

PH 102 Exam III SOLUTION

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Part I: Multiple choice (50%)

1. Answer all multiple choice problems.
2. No partial credit will be given for multiple choice questions.

1. What energy photon emitted is when an electron in a hydrogen atom goes from the $n=5$ energy level to the $n=2$? energy level?

- 10.20 eV
 13.60 eV
 6.80 eV
 2.86 eV

The energy of the emitted photon has to match the energy difference between the $n=1$ and $n=2$ levels:

$$\begin{aligned}E_n &= -13.6 \text{ eV} \cdot \left(\frac{1}{n^2}\right) \\E_i &= E_f + hf \\E_5 &= E_2 + hf \\\Delta E &= E_f - E_i = E_2 - E_5 = -hf \\\Delta E &= -13.6 \text{ eV} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = -13.6 \text{ eV} \left(\frac{1}{2^2} - \frac{1}{5^2}\right) = -13.6 \text{ eV} \left(\frac{1}{4} - \frac{1}{25}\right) \approx -2.86 \text{ eV} = -hf \\hf &= 2.86 \text{ eV}\end{aligned}$$

Thus the initial state is 2.86 eV *higher* in energy than the final state, so the photon has to carry away 2.86 eV to have conservation of energy.

2. Calculate the de Broglie wavelength of a 0.145 kg baseball moving at a speed of 45.2 m/s (≈ 101 mph).

- 1.01×10^{-34} m
 2.62×10^{-24} m
 3.17×10^{-9} m
 4.58×10^{-2} m

The de Broglie wavelength is just Planck's constant divided by momentum. Since the baseball is "only" moving at ≈ 101 mph, it is non-relativistic, so momentum is just mass times velocity.

$$\begin{aligned}\lambda &= \frac{h}{mv} \\&= \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(0.145 \text{ kg})(45.2 \text{ m/s})} \approx 1.01 \times 10^{-34} \text{ m}\end{aligned}$$

3. An inverted image of an object is viewed on a screen from the side facing a converging lens. An opaque card is then introduced covering *only the upper half* of the lens. What happens to the image on the screen?

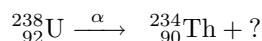
- Half the image would disappear.
- Half the image would disappear and be dimmer.
- The entire image would appear and remain unchanged.
- The entire image would appear, but would be dimmer.

Look at the ray diagrams once more ... even if the upper half of the lens is covered, there are still rays which go through the bottom of the lens to form the image. Half of them, in fact! The image will still be formed by those rays going through the bottom of the lens, but dimmer since half of the rays never make it to the image location. Have a look at the ray diagram, and I think it will be clear why this is true.

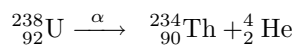
4. When ${}_{92}^{238}\text{U}$ decays to ${}_{90}^{234}\text{Th}$, what is emitted?

- beta particle
- gamma ray
- alpha particle
- deuteron

We have to balance a nuclear reaction just like a chemical reaction, except that both atomic mass and atomic number (upper and lower numbers) must balance. What we really have is this:



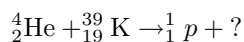
In order to balance this reaction, we need something on the right-hand side with an atomic mass of 4, and an atomic number of 2 – an alpha particle (or ${}^4_2\text{He}$).



5. When an alpha particle hits a ${}_{19}^{39}\text{K}$ nucleus, one of the products is a proton. The other product is:

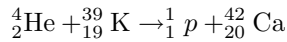
- ${}_{20}^{42}\text{Ca}$
- ${}_{18}^{36}\text{Ar}$
- ${}_{17}^{38}\text{Cl}$
- None of the above

Just like the last problem, balance the atomic numbers and masses in the reaction. We have this:



The left-hand side of this equation has a total mass of 43, and a total atomic number of 21. The right-hand side has one each, so to balance the reaction, we need something on the right-hand side with mass 42, and

atomic number 20. Inspection of the periodic table reveals that Calcium (Ca) is the element with atomic number 20.



6. An x-ray photon is scattered by an electron. What happens to the frequency of the *scattered photon*, relative to that of the incident photon?
- it increases
 - it decreases
 - it does not change
 - photons cannot be scattered by electrons

Basically: conservation of energy.

$$E_i = hf_i$$

$$E_f = hf_f + KE_{e^-} = E_i = hf_i$$

The incident photon scatters off of an electron, and imparts it some kinetic energy. Therefore, the scattered photon has a lower energy (since it lost some energy to the electron). Since its energy is lower, and the energy of a photon is just hf , the scattered photon has to have a lower frequency.

7. A nonrelativistic electron and a nonrelativistic proton are moving and have the same de Broglie wavelength. Which of the following are also the same for the two particles?
- momentum
 - frequency
 - kinetic energy
 - speed

The de Broglie wavelength is $\lambda = h/p$, and non-relativistically, momentum is just $p = mv$. If the proton and electron have the same wavelength, since Planck's constant is, well, *constant*, then the two must have the same momentum.

