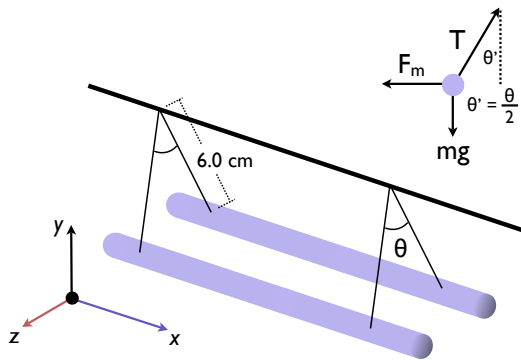# PH 102 Exam II SOLUTION

20Mar07 LeClair

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INSTRUCTIONS

1. **Solve 8 problems out of the 12 below.** All problems have equal weight.

2. Clearly mark the problems you choose by filling in the adjacent circle.

3. **Show as much work as possible for partial credit.**

4. Solve the problems on separate sheets. Staple your sheets to the exam when finished.

</div>

○ **1**. Two long parallel wires, each with a mass per unit length of $\lambda = m/l = 0.040\,\text{kg/m}$, are supported in a horizontal plane by 6.0 cm strings, as shown at left. Each wire carries the same current I, causing the wires to repel one another, which causes the supporting strings to make an angle $\theta = 16°$ with one another.

Are the currents in the same direction or opposing? Find the magnitude of each current.

Hint: consider the free-body diagram for one of the wires in the upper right. If a wire has mass m and length l, $\lambda = m/l$.

On the free-body diagram in the exam, I mistakenly labeled the angle between tension and the vertical as θ when it should have been $\theta/2$. I did not count off if you used the wrong angle, since it was my fault. Not many people attempted this problem in any case.

Basically, we have two current-carrying wires, both carrying the same current I, and we know the magnitude of the force per unit length between them must be:

$$\frac{F_m}{l} = \frac{\mu_0 I^2}{2\pi d} \tag{1}$$

Here d is the *lateral* separation of the wires. If the wires are hanging as shown, then the magnetic force between the wires must be balancing the weight of each wire and the tension in the strings holding them up. Therefore **the magnetic force must be repulsive, and the currents in the opposite direction.**

Notice the free-body diagram in the upper right corner. In equilibrium, the sum of all forces is zero. Take $+x$ to the right, and $+y$ upward. Then we can write down the net forces in the x and y directions. **For convenience, we will say from now on $\theta' \equiv \theta/2$, since we only need the half angle to do the problem.**

$$
\begin{aligned}
\Sigma F_x &= T\sin\theta' - F_m = 0 & \Rightarrow & \quad F_m = T\sin\theta' \\
\Sigma F_y &= T\cos\theta' - mg = 0 & \Rightarrow & \quad T = \frac{mg}{\cos\theta'}
\end{aligned}
$$

Plug the expression for T into the one for F_m, and divide by the length of the wires, which we will call l, so we can equate this result to Eq. 1:

$$\frac{F_m}{l} = \frac{m}{l} g \tan \theta' = g\lambda \tan \theta' = \frac{\mu_0 I^2}{2\pi d}$$

Note the substitution $\lambda = m/l$ above. Now the question is, what is d, the separation of the wires? Simple plane geometry relates the separation d to the length of the support wires (the 6 cm, we'll call this h), and the angle θ': $d = 2h \sin \theta'$. Put that into the equation above and solve for I ...

$$\frac{|\vec{F}_m|}{l} = \lambda g \tan \theta' = \frac{\mu_0 I^2}{2\pi d} = \frac{\mu_0 I^2}{4\pi h \sin \theta'}$$

$$I^2 = \frac{4\pi}{\mu_0} g\lambda h \sin \theta' \tan \theta'$$

Putting in the numbers given, you should get $I \approx 67.8\,\text{A}$. And as noted above, the currents are in opposite directions.

○ **2**. Sodium ions (Na^+) move at $0.85\,\text{m/s}$ through a bloodstream in the arm of a person standing near a large magnet. The magnetic field has a strength of $|\vec{B}| = 1.2\,\text{T}$ and makes an angle of $73°$ with the motion of the sodium ions. The arm contains $120\,\text{cm}^3$ of blood, with $3.0 \times 10^{20}\,\text{Na}^+$ ions per cubic centimeter.

If no other ions were present in the arm, what would be the magnetic force on the arm?

