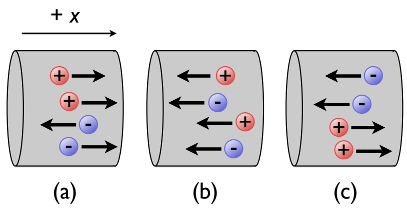
What we are really after is the resistance, however, which we can find with Ohm's law:

$$R = \frac{\Delta V}{I} = \frac{\Delta V}{nqAv_d} \propto \frac{1}{nv_d}$$

So the resistance is inversely proportional to the carrier density and drift velocity. Let's say the initial resistance is R_0, and the resistance after changing n and v_d is just R. If we decrease the number of carriers by 100 times, the resistance goes *up* by 100 times. If we increase the drift velocity by 5 times, the resistance goes *down* by 5 times.

$$\begin{aligned} R_o &\propto \frac{1}{nv_d} \\ R &\propto \frac{1}{\left(\frac{n}{100}\right)(5v_d)} = \frac{1}{\frac{nv_d}{20}} = \frac{20}{nv_d} \\ \Longrightarrow R &= 20R_o \end{aligned}$$

Even though we don't know what the actual resistance R_0 is, we can say that R is twenty times more. The one tricky step here is to write down the proper relationship between *resistance* and the given quantities, not just the relationship between *current* and the given quantities.



15. Rank the relative currents in figures a, b, and c from lowest to highest. Assume positive current corresponds to positive charges flowing to the right, and that all charges move at the same velocity.

- ○ $a < b < c$
- ⊗ $b < a < c$
- ○ $c < b = a$
- ○ $b < c < a$

Percent answering correctly: 85.4%
Similar to: Quiz 4, #4, last year's quizzes, notes.
Average score on Quiz 4, #4: 71%

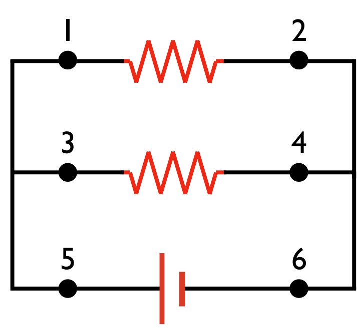
There are really only three rules to keep in mind: (1) a negative charge moving in one direction is the same thing as a positive charge moving in the opposite direction, (2) a positive and negative charge moving in the same direction cancel out, and (3) two charges of the same sign moving in the opposite direction cancel out. With that in mind ...

In (a), we have two positive charges moving to the right, for a contribution of +2. The two negative charges are moving in opposite directions, and therefore cancel each other, so the relative current is just +2. Equivalently, one could say that the negative charge moving to the right cancels one of the positive charges out, which leaves only one positive charge moving to the right, and a negative charge moving to the left. These both give a +1 contribution, for again a total of +2.

In (b), both positive charges are next to a negative charge moving in the same direction, so everything cancels out. The net current is zero. Put another way, the two positive charges are moving to the left, and give a −1 contribution each. The two negative charges are moving to the left, which would be the same as two positive charges moving to the right. Each of those gives a +1 contribution, leading to zero overall.

In (c), the two negative charges moving to the left can be thought of as two positive charges moving to the right. That means we have effectively a total of four positive charges moving to the right, for a net of +4. Overall, we have $c=4$, $b=0$, and $a=2$, so the order must be $b<a<c$.

16. Rank the currents at points 1, 2, 3, 4, 5, and 6 from *highest to lowest*. The two resistors are identical.



- ○ 5, 1, 3, 2, 4, 6
- ○ 5, 3, 1, 4, 2, 6
- ○ 5=6, 3=4, 1=2
- ⊗ 5=6, 1=2=3=4
- ○ 1=2=3=4=5=6

Percent answering correctly: 52.1%
Similar to: n/a

We only need to remember three things to figure this one out: (1) when a current encounters a junction, it splits up to take each path in amounts inversely proportional to the resistance of the path, (2) the current through a single loop of a circuit is the same everywhere, and (3) related to the last point, charge must be conserved, such that the same number of charges entering a wire have to leave it.

First, think about a current leaving the battery at point 5 and traveling clockwise around the circuit. The current reaches the junction leading to points 1 and 3, and must split up to take both paths. Since both paths have the same resistance (the resistors are equivalent, remember), the current will spit up equally between the two. Therefore, the current is the same at points 1 and 3.

The current in the path from 1-2 or 3-4 is in just a single wire, and the current can't change. Conservation of charge requires that every charge entering point 1 leaves through point 2 (and the same for points 3 and 4). Therefore, the currents at points 1 and 2 are equal, and so are those at points 3 and 4. Putting everything so far together, the current is the same at 1, 2, 3, and 4.

What about the currents at points 5 and 6? Conservation of charge again requires that the charges leaving the battery at 5 must eventually come back through point 6 – no charge can be gained or lost when going around the loop. Therefore, the currents at points 5 and 6 must be the same. Further, since the whole current leaving the battery at point 5 splits up into two separate (and equal) currents at points 1 and 3, the current at point 5 must be larger than the current at points 1 and 3. Therefore, overall the ranking from highest to lowest must be 5=6, 1=2=3=4.

17. A flashlight uses a 1.5 V battery with a negligible internal resistance to light a bulb rated for a maximum power of 1 W. What is the maximum current through the bulb? Assume that the battery has more than enough capacity to drive this current, *i.e.,* it is ideal.

- ⊗ 0.67 A
- ◯ 1.50 A
- ◯ 2.25 A
- ◯ 0.50 A

Percent answering correctly: 79.2%
Similar to: example exam

Basically, all we need to remember is the relationship between power $\mathscr{P}$, current I, and voltage ΔV:

$$\begin{aligned}
\mathscr{P} &= I\Delta V \\
1\,\text{W} &= I\,(1.5\,\text{V}) \\
\Longrightarrow I &= \frac{1\,\text{W}}{1.5\,\text{V}} \approx 0.67\,\text{A}
\end{aligned}$$

18. A 9 V battery with a 1 Ω internal resistance is connected to a 10 Ω resistor. What is the actual voltage across the 10 Ω resistor? Assume that the battery behaves as an ideal voltage source of 9 V in series with its internal resistance.

- ○ 9.9 V
- ⊗ 8.2 V
- ○ 0.9 V
- ○ 4.5 V

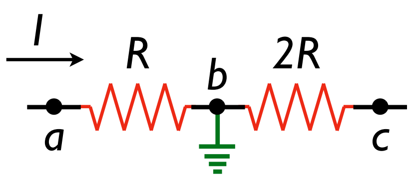
Percent answering correctly: 79.2%
Similar to: in-class examples

If we treat the battery as a perfect voltage source in series with its internal resistance, then the whole circuit under consideration is a perfect source of 9 V, a 1 Ω resistor, and a 10 Ω resistor all in *series*. The fact that they are all in series means they all have the same current. The internal resistance and the 10 Ω load resistance in series are equivalent to a single 11 Ω resistor, which means that effectively a perfect 9 V battery is connected to a single 11 Ω resistor. In that case, we can find the voltage across the 10 Ω resistor by first finding the current in the single loop of the circuit:

$$I = \frac{\Delta V}{R_{\text{eq}}} = \frac{9\,\text{V}}{11\,\Omega} \approx 0.818\,\text{A}$$

The voltage across the 10 Ω resistor is then just given by Ohm's law:

$$\Delta V_{10\,\Omega} = I(10\,\Omega) \approx 8.18\,\text{V}$$



19. A current I flows through two resistors in series of values R and $2R$. The wire connecting the two resistors is connected to ground at point b. Assume that these resistors are part of a larger complete circuit, such that the current I is constant in magnitude and direction. What is the electric potential relative to ground at points **a** and **c**, V_a and V_c, respectively? *Hint: what is the potential of a ground point?*

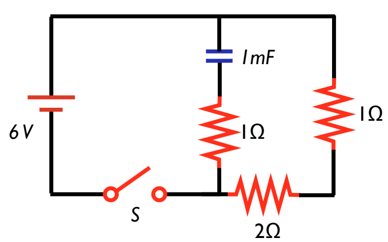
- ○ $V_a = -IR$, $V_c = -2IR$
- ○ $V_a = 0$, $V_c = -3IR$
- ○ $V_a = +IR$, $V_c = +2IR$
- ⊗ $V_a = +IR$, $V_c = -2IR$

Percent answering correctly: 68.8%
Similar to: in-class examples, course notes

What we have to remember here is that grounding a point in circuit *defines its potential to be zero*, so $V_b = 0$. First, consider the resistor R. If there is a current I flowing through it from left to right, we know that the potential *difference* between points a and b must be $\Delta V_{ba} = V_b - V_a = -IR$. That is, the presence of a current I means that there is a *drop* of potential for charges going across the resistor. If we know that the potential at b is zero due to the ground point, $V_b = 0$, then in order to satisfy $\Delta V_{ba} = V_b - V_a = -IR$, we have to have $V_a = +IR$.