None of the others are true. If the proton and electron have the same wavelength, then their momentums *must* be the same. However, since the proton and electron have wildly different masses, they cannot then also have same velocity (speed). It is then also not necessarily true that they have the same kinetic energy either. Similarly, the frequency need not be the same. The relationship between wavelength and frequency is $v = \lambda f$, and we can be certain that if the electron and proton have the same momentum, they will not have the same velocity, and hence cannot have the same frequency. However, if they have the same wavelength, they *must* have the same momentum.

8. A light-emitting diode (LED) emits blue photons of wavelength 480 nm. What would be the minimum voltage you would expect to apply to the LED before it emits light? (Hint: we assume all of the potential energy of one electron is converted into light.)

- 2.6 V
 3.5 V
 1.2 V
 0.82 V

We assume that one electron, with potential energy $e\Delta V$ creates one photon of energy $hf = hc/\lambda$. Using our handy formula sheet and conservation of energy:

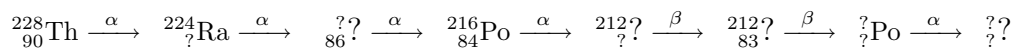
$$\begin{aligned} e\Delta V &= \frac{hc}{\lambda} \\ \Delta V &= \frac{hc}{e\lambda} \\ &= \frac{1240 \text{ eV} \cdot \text{nm}}{480 \text{ nm}} \\ &\approx 2.6 \text{ V} \end{aligned}$$

If you don't understand where the 1240 came from, just start with the second line, and plug in all the constants with their usual values and units, and it should work out just fine ...

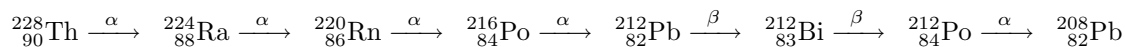
Part II: Problems (50%)

1. Solve 2 problems out of the 5 below. All problems have equal weight.
2. Clearly mark the problems you choose by filling in the adjacent circle.
3. Show as much work as possible for partial credit.
4. Solve the problems on separate sheets. Staple your sheets to the exam when finished.

1. Fill in the missing elements, atomic numbers, and atomic masses (denoted by question marks) in the following radioactive decay series.



Every time you have an alpha decay, subtract 4 from the mass and 2 from the atomic number. For each beta decay, add one to the atomic number, and do nothing with the mass. These are all β^- decays.



2. A hydrogen atom has a radius of ~ 0.05 nm. (a) Assuming we know the position of an electron in a hydrogen atom to an accuracy of 1% of this radius, estimate the uncertainty in the velocity of the electron. How does this value compare to c ? (b) Compare this value to the uncertainty in the velocity of a ball of mass 0.2 kg and radius 0.05 m whose position is known within 1% of its radius.

The uncertainty principle ... we are saying that the uncertainty in position Δx is $0.01r$, or 1% of the radius. Plug this into the uncertainty relationship, and solve for v ...

$$\begin{aligned}\Delta x \Delta p &= (0.01r) \Delta p = 0.01r (m_e \Delta v) \geq \frac{\hbar}{2} \\ \Delta v &\geq \frac{\hbar}{0.02rm_e} \approx 5.8 \times 10^7 \text{ m/s} \approx 0.19c\end{aligned}$$

For the second part, just do the same thing! The only difference is now the mass is 0.2 kg instead of one electron mass, and the radius is 0.05 meters instead of nanometers. You should get:

$$\Delta v \approx 2.6 \times 10^{-31} \text{ m/s}$$

In other words, the uncertainty is completely and utterly negligible until we get to very, very small distances.

- **3.** A molecule is known to exist in an unstable higher energy configuration for $\Delta t = 10 \text{ nsec}$, after which it relaxes to its lower energy stable state by emitting a photon. **(a)** What uncertainty in the frequency Δf of the emitted photon is implied? **(b)** If this state is being probed with Nuclear Magnetic Resonance (NMR) at a frequency of $f \approx 200 \text{ MHz}$, what is the relative uncertainty in the measurement, $\Delta f/f$?

This one is energy-time uncertainty, remembering that photon energy is just hf , and that h is a constant. The energy uncertainty for a photon is $h\Delta f$, since h by definition can have no uncertainty.

$$\begin{aligned}\Delta E \Delta t &\geq \frac{\hbar}{2} \\ (h\Delta f) \Delta t &\geq \frac{\hbar}{2} = \frac{h}{4\pi} \\ \Delta f &\geq \frac{1}{4\pi\Delta t} \approx 7.96 \times 10^6 \text{ Hz} = 7.96 \text{ MHz}\end{aligned}$$

Note that one could get this directly from Fourier analysis - it is true for any waveform, and can be derived without appealing to quantum mechanics at all. The relative broadening in the linewidth (lifetime broadening) is then just:

$$\frac{\Delta f}{f} \approx \frac{7.96 \text{ MHz}}{200 \text{ MHz}} \approx 0.038 = 3.8\%$$

- **4.** Calculate the binding energy *in MeV* of a deuteron (the atom ${}^2_1\text{H}$), given that its atomic mass is 2.014102 u . Note that $m_{p^+} = 1.007825 \text{ u}$, and $m_{n^0} = 1.008665 \text{ u}$.

See Example 29.2 in the book. The binding energy is the difference in the mass of the bare protons and neutrons and the nucleus itself, all times c^2 to get an energy. For a deuteron, we have one neutron and one proton, so we add those two masses together, and subtract off the mass of the deuteron itself:

$$\begin{aligned}
\text{Binding Energy} &= \left[\sum_{\substack{\text{all } p^+ \\ \text{and } n^0}} mc^2 \right] - m_{\text{atom}}c^2 \\
&= \left[(1 p^+) \cdot \left(1.007825 \frac{\text{u}}{p^+} \right) + (1 n^0) \cdot \left(1.008665 \frac{\text{u}}{n^0} \right) \right] c^2 - 2.014102 \text{ u} \cdot c^2 \\
&= 0.002388 \text{ u} \cdot c^2 \\
&= 0.002388 \text{ u} \cdot c^2 \left(\frac{931 \text{ MeV}}{\text{u} \cdot c^2} \right) \\
&= 2.224 \text{ MeV}
\end{aligned}$$

Note that in that last step, we used the (given) conversion of 931 MeV/u·c² to get from a mass difference to an energy ...

- 5. In a coordination compound, the so-called “crystal field” gives rise to a difference in energy levels for some of the electrons in a transition metal ion. That is, electrons can occupy one of two states, separated by the crystal field splitting energy Δ .

The octahedral complex $[\text{Cr}(\text{NH}_3)_6]^{3+}$ has a crystal field splitting of $\Delta_o \sim 2.16 \text{ eV}$, while $[\text{Co}(\text{NH}_3)_6]^{3+}$ has $\Delta_o \sim 2.84 \text{ eV}$. What color are these compounds? Make use of the table below. If a compound absorbs a certain color of light, it exhibits the color *complementary* to the color of absorbed light.

Table 1: Absorbed wavelength λ and observed color

λ (nm)	absorbed color	observed color
400	violet	greenish-yellow
450	blue	yellow
490	blue-green	red
570	yellow-green	violet
580	yellow	dark blue
600	orange	blue
650	red	green

Ignore the complexity and weird terminology. Crystal field splitting just means an energy difference between two energy levels, which happens to come from the symmetry of the crystal, but that is irrelevant for the present problem. We basically have two energy levels, separated by Δ_o and we can change between them by emitting a photon. All we have to do is figure out what photon energy corresponds to Δ_o for each compound, and that is the color of photon that is absorbed. Then we can use the table to figure out the complementary observed color.

For any energy level spacing Δ_o , the energy of an emitted photon has to match this, so:

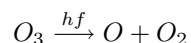
$$\Delta_o = E_{\text{photon}} = \frac{hc}{\lambda} \quad \implies \quad \lambda = \frac{hc}{\Delta_o}$$

That's it. Really. Well, that and the table, which is just graphic design in some sense. Anyway, for the Cr-based coordination compound, you should find $\lambda = 575 \text{ nm}$, which means yellow or yellow/green is *absorbed*, so **violet or dark blue** is the observed color.

For the Co-based compound, you should find $\lambda = 437 \text{ nm}$, which means violet or blue light is *absorbed*, so **green/yellow or yellow** is the observed color.

BONUS (worth 1 normal question):

1. The energy required to break one O=O bond in ozone (O_3 , O=O=O) is about 500 kJ/mol. What is the maximum wavelength of the photon that has enough energy to photodissociate ozone by breaking one of the O=O bonds? You must show your work to receive bonus points. Note that Avogadro's number is $N_A = 6.02 \times 10^{23}$ things/mol.



We have a lot of units to convert. First, let's convert the bond energy into eV per molecule, instead of kJ/mol:

$$E_{\min} = \frac{500 \times 10^3 \text{ J}}{\text{mol}} \cdot \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} \cdot \frac{1 \text{ mol}}{6.02 \times 10^{23} \text{ molecules}} = 5.18 \text{ eV/molecule}$$

So this is already the bond dissociation energy per molecule, really just a unit conversion (and an example of how chemists and physicists sometimes use different language to describe the same things). Now we just need to figure out what photon has the 5.18 eV required to break the oxygen double bond in one ozone molecule:

$$\begin{aligned} E_{\min} &= hf_{\max} = \frac{hc}{\lambda_{\min}} \\ \lambda_{\max} &= \frac{hc}{E_{\min}} = \frac{1240 \text{ eV} \cdot \text{nm}}{5.18 \text{ eV}} \\ \lambda_{\max} &\approx 239 \text{ nm} \end{aligned}$$

Since energy and wavelength are *inversely proportional*, the minimum energy gives the maximum wavelength - energy goes up as wavelength goes down. This wavelength is in the deep UV region, and is harmful in the cancer-causing sort of way. The fact that ozone absorbs UV is one simple reason why the ozone layer is so important - it protects us from harmful ionizing UV rays.