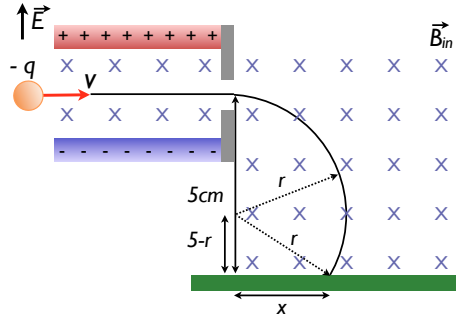# PH 102 Exam II SOLUTION

○ **1**. Consider the mass spectrometer shown at left. The electric field between the plates of the velocity selector is $|\vec{\mathbf{E}}| = 1000\,\text{V/m}$, and the magnetic fields in both the velocity selector and the deflection chamber have magnitudes of 0.1 T.

Calculate the *position x* at which a singly charged ion with mass $m = 7.3 \times 10^{-26}\,\text{kg}$ (corresponding to CO_2) will hit the detector plate.

Percent choosing to answer: 91%
Average score: 77%
Similar to: A variation on homework 5, problem 5; practice exam problems.
Average score on HW5, #5: 69.8%

I think you are all familiar with the mass spectrometer at this point. In the left-most region, there are both $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ fields. The electric force and the magnetic force act in opposite directions. Since the ion's velocity is constant, there must be zero acceleration. If there is zero acceleration, the sum of all forces must be zero:

$$\Sigma F = F_e - F_m = qE - qvB = 0 \qquad \Rightarrow \qquad qE = qvB \qquad \Rightarrow \qquad v = \frac{E}{B}$$

Next, in the region of purely magnetic field on the right, we have only a magnetic force. But, if the path of the ion is circular, then the sum of all forces must equal the centripetal force:

$$\Sigma F = F_m = qvB = \frac{mv^2}{r} \qquad \Rightarrow \qquad r = \frac{mv}{qB}$$

Now we can use the fact that $v = E/B$ and simplify. Again we note that a singly-charged ion has a charge $q = e$:

$$r = \frac{mv}{qB} = \frac{m}{qB}\left(\frac{E}{B}\right) = \frac{mE}{qB^2} \approx 4.56\,\text{cm}$$

This gives us the radius of curvature in the region of only magnetic field. To get the actual position on the detector plate x, we need to use some geometry. Looking at the (revised) figure, we can see that the length x is one leg of a right triangle, the other is $5-r$, and the hypotenuse is r. From this, we can find x:

$$\begin{aligned}
x^2 + (5-r)^2 &= r^2 \\
x^2 + 25 - 10r + r^2 &= r^2 \\
x^2 &= 10r - 25 \\
x &= \pm\sqrt{10r - 25} \approx 4.5\,\text{cm}
\end{aligned}$$

Clearly, on the positive solution for x makes any sense in this context.

○ 2.   A wire with a *weight* per unit length of $0.10\,\text{N/m}$ is suspended directly above a second wire. The top wire carries a current of $30\,\text{A}$ and the bottom wire carries a current of $60\,\text{A}$. Find the distance of separation between the wires so that the top wire will be held in place by magnetic repulsion.

Percent choosing to answer: 78%
Average score: 69%
Similar to: Homework 5, problem 10.
Average score on HW5, #10: 79.1%

Since we have two parallel wires with currents flowing, we know we are going to have a magnetic force between. Now the problem says that the magnetic force is repulsive (which implies that the currents are in opposite directions), which it must be in order for the the top wire to be held in place *against the force of gravity*. This means we want to balance the gravitational force on the top wire, acting downward, against the repulsive magnetic force between the two wires, acting upward on the top wire.

The top wire is quoted to have a *weight* per unit length of $0.10\,\text{N/m}$, which we will call χ. A *weight* is already a force, mass times gravity, so the problem gives you the gravitational force per unit length $\chi = mg/l$ for some section of wire of length l and mass m. We can relate the more common *mass per unit length* $\lambda = m/l$ and the *weight per unit length* easily: $\lambda g = \chi$. Since the force between two parallel current carrying wires is also expressed in terms of force per unit length, we are nearly done.

Given the currents in the top and bottom wire (I_1 and I_2, respectively), the weight per unit length (χ), and the separation between the wires (d), we just have to set the weight per unit length equal to the magnetic force per unit length, and solve for d:

$$\begin{aligned}
\text{weight per length} &= \text{magnetic force per length} \\
\chi \equiv \frac{mg}{l} &= F_B \\
\chi &= \frac{\mu_0 I_1 I_2}{2\pi d} \\
\implies d &= \frac{\mu_0 I_1 I_2}{2\pi \chi}
\end{aligned}$$

Plugging in the numbers we're given ...

$$d = \frac{\left[4\pi \times 10^{-7}\,\mathrm{N/A^2}\right][30\,\mathrm{A}][60\,\mathrm{A}]}{2\pi \cdot 0.10\,\mathrm{N/m}}$$

$$= \frac{2 \cdot 1800 \cdot 10^{-7}}{0.1}\,\mathrm{m}$$

$$\approx 3.6 \times 10^{-3}\,\mathrm{m}$$

Of course, the *units* work out much easier if you know that μ_0 can be expressed in T·m/N or N/A^2, the two are equivalent: $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}$ and $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2}$. This equivalence makes some sense – the first set of units comes from thinking about the field created by a current-carrying wire, while the second comes from thinking about the force between *two* current-carrying wires.
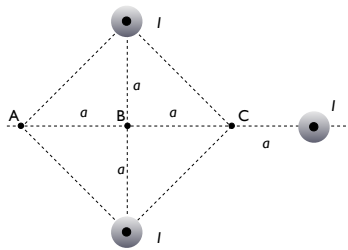
○ 3. Three long parallel conductors carry currents coming *out of the page* of $I = 2.8\,\mathrm{A}$, as shown in the figure at left. Given $a = 2.0\,\mathrm{cm}$, find the magnitude and direction of the magnetic field at all three points A, B, and C.

Percent choosing to answer: 46%
Average score: 27%
*Similar to:* Homework 5, problem 7; practice text verbatim.
Average score on HW5, #7: 72.7%

There is nothing special to do here, except calculate the field at a given point due to each individual wire, and add the results together to get the field due to all three wires. Of course, you have to add the fields as *vectors*, which makes this much more fun. In order to do that, let's define our problem a bit better. Let's call our y axis the line connecting point B with the upper and lower wires, and our x axis the line connecting points A, B, and C. We will label the upper-most wire "1", the lower-most wire "2", and the farthest right wire "3."

We can notice right away that the current is out of the page for all three wires, which means that the field will circulate *counterclockwise*, and be constant on circles centered around each wire. Further, for any point along the x axis, the x components of the fields from wires 1 and 2 are *always* going to be equal and opposite. This can be deduced from symmetry alone - they are at the same x position, the same distance away from the x axis, and carry the same current in the same direction. This means already that the field at all three points will have *no y component*. But we get ahead of ourselves ...

Point A: First, we should think about the direction of the fields. Around each wire, draw a circle centered on the wire, which goes through point A. Since the wires go out of the page, the field is constant on this circle, and it circulates counterclockwise. The direction of the field is tangent to the circle for each wire. Since the circles around wires 1 and 2 have the same radius, this means that the fields from wires 1 and 2 have the same magnitude.

From the symmetry of the problem (or basic geometry), one can see that the x components of the fields from wires 1 and 2 are equal and opposite, leaving only a downward component. The field from wire 3 is purely down, and has no x component to begin with. Thus, the total field is purely downard, and we only need to add up the y components of the fields from each wire. The figure below might help you see this:
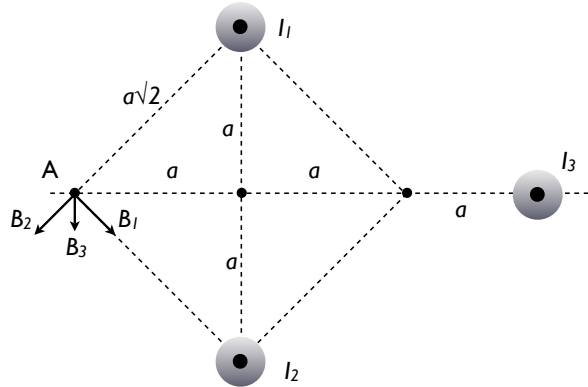
We also know - again from symmetry or basic geometry - that the y components of the field from wires 1 and 2 must be the same. Both wires are a distance $a\sqrt{2}$ from point A. First, let's calculate the *magnitude* of the field from wires 1 and 2:

$$|\vec{\mathbf{B}}_{1,A}| = |\vec{\mathbf{B}}_{2_A}| = \frac{\mu_0 I}{2\pi a \sqrt{2}}$$

The y component of the field is no problem - all the relevant angles are 45 and 90 degrees:

$$
\begin{aligned}
B_{1,A,y} = B_{2,A,y} &= |\vec{\mathbf{B}}_{1,A}| \sin\left(-45°\right) \\
&= \frac{\mu_0 I}{2\pi a \sqrt{2}} \sin\left(-45°\right) \\
&= \frac{\mu_0 I}{2\pi a \sqrt{2}} \left(\frac{-1}{\sqrt{2}}\right) \\
&= \frac{-\mu_0 I}{2\pi a \cdot 2} \\
&= \frac{-\mu_0 I}{4\pi a}
\end{aligned}
$$

Here we used the fact that $\sin\left(-45°\right) = 1/\sqrt{2}$. For wire 3, since the field is purely in the $-y$ direction, we just need to calculate the magnitude of the field itself. No problem, we just note that wire 3 is actually a distance $3a$ from point A:

$$B_{3,A,y} = |\vec{\mathbf{B}}_{3,A}| = \frac{-\mu_0 I}{2\pi a \cdot 3} = \frac{-\mu_0 I}{6\pi a}$$

Finally, to get the total field at point A, which is purely in the $-y$ direction, we just add all the y components together:

$$
\begin{aligned}
|\vec{\mathbf{B}}_{A,tot}| &= B_{1,A,y} + B_{2,A,y} + B_{3,A,y} \\
&= \frac{-\mu_o I}{\pi a}\left[\frac{1}{4} + \frac{1}{4} + \frac{1}{6}\right] \\
&= -\frac{2}{3}\frac{\mu_o I}{\pi a} \\
&\approx -37\,\mu\text{T}
\end{aligned}
$$

Again, the direction is purely downward in the $-y$ direction. Take care that the distance a is converted to meters ...

Point B: At point B, things are even simpler. Since point B is perfectly between wires 1 and 2, the fields from wires 1 and two perfectly cancel each other. All we are left with is the field from wire 3, a distance $2a$ from point B, which is again purely in the $-y$ direction:

$$
B_{B,tot} = B_{3,B,y} = \frac{-\mu_0 I}{2\pi \cdot 2a} = -\frac{1}{4}\frac{\mu_o I}{\pi a} \approx -14\,\mu\text{T}
$$

Point C: This is similar to point A in fact. Again, the x components of the fields from wires 1 and 2 must cancel, and again the field from wire 3 has only a y component. Wires 1 and 2 are a distance $a\sqrt{2}$ from point C, so the y components of their fields will be exactly the same as they were for point A, except that now the field points up instead of down:

$$
B_{1,C,y} = B_{2,C,y} = \frac{\mu_0 I}{4\pi a}
$$

The field from wire 3 still points down, and has only a y component as well. Wire 3 is a distance a from point C, so its field is:

$$
B_{3,C,y} = \frac{-\mu_0 I}{2\pi a}
$$

If we add up the total field, which still has only y components:

$$
B_{C,tot} = B_{1,C,y} + B_{2,C,y} + B_{3,C,y} = \frac{\mu_0 I}{4\pi a} + \frac{\mu_0 I}{4\pi a} + \frac{-\mu_0 I}{2\pi a} = \left[\frac{1}{4} + \frac{1}{4} - \frac{1}{2}\right]\frac{\mu_0 I}{\pi a} = 0
$$

So all three fields exactly cancel, and the field is precisely zero at point C. Which one could have guessed from symmetry alone ...

⃝ 4. A circular coil enclosing an area of $105\,\text{cm}^2$ is made of 200 turns of copper wire. The wire making up the coil has resistance of $7.0\,\Omega$, and the ends of the wire are connected to form a closed circuit. Initially, a $2.0\,\text{T}$ uniform magnetic field points perpendicularly upward through the plane of the coil. The direction

of the field then reverses so that the final magnetic field has a magnitude of $2.0\,\mathrm{T}$ and points downward through the coil. If the time required for the field to reverse directions is $0.15\,\mathrm{s}$, what average current flows through the coil during that time?

Percent choosing to answer: 65%
Average score: 43%
Similar to: HW 6 problem 2; Practice problems.
Average score on HW 6 problem 2: 80%

First, we know that the changing magnetic field through the coil will give an induced voltage *via* Faraday's law. From the induced voltage, and the given resistance, we can find the current using Ohm's law. First, since the coil geometry is fixed, the voltage induced in a coil of N turns of wire of area A due to a changing B field is just:

$$\Delta V = -N\frac{\Delta \Phi_B}{\Delta T} = NA\frac{\Delta B}{\Delta t}$$

So the first question is: what is $\Delta B/\Delta t$? The field changes from $+2$ to $-2\,\mathrm{T}$ in $0.15\,\mathrm{s}$, so we just calculate it. You did take care that the field changes sign, right?

$$\frac{\Delta B}{\Delta t} = \frac{2\,\mathrm{T} - (-2\,\mathrm{T})}{0.15\,\mathrm{s}} = \frac{4}{0.15}\,\mathrm{T/s}$$
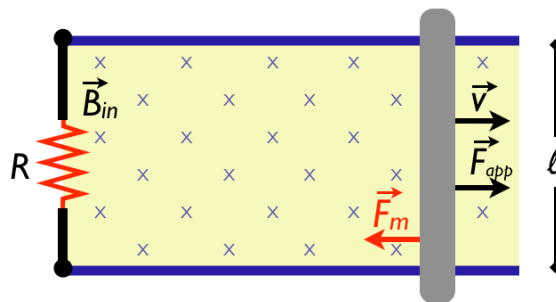
Now that we have that, we note that Ohm's law gives us the current from the induced voltage and resistance. You of course remembered to change the area to square *meters*, not centimeters.

$$
\begin{aligned}
I &= -\frac{\Delta V}{R} = \frac{NA}{R}\left(\frac{\Delta B}{\Delta t}\right)\\
&= -\frac{200 \cdot \left(0.0105\,\mathrm{m}^2\right)}{7\,\Omega}\left(\frac{4\,\mathrm{T}}{0.15\,\mathrm{s}}\right)\\
&= -8\,\mathrm{A}
\end{aligned}
$$

If you used all SI units, your answer is in amps. In order to *verify* this, you need to 'recall' that $1\,\mathrm{T{\cdot}m}^2 = 1\,\mathrm{V{\cdot}s} = 1,\mathrm{Wb}$, and $1\,\Omega = 1\,\mathrm{V/A}$.

○ 5. A conducting rod of length l moves on two (frictionless) horizontal rails, as shown to the right. A constant force of magnitude $|\vec{\mathbf{F}}_{\mathrm{app}}| = 1.0\,\mathrm{N}$ moves the bar at a uniform speed of $|\vec{\mathbf{v}}| = 2.0\,\mathrm{m/s}$ through a magnetic field $\vec{\mathbf{B}}$ directed into the page. The resistor has a value $R = 8.0\,\Omega$.

(a) What is the current through the resistor R?
(b) What is the mechanical power delivered by the constant force?

When the conducting rod moves to the right, this serves to *increase* the flux as time passes (A increases while B stays constant), so any induced current wants to stop this change and *decrease* the magnetic flux. Therefore, the induced current will act in such a way to oppose the external field (*i.e.*, the field due to the induced current will be opposite to the external field). This must be a **counterclockwise** current.

To find the current, we only need to find the motionally-induced voltage and then apply Ohm's law. The rod is just a bar of length l moving at velocity v in a magnetic field B. This gives us an voltage ΔV, and Ohm's law gives us I:

$$\Delta V = Blv = IR \qquad \Longrightarrow I = \frac{Blv}{R}$$

Keep in mind that v is *velocity*, while ΔV is *voltage*. Of course, the problem is now that we don't know B. We *do* know that the external and magnetic forces must balance for the rod to have a constant velocity however. Constant velocity implies zero acceleration, which implies no net force. If this is to be true, the applied force must exactly balance the magnetic force on the rod. For a conductor of length l carrying a current I in a field B, we know how to calculate the magnetic force

$$F_m = BIl = F_{\text{app}} \qquad \Rightarrow \qquad B = \frac{F_{\text{app}}}{Il}$$

Plug that into the first equation:

$$I = \frac{\Delta V}{R} = \frac{Blv}{R} = \frac{F_{\text{app}}lv}{IlR} = \frac{F_{\text{app}}v}{IR} \qquad \Rightarrow \qquad I^2 = \frac{Fv}{R}$$

You should get $I = 0.5\,\text{A}$.

What about the power? Conservation of energy tells us that the mechanical power delivered must be the same as the power dissipated in the resistor, or $\mathscr{P}_F = \mathscr{P}_R = I^2 R$. You should get $2\,\text{W}$.

Alternatively, you note that power delivered by a force is $\mathscr{P}_F = Fv\cos\theta$, where θ is the angle between the force and velocity. In this case, $\theta = 0$, so $\mathscr{P}_F = Fv = 3\,\text{W}$.

Super sneaky way to do everything at once: recall in the first place that the power supplied by the force must equal the power dissipated in the resistor: $\mathscr{P}_F = I^2 R = Fv$. You know F and v, so you can calculate the power, and you also know R, so just solve for I. All wrapped up in one nice neat little package.

○ 6.　　An aluminum ring of radius $5.0\,\mathrm{cm}$ and resistance $1.0\times 10^{-4}\,\Omega$ is placed around the top of a long air-core solenoid with $n = 996$ turns per meter and a smaller radius of $3.0\,\mathrm{cm}$, as in the figure. If the current in the solenoid is increasing at a constant rate of $266\,\mathrm{A/s}$, what is the induced current in the ring?
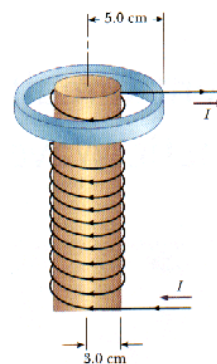
Assume that the magnetic field produced by the solenoid over the area at the end of the solenoid is one-half as strong as the field at the center of the solenoid. Assume also that the solenoid produces a negligible field outside its cross-sectional area.

　　Clearly, this is a magnetic induction problem. We know that a current through the soleniod produces a magnetic field, therefore a time-varying current creates a time-varying magnetic field. This time-varying magnetic field is felt by the aluminum ring, and by Faraday's law, a voltage is induced. First, we can write down Faraday's law for the voltage induced around the aluminum ring due to the time-varying field of the solenoid, noting that the ring only picks up *one half* of the solenoid's field:

$$
\begin{aligned}
\Delta V_{\mathrm{ring}} &= N_{\mathrm{ring}}\frac{\Delta \Phi_{\mathrm{ring}}}{\Delta t} \\
&= \frac{\Delta\left(\frac{1}{2}B_{\mathrm{sol}}\cdot A_{\mathrm{ring}}\right)}{\Delta t} \\
&= \frac{1}{2}A_{\mathrm{ring}}\frac{\Delta B_{\mathrm{sol}}}{\Delta t}
\end{aligned}
$$

　　Here we made use of the fact that the area of the aluminum ring is a constant, and being a solid ring, it has only one turn ($N_{\mathrm{ring}} = 1$). How do we find $\frac{\Delta B_{\mathrm{sol}}}{\Delta t}$? We know the rate that the *current* through the solenoid changes, $\frac{\Delta I_{\mathrm{sol}}}{\Delta t}$, so all we need is the relation between current and B field for a solenoid:

$$
\begin{aligned}
B_{\mathrm{sol}} &= \mu_0 n I \\
\Longrightarrow \frac{\Delta B_{\mathrm{sol}}}{\Delta t} &= \frac{\Delta\left(\mu_0 n_{\mathrm{sol}} I\right)}{\Delta t} \\
\frac{\Delta B_{\mathrm{sol}}}{\Delta t} &= \mu_0 n_{\mathrm{sol}}\frac{\Delta I_{\mathrm{sol}}}{\Delta t}
\end{aligned}
$$

　　Here n_{sol} is the number of turns *per unit length* of the solenoid. Now we can plug this into our expression for ΔV_{ring}, and we are nearly done:

$$
\begin{aligned}
\Delta V_{\mathrm{ring}} &= \frac{1}{2}A_{\mathrm{ring}}\frac{\Delta B_{\mathrm{sol}}}{\Delta t} \\
&= \frac{1}{2}A_{\mathrm{ring}}\mu_0 n_{\mathrm{sol}}\frac{\Delta I_{\mathrm{sol}}}{\Delta t}
\end{aligned}
$$

The only thing left is to relate the current in the ring to the induced voltage, *via* Ohm's law, and plug in the numbers.

$$
\begin{aligned}
I_{\text{ring}} &= \frac{\Delta V_{\text{ring}}}{R_{\text{ring}}} \\
&= \frac{\mu_0 A_{\text{ring}} n_{\text{sol}}}{2 R_{\text{ring}}} \frac{\Delta I_{\text{sol}}}{\Delta t} \\
&= \frac{\left(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}\right) \cdot \left(\pi \left(0.03\,\text{m}\right)^2\right) \cdot \left(996\,\text{m}^{-1}\right)}{2 \cdot 1.0 \times 10^{-4}\,\Omega}\,(266\,\text{A/s}) \\
&= 4.7\,\frac{\text{T}\cdot\text{m}\cdot\text{m}^2\cdot\text{m}^{-1}\cdot\text{A}}{\Omega\cdot\text{s}\cdot\text{A}} \\
&= 4.7\,\frac{\text{T}\cdot\text{m}^2}{\text{V}\cdot\text{s}}\cdot\text{A} \\
I_{\text{ring}} &= 4.7\,\text{A}
\end{aligned}
$$

Again, note $1\,\text{T}\cdot\text{m}^2 = 1\,\text{V}\cdot\text{s}$.

○ 7.   A helium-neon laser delivers 1.05×10^{18} photons/sec in a beam diameter of $1.75\,\text{mm}$. Each photon has a wavelength of $601\,\text{nm}$.

(a) Calculate the amplitudes of the electric and magnetic fields inside the beam.
(b) If the beam shines perpendicularly onto a perfectly reflecting surface, what force does it exert?
(c) If the perfectly reflecting surface is a block of aluminum with mass $m = 1\,\text{g}$, how long will it take for the incident photons to accelerate it to a velocity of $1\,\text{m/s}$? Assume the beam does not diverge, air resistance and gravity can be neglected.

Percent choosing to answer: 30%
Average score: 11%
Similar to: practice exam questions (somewhat); in-class examples

(a) This is a multi-step problem, and requires a bit of thought - there is no one formula to start with. First, we know we can relate the amplitudes of the E and B fields to the intensity of light $\mathcal{I}$. Second, we know that the the ratio of the E and B field has to give the speed of light:

$$
\begin{aligned}
\mathcal{I} &= \frac{E_{\text{max}}^2}{2\mu_0 c} \qquad \text{and} \\
c &= \frac{|\vec{\mathbf{E}}|}{|\vec{\mathbf{B}}|}
\end{aligned}
$$

Now, how does this help us? We first have to think about what intensity $\mathcal{I}$ is - energy per unit time per unit area. Since light energy of a single wavelength - like we have here - comes as individual photons, the energy delivered per second has to be the number of photons per second times the energy per photon:

$$\mathcal{I} = \left[\frac{\text{photons}}{\text{time}}\right]\left[\frac{\text{energy}}{\text{photon}}\right]\left[\frac{1}{\text{Area}}\right]$$

Now we know the energy per photon has to be:

$$E_{\text{photon}} = hf = \frac{hc}{\lambda}$$

Here we used the relation between wavelength λ, frequency f, and the speed of light c: $\lambda f = c$. We are given the number of photons per second, let's call this N. Putting together our formulas:

$$\begin{aligned}
\mathcal{I} &= \left[\frac{\text{photons}}{\text{time}}\right]\left[\frac{\text{energy}}{\text{photon}}\right]\left[\frac{1}{\text{Area}}\right] \\
&= N \cdot \left(\frac{hc}{\lambda}\right) \cdot \frac{1}{\text{Area}} \\
&= \frac{E_{\text{max}}^2}{2\mu_0 c}
\end{aligned}$$

Here A is just the area of the beam, which is easily found from the diameter given. Now we know everything above except E_{max}, so we can solve for that:

$$\begin{aligned}
E_{\text{max}}^2 &= \frac{2\mu_0 hc^2}{\lambda} \cdot N \cdot \frac{1}{\pi\,(d/2)^2} \\
&= \frac{\left(4\pi \times 10^{-7}\,\text{N/A}^2\right)\cdot\left(6.63 \times 10^{-34}\,\text{J}\cdot\text{s}\right)\cdot\left(3 \times 10^8\,\text{m/s}\right)^2}{601 \times 10^{-9}\,\text{m}} \cdot \left(1.05 \times 10^{18}\,1/\text{sec}\right) \cdot \frac{1}{\pi\,\left(8.75 \times 10^{-4}\,\text{m}\right)^2} \\
&= 1.09 \times 10^8\,\frac{\text{N}\cdot\text{J}}{\text{A}^2\cdot\text{m}\cdot\text{s}^2} = 1.09 \times 10^8\,\frac{\text{N}\cdot\text{J}\cdot\text{s}}{\text{C}^2\cdot\text{m}} = 1.09 \times 10^8\,\frac{\text{N}^2}{\text{C}^2}\cdot\frac{\text{J}}{\text{N}\cdot\text{m}} \\
&= 1.09 \times 10^8\,\frac{\text{N}^2}{\text{C}^2} \\
\Longrightarrow E_{\text{max}} &= 1.0 \times 10^4\,\text{N/C} = 1.0 \times 10^4\,\text{V/m}
\end{aligned}$$

Here we used the conversions $1\,\text{J} = 1\,\text{N}\cdot\text{m}$, $1\,\text{C} = 1\,\text{A/s}$, and $1\,\text{N/C} = 1\,\text{V/m}$. We can now find B_{max} easily, once again noting that $\text{V}\cdot\text{s} = \text{T}\cdot\text{m}^2$:

$$\begin{aligned}
B_{\text{max}} &= \frac{E_{\text{max}}}{c} \\
&= \frac{1.0 \times 10^4\,\text{V/m}}{\left(3 \times 10^8\,\text{m/s}\right)^2} \\
&= 3.5 \times 10^{-6}\,\frac{\text{V}\cdot\text{s}}{\text{m}^2} \\
B_{\text{max}} &= 35\,\mu\text{T}
\end{aligned}$$

(b) So. How do we get from intensity to pressure? Photons have momentum, remember from relativity

that $E = |\vec{p}|c$. Using this and the formula for intensity above, we figured out that for a *perfectly reflecting* surface, the radiation pressure can be found *via*

$$P_{\text{refl}} = \frac{2I}{c} = \frac{E_{\text{max}}^2}{\mu_0 c^2}$$

From the known pressure P, we can easily get the force, since pressure is force per unit area:

$$
\begin{aligned}
F &= P \cdot A \\
&= \frac{E_{\text{max}}^2 A}{\mu_0 c^2} \\
&= \frac{\left(1.0 \times 10^4 \, \text{N/C}\right)^2}{4\pi \times 10^{-7} \, \text{N/A}^2 \cdot \left(3 \times 10^8 \, \text{m/s}\right)^2} \cdot \left(\pi \left(8.75 \times 10^{-4} \, \text{m}\right)^2\right) \\
&= 2.3 \times 10^{-9} \, \frac{\text{N}^2 \cdot \text{A}^2 \cdot \text{s}^2 \cdot \text{m}^2}{\text{N} \cdot \text{C}^2 \cdot \text{m}^2} \\
F &= 2.3 \times 10^{-9} \, \text{N}
\end{aligned}
$$

Once again, we need the conversion C=A/s.

(c) Given a (constant) net force, we have a (constant) net acceleration. Given a constant acceleration, we can find the velocity at any later time. First, the acceleration:

$$a = \frac{F}{m} = \frac{2.3 \times 10^{-9} \, \text{N}}{10^{-3} \, \text{kg}} = 2.32 \times 10^{-6} \, \text{m/s}^2$$

Remember that $1 \, \text{N} = 1 \, \text{kg} \cdot \text{m/s}^2$. Assuming the block to be initially at rest ($v_0 = 0$), ignoring air resistance, gravity, friction, *etc.*, we now find the time it takes the block to reach a final velocity of $v_f = 1 \, \text{m/s}$.

$$
\begin{aligned}
v_f &= v_0 + a \cdot \Delta t \\
1 \, \text{m/s} &= 0 + a \cdot \Delta t \\
\Longrightarrow \Delta t &= \frac{v_f}{a} = \frac{1 \, \text{m/s}}{2.32 \times 10^{-6} \, \text{m/s}^2} \\
\Delta t &= 4.3 \times 10^5 \, \text{s} \approx 5 \, \text{days}
\end{aligned}
$$

○ 8. You are given 2 resistors, 1 capacitor, and 1 inductor.

(a) Using only these components, construct both a low and high pass filter.
**(b)** Sketch the frequency response for each.

Percent choosing to answer: 48%
Average score: 34%
Similar to: lengthy in-class examples, examples in course notes

Have a look at the figures below. Remember that an inductor presents a high resistance to high frequency signals, and lets low frequency signals through easily. Therefore, for the high-pass filter, we use the inductor to "short out" low frequency signals to the ground, and let high frequency signals through to the output.

Capacitors, on the other hand, present a high resistance to *low* frequency signals, and let *high* frequency signals through easily. Therefore, for the low-pass filter, we use the capacitor to "short out" high frequency signals to ground, and let low frequency signals through to the output. The frequency response is as illustrated schematically.

See the course notes packet for further details, including how to select the actual component values to determine the range of frequencies filtered.

low-pass

high-pass
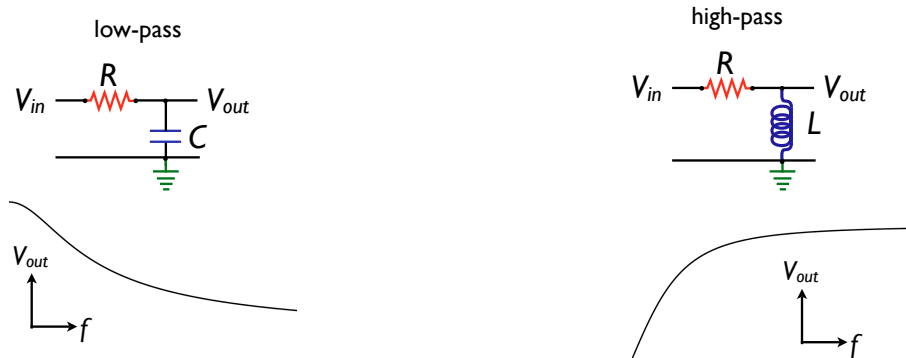
○ 9. A cylindrical cistern, constructed below ground level, is 2.9 m in diameter and 2.0 m deep and is filled to the brim with a liquid whose index of refraction is 1.5. A small object rests on the bottom of the cistern at its center. How far from the edge of the cistern can a girl whose eyes are 1.2 m from the ground stand and still see the object?

Percent choosing to answer: 80%
Average score: 68%
Similar to: HW 8, number 3
Average score on HW 8, number 3: 90.2%

First thing you need to do on a problem like this: draw a little picture to figure out what's going on. Really, it helps. Below is my attempt.

Now the problem is a bit more clear. The light from the bottom of the cistern goes up through the water, and at the water-air interface, is refracted *away* from the normal. Using the angle and distance definitions in the figure, we can first use geometry to find expressions for the two angles θ_1 and θ_2:
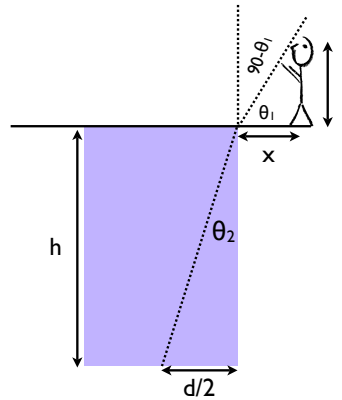
$$\tan\theta_1 = \frac{l}{x}$$

$$\tan\theta_2 = \frac{d/2}{h} = \frac{d}{2h} = \frac{2.9}{4}$$

$$\implies \theta_2 = 35.9°$$

Next, we can use the law of refraction to find another relation between the two angles θ_1 and θ_2. The index of refraction of the liquid in the cistern is $n_{\text{liquid}} = 1.5$ and the index of refraction of air is $n_{\text{air}} = 1$:

$$n_{\text{air}}\sin(90 - \theta_1) = n_{\text{liquid}}\sin\theta_2$$

Now we can solve for θ_1:

$$\theta_1 = 90 - \sin^{-1}(n_{\text{liquid}}\sin\theta_2)$$
$$= 90 - 61.7°$$
$$\theta_1 = 28.3°$$

Given θ_1, we can readily find x using our very first equation:

$$x = \frac{l}{\tan\theta_1}$$
$$= \frac{1.2\,\text{m}}{0.538}$$
$$= 2.23\,\text{m}$$

○ 10. Refer to the figure below. Red and blue light are incident on a glass-air interface, from the glass side, at an angle of incidence θ_i. The index of refraction for red light is $n_{\text{red}} = 1.50$ and $n_{\text{blue}} = 1.52$ for blue

light. If θ_i is greater than some critical angle θ_c, the transmitted beam contains only red light.

    (a) What is the minimum angle of incidence θ_c such that only red light emerges?
    (b) What is the corresponding minimum *refracted* angle θ_r?
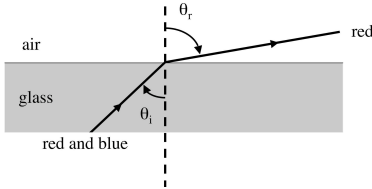
Percent choosing to answer: 80.4%
Average score: 56.3%
Similar to: HW 8, problem 4; 2005 physics graduate qualifier
Average score on HW 8 problem 4: 75.6

In order for only red light to come out, we have to have the blue light totally internally reflected within the glass, but *not* the red. This is indeed possible for some range of angles, since the index of refraction for blue light is higher. Total internal reflection for blue light takes place when:
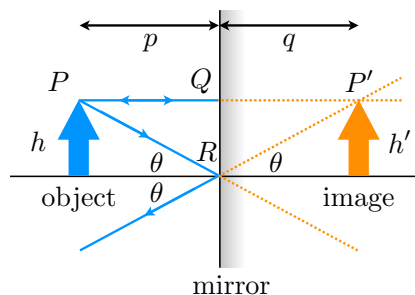
$$
\begin{aligned}
n_{\text{blue}} \sin \theta_c &= n_{\text{air}} \sin 90^\circ = 1 \qquad \text{or} \\
\sin \theta_c &= 1/n_{\text{blue}} = 1/1.52 \\
\theta_c &= \sin^{-1}\left[\frac{1}{n_{\text{blue}}}\right] = \sin^{-1} 0.658 \\
\Longrightarrow \theta_c &= 41.1^\circ
\end{aligned}
$$

Since this θ_c is now also the same minimum angle of incidence for the red light, the minimum *refracted* angle θ_r of red light is given by:

$$
\begin{aligned}
n_{\text{red}} \sin \theta_c &= n_{\text{air}} \sin \theta_r = \sin \theta_r \\
\sin \theta_c &= 1/n_{\text{blue}} \qquad \text{from above} \\
\sin \theta_r &= n_{\text{red}} \cdot 1/n_{\text{blue}} = 1.50/1.52 = 0.987 \\
\theta_r &= \sin^{-1}\left[\frac{n_{\text{red}}}{n_{\text{blue}}}\right] = \sin^{-1} 0.987 \\
\Longrightarrow \theta_r &= 80.7^\circ
\end{aligned}
$$

One does not necessarily need the substitution $\sin \theta_c = 1/n_{\text{blue}}$ above – in practice at that point you already know θ_c, so you can just calculate $\sin \theta_c$ directly. It does make the result much more elegant and comprehensible though – one sees that the two angles calculated above are not really independent, but both simply determined by the refractive indicies. So yes, there are style points in physics too.

○ 11. Use the figure at right to give a geometrical proof that the virtual image formed by a flat mirror is the same distance behind the mirror as the object is in front of it, and of the same height as the object.

There was a lot of leeway given on this on - a bulletproof, strict geometrical proof was not necessary to get most of the credit, so long as your logic was correct and you made a reasonable case. I'll sketch how one may go about a proof below, just to give you an idea.

The two angles on the left side of the mirror labeled θ are equal based on the law of reflection. Using wave optics, we can prove that this must be so, but you were given the law of reflection, and may take it as fact. The angle θ on the right side of the mirror must also be the same, since it is an alternate interior angle of the lower left θ. You were given this much, and could assume that all the θ angles are identical and start from there.

Now, the line PQP′ connecting the tip of the image and object arrows is, by construction, perpendicular to the mirror itself, and therefore parallel to the horizontal line connecting the bases of the arrows as well. At this point, it is already obvious that $h=h'$ in fact.

Given that PQP′ and the mirror are perpendicular by construction, then ∠PQR and ∠P′QR are right angles. Further, since PQP′ and the axis connecting the bases of the arrows are parallel, then the angle θ at point R and ∠PRQ must sum to a right angle: $\theta+$∠PRQ$=90$. The same must be true for ∠P′RQ and θ: $\theta+$∠P′RQ$=90$. Therefore, $\theta+$∠PRQ$=\theta+$∠P′RQ or ∠PRQ$+$∠P′RQ.

Now consider the triangles △PQR and △P′QR. These two triangles have two equivalent angles (∠PRQ$=$∠P′RQ and ∠PQR$=$∠P′QR) which bound a shared side (QR), and by the angle-side-angle (ASA) theorem, the two triangles are congruent. Therefore, $h=h'$, the image height is the same as the object height, and PQ$=$P′Q, the image is as far behind the mirror as the object is in front of it.

○ 12. While looking at her image in a cosmetic mirror, Dina notes that her face is highly magnified when she is close to the mirror, but as she backs away from the mirror, her image first becomes blurry, then disappears when she is about 38.0 cm from the mirror, and then inverts when she is beyond 38.0 cm.

(a) What type of mirror does Dina have?
(b) What is the focal length of the mirror?

(c) What is the radius of curvature of the mirror?

The fact that the image chagnes from upright to inverted immediately tells us that Dina has a **concave** spherical mirror. A convex mirror always gives an upright image, as does a flat mirror. The point at which the image (briefly) disappears and inverts is the focal length, so $f = 38$ cm. For spherical mirrors, we know that $f = 2R$, the radius of curvature is just twice as big: $R = 76$ cm.

The image below may jog your memory a bit. Right at the focal point, when the image goes from upright and enlarged to inverted and enlarged, the image disappears.