Similar reasoning works to find the potential at point c. If there is a current I flowing through a resistor $2R$, then the potential must *decrease* by $2IR$ when moving across the resistor. Thus, we must have $\Delta V_{cb} = V_c - V_b = -2IR$. Again, since $V_b = 0$ due to the ground point, we must have $V_c = -2IR$.

Remember, when we talk about the potential difference across resistors and batteries in circuits, we are really talking about the *difference* in electric potential between two points. We can only talk about the actual absolute electric *potential* when we have defined some point of reference. A ground point effectively defines some point in the circuit to have $V = 0$, and once we have chosen a ground point *then* we can meaningfully talk about the absolute potential at a certain point in a circuit. Without the ground point, only the *relative* potentials of points a, b, and c would be known, not the *absolute* potentials.



20. The switch S is suddenly closed in the circuit at left. The capacitor is uncharged before the switch is closed. After a very long time, what will be the steady-state current in the $2\,\Omega$ resistor? *Hint: what is the capacitor doing after a long time?*

- ○ 4 A
- ○ 3 A
- ⊗ 2 A
- ○ 1 A

Percent answering correctly: 81.3%
Similar to: practice exam.

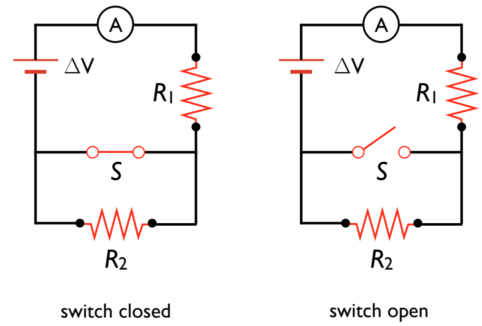
After a long enough time, the capacitor will be completely charged. A current only flows in a capacitor while it is charging or discharging. Even during charging and discharging, the current steadily decreases with time until the capacitor is completely full or empty, respectively. Since the problem says a "very long time" and "steady-state current," we are to assume that the capacitor is no longer charging – if it were, the current would not be steady, but decreasing, and after a long enough time, the capacitor should be fully charged anyway.

If the capacitor is fully charged and no current flows through it, then there is also no current through the $1\,\Omega$ resistor in series with it. If there is no current through the resistor either, then there is no voltage drop across it, and that whole branch of the circuit actually does nothing. Remember, if no current flows through a path in a circuit, it isn't doing anything except possibly storing energy. Portions of a circuit with no current can almost always be neglected when analyzing the rest of the circuit.

If the $1\,\text{mF}$-$1\,\Omega$ branch of the circuit can be neglected, then the only things left are a single $6\,\text{V}$ battery, a $1\,\Omega$ resistor, and a $2\,\Omega$ resistor, all in series. Finding the current now is a simple matter, since the $1\,\Omega$ and $2\,\Omega$ resistors in series just make an equivalent resistance of $3\,\Omega$. Effectively, we have a single battery and resistor, for which we can easily calculate the current:

$$I = \frac{\Delta V}{R_{eq}} = \frac{6\,\text{V}}{3\,\Omega} = 2\,\text{A}$$

21. Refer to the figures at right. What happens to the reading on the ammeter when the switch S is opened? Assume the wires and switch are perfect, and have zero resistance.



- ◯ The reading goes up.
- ⊗ The reading goes down.
- ◯ The reading does not change.
- ◯ More information is needed.

Percent answering correctly: 56.3%
Similar to: practice exam

When the switch is closed, we have R_2 in parallel with a switch. Switches (ideally) have zero resistance, so all the current goes through the switch and none goes through R_2 – if we calculate the equivalent resistance between R_2 in parallel with zero, the equivalent resistance is still zero. Thus, the battery is connected effectively only to R_1, and there is a current of:

$$I_{\text{closed}} = \frac{\Delta V}{R_1}$$

When the switch is opened, resistors R_1 and R_2 are now in series, so that the total circuit resistance is larger than when the switch was closed. As a result, the current decreases, since the applied voltage is the same in both cases. The total current is now:

$$I_{\text{open}} = \frac{\Delta V}{R_1 + R_2} < \frac{\Delta V}{R_1} = I_{\text{closed}}$$

No matter what R_1 and R_2 are, since resistances are always positive, the current has to be smaller when the switch is open.

22. The basic rules we have used for analyzing circuits are: **(1)** the sum of voltage sources and drops around a closed circuit loop is zero, and **(2)** the amount of current entering a junction has to equal the amount of current leaving the junction. These rules result from two basic physical laws. What are they?

- ◯ Conservation of Energy and Charge Quantization
- ◯ Conservation of Energy and Conservation of Momentum
- ⊗ Conservation of Charge and Conservation of Energy
- ◯ Coulomb's law and Conservation of Charge

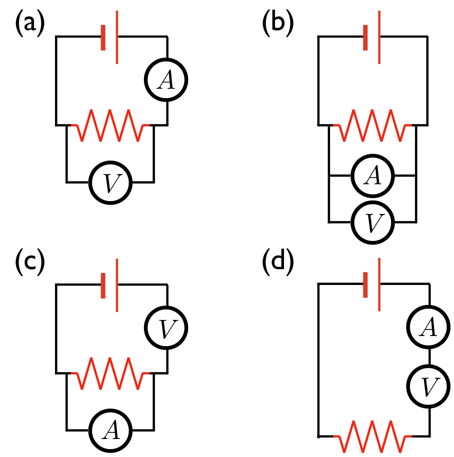
Percent answering correctly: 87.5%
Similar to: in-class examples, course notes, practice exam,

Conservation of energy tells us that the sum of voltage drops and sources around any closed loop has to be zero. Voltage is electrical potential energy per unit charge, and since the electric force is conservative, the change in electrical potential energy has to be zero around *any* closed path, not just in a circuit. Conservation of charge tells us that the current entering an element has to be the same as the current leaving it, and more generally that the sum of currents entering a *junction* must be the sum of the currents leaving it.

Conservation of momentum played no role in the two rules stated. It *did* help us derive Ohm's law in a simple way, but it does not lead us to the rules above. Coulomb's law does not directly lead us to rule (1) or (2) – it deals with electric *force*, whereas rule (1) deals with electric *potential*. At the very least, we need Coulomb's law plus a bit of calculus to get rule (1), and it will not get us rule number (2). Finally, charge quantization does not imply conservation of charge. Charge quantization just says that charge comes in discrete units of e, it does not tell us that charges cannot be created or destroyed.

23. Refer to the figure at right. Which circuit properly measures the current and voltage for the resistor? You may assume that the voltmeters and ammeters are perfect, and the battery is ideal.

- ⊗ circuit (a)
- ○ circuit (b)
- ○ circuit (c)
- ○ circuit (d)



Percent answering correctly: 79.2%
Similar to: in-class examples, course notes

Remember: voltmeters have enormous internal resistances, and must be in *parallel* with what they are measuring. Ammeters have tiny internal resistances, and must be in *series* with what they are measuring. Based on this alone, (a) is the only correct diagram.

Circuit (b) is wrong because the ammeter is connected in parallel with the resistor. The ammeter's resistance is sufficiently low (zero, ideally) that it will 'steal' all of the current from the resistor instead of measuring it. The same effect could be had by just connecting a short-cut wire across the resistor – the ammeter effectively takes it out of the circuit by providing a far lower resistance path, such that little current will actually go through the resistor. The fact that a low equivalent resistance is connected to the battery means a large current will flow, quickly draining the battery. The voltmeter is connected correctly, but in this case it will basically only measure the voltage drop across the ammeter itself.

Circuit (c) is wrong because the ammeter is in series *and* the voltmeter is in parallel. The enormous resistance of the voltmeter (infinite, ideally) means that almost all of the battery's voltage will be dropped across the voltmeter itself, and almost none will be left for the ammeter and resistor. Since the ammeter effectively short-circuits the resistor anyway, this circuit will measure neither I nor ΔV correctly.

Circuit (d) is wrong because again the voltmeter is in series. The ammeter is correct, but the high resistance of the voltmeter will prevent all but the most miniscule currents from flowing anyway, so there will be nothing to measure!

24. A potential difference of 11 V is found to produce a current of 0.45 A in a 3.8 m length of wire with a uniform radius of 3.8 **mm**. What is the resistivity of the wire?

○ $200\,\Omega\cdot\text{m}$
○ $2.9\,\Omega\cdot\text{m}$
○ $2.0\times10^{6}\,\Omega\cdot\text{m}$
⊗ $2.9\times10^{-4}\,\Omega\cdot\text{m}$

Percent answering correctly: 83.3%
Similar to: HW 4, #5; course notes
Average score on HW 4, #5: 95.2

We first need to know the relation between resistivity and resistance, which includes the cross-sectional area of the wire A and its length l:

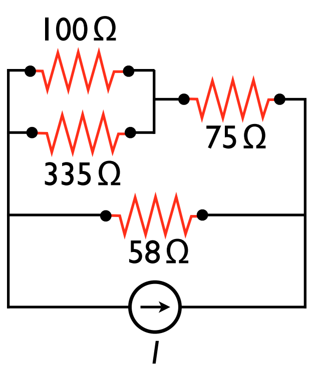
$$R=\frac{\varrho l}{A}\qquad\text{or}\qquad\varrho=\frac{RA}{l}$$

And then we add in the relation between current, voltage, and resistance, *viz.* $R=\Delta V/I$.

$$\varrho=\frac{RA}{l}=\frac{\left(\frac{\Delta V}{I}\right)A}{l}=\frac{\Delta V\cdot A}{I\cdot l}$$

The wire is said to have a uniform radius, which can only be true if its cross section is circular. The area of the circular cross section is then just $A=\pi r^2$. Making sure we keep track of the units, we just plug everything in and run the numbers:

$$\varrho=\frac{\Delta V\cdot A}{I\cdot l}=\frac{11\,\text{V}\cdot\pi\left(3.8\times10^{-3}\,\text{m}\right)^2}{0.45\,\text{A}\cdot3.8\,\text{m}}=2.9\times10^{-4}\,\frac{\text{V}\cdot\text{m}}{\text{A}}=2.9\times10^{-4}\,\Omega\cdot\text{m}$$



25. What is the equivalent resistance of the arrangement of resistors at left? You do not need to include the current source in your analysis

⊗ $42\,\Omega$
○ $122\,\Omega$
○ $175\,\Omega$
○ $31\,\Omega$

Percent answering correctly: 81.3%
Similar to: in-class examples, course notes, practice exam

Where to start? The only pure series or parallel combination initially are the 100 and 335 Ω resistors, which are simply in parallel. We can replace these two resistors with one equivalent resistor:

$$R_{\text{eq},100\&335} = \frac{100 \cdot 335}{100 + 335}\,\Omega \approx 77.0\,\Omega$$

Now this equivalent resistor is purely in series with the $75\,\Omega$ resistor. That means it and the $75\,\Omega$ resistor can both be replaced by an equivalent resistance:

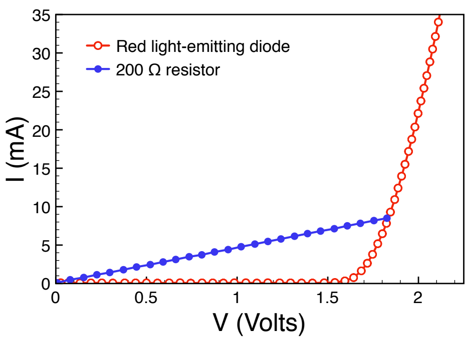
$$R_{\text{eq},100\&335\&75} = R_{\text{eq},100\&335} + 75\,\Omega = 152\,\Omega$$

Finally, this equivalent resistance – which replaces the 100, 335, and $75\,\Omega$ resistors – is purely in parallel with the only remaining resistor, the $58\,\Omega$ resistor. The overall equivalent resistance is then readily found:

$$R_{\text{eq, total}} = \frac{R_{\text{eq},100\&335\&75} \cdot 58\,\Omega}{R_{\text{eq},100\&335\&75} + 58\,\Omega} \approx 42\,\Omega$$

BONUS QUESTION

(worth as much as one normal question)



26. The figure at right shows the current-voltage relationship for a light-emitting diode (LED) and a resistor. When the voltage is 1.7 V, which has the **higher resistance**? *Hint: what does the slope of this plot mean?*

- ○ The resistor.
- ⊗ The LED.
- ○ Cannot be determined.
- ○ They have the same resistance.

Percent answering correctly: 79.2%
Similar to: course notes example

Resistance is just voltage divided by current. If we pick a constant voltage of 1.7 V, then which ever component has a *lower* current has a *higher* resistance. At 1.7 V, the curve for the LED is well below that of the resistor, so the LED has a much smaller current at the same voltage, and thus a higher resistance.