Straight off of the practice problems ... We have a stream of singly-charged Na^+ ions (*i.e.*, $q = e = 1.6 \times 10^{-19}\,\text{C}$) moving at a velocity v at an angle θ to a magnetic field B. We know the force on a *single* ion is:

$$F_{\text{single}} = evB \sin \theta$$

The total force is just the force per ion times the number of ions. We have a density of ions $\rho_{Na^+} = 3 \times 10^{20}\,\text{cm}^3$, and a volume of $V = 120\,\text{cm}^3$. The total number of ions is then just $N_{Na^+} = V \rho_{Na^+} = 3.6 \times 10^{22}$. Note that we don't really have to convert cm^3 to m^3, since the units cancel. Put that all together:

$$F_{\text{tot}} = (N_{Na^+})(F_{\text{single}}) = (N_{Na^+})(evB \sin \theta) \approx 5630\,\text{N}$$

I think you are all familiar with the mass spectrometer at this point. In the left-most region, there are both $\vec{E}$ and $\vec{B}$ fields. The electric force and the magnetic force act in opposite directions. Since the ion's velocity is constant, there must be zero acceleration. If there is zero acceleration, the sum of all forces must be zero:

$$\Sigma F = F_e - F_m = qE - qvB = 0 \qquad \Rightarrow \qquad qE = qvB \qquad \Rightarrow \qquad v = \frac{E}{B}$$

Next, in the region of purely magnetic field on the right, we have only a magnetic force. But, if the path of the ion is circular, then the sum of all forces must equal the centripetal force:

$$\Sigma F = F_m = qvB = \frac{mv^2}{r} \qquad \Rightarrow \qquad r = \frac{mv}{qB}$$
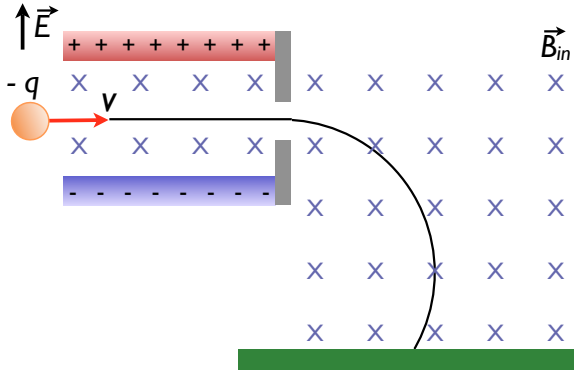
3. Consider the mass spectrometer shown at left. The electric field between the plates of the velocity selector is $|\vec{\mathbf{E}}| = 1000\,\text{V/m}$, and the magnetic fields in both the velocity selector and the deflection chamber have magnitudes of $1.0\,\text{T}$.

Calculate the *radius* of the circular path in the deflection chamber for a singly charged ion with mass $m = 7.3 \times 10^{-26}\,\text{kg}$ (corresponding to CO_2).

Now we can use the fact that $v = E/B$ and simplify. Again we note that a singly-charged ion has a charge $q = e$:

$$r = \frac{mv}{qB} = \frac{m}{qB}\left(\frac{E}{B}\right) = \frac{mE}{qB^2}$$

Plug in the numbers given (no unit conversions for once), and you find $r = 4.6 \times 10^{-4}\,\text{m}$, or $r = 0.46\,\text{mm}$.

4. Assume that the Sun delivers an average power ($\mathscr{P}$) per unit area (A) of about $\mathcal{I} \equiv \mathscr{P}/A = 1.00 \times 10^3\,\text{W/m}^2$ to Earth's surface.

(a) Calculate the total power incident on a flat tin roof $7.17\,\text{m}$ by $21.1\,\text{m}$. Assume that the radiation is incident normal (perpendicular) to the roof.

(b) Calculate the peak electric and magnetic fields of the light.

If $\mathcal{I}$ is just power per unit area, then the first part is easy:

$$\mathscr{P} = \mathcal{I}A = \left(1.00 \times 10^3\,\text{m}^2\right)\left(7.17 \times 21.1\,\text{m}^2\right) = 1.51 \times 10^5\,\text{W}$$

For the second part, we note the following relationship:

$$\mathcal{I} = \frac{E_{\text{max}}^2}{2\mu_0 c}$$

Use the above to get E_{max}, then use the fact that $c = E_{\text{max}}/B_{\text{max}}$. You should get $E_{\text{max}} = 868\,\text{V/m}$, $B_{\text{max}} = 2.89\,\mu\text{T}$

○ **5**. Using an electromagnetic flowmeter (see figure), a heart surgeon monitors the flow rate of blood through an artery. Electrodes A and B make contact with the outer surface of the blood vessel, which has inside **diameter** 3.2 mm. Permanent magnets outside the blood vessel create a magnetic field perpendicular to the blood flow direction. For a magnetic field strength of $|\vec{\mathbf{B}}| = 0.037\,\text{T}$, a potential difference of $\Delta V = 160\,\mu\text{V}$ appears between the electrodes.
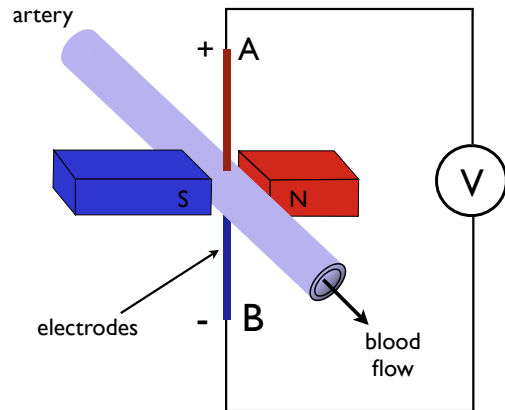
(a) Calculate the speed of the blood.
(b) Does the sign of the potential difference depend on whether the mobile ions in the blood are predominantly positively or negatively charged?

From the homework. The presence of a magnetic field perpendicular to the movement of ions in the blood means that they experience a magnetic force.

$$F_m = qvB$$

The force for positive ions is pointing up, and for negative ions it is pointing down. This serves to separate spatially the positive and negative charges - positive charges move toward electrode A, and negative charges move to point B. This continues until the ions reach the surface of the artery, at which point they are separated by the diameter of the artery d.

If the charges are separated spatially by a distance d, then this gives rise to an electric field E, and a potential difference $\Delta V = Ed$. At this point, the electric and magnetic forces are balanced. Set the magnetic and electric fields equal to one another, use the expression for ΔV, and take care with units:

$$
\begin{aligned}
F_m &= qvB = F_e = qE = q\left(\frac{\Delta V}{d}\right) \\
\Rightarrow v &= \frac{q\Delta V}{qBd} = \frac{\Delta V}{Bd} = 1.35\,\text{m/s}
\end{aligned}
$$
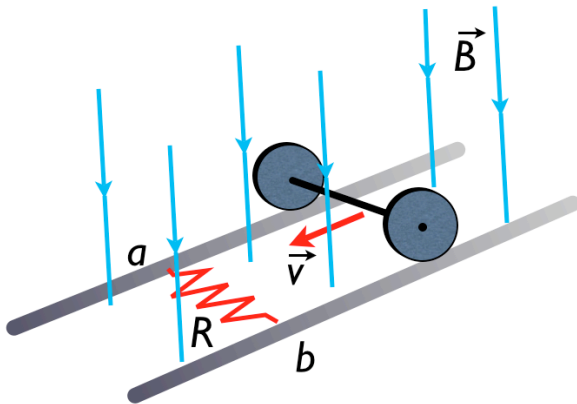
○ **6**. In the figure, the rolling axle, 1.50 m long, is pushed along horizontal rails at a constant speed $|\vec{\mathbf{v}}| = 4.00\,\text{m/s}$. A resistor $R = 0.4\,\Omega$ is connected to the rails at points a and b, directly opposite each other. (The wheels make good electrical contact with the rails, so the axle, rails, and R form a closed-loop circuit. The only significant resistance in the circuit is R.) A uniform magnetic field $|\vec{\mathbf{B}}| = 0.1500\,\text{T}$ is directed vertically downwards.

(a) Find the induced current I in the resistor.
(b) What horizontal force (magnitude **and** direction) is required to keep the axle rolling at constant speed? *Hint: ignore everything but the axle.*

When the axle moves to the left, this serves to *decrease* the flux as time passes, so any induced current wants to stop this change and *increase* the flux. Therefore, the induced current will act in such a way to reinforce

the external field (*i.e.*, the field due to the induced current will be in the same direction as the external field). This must be a **clockwise** current.

To find the current, we only need to use motional emf. The axle is just a bar of length l moving at velocity v in a magnetic field B. This gives us an emf $\mathcal{E}$, and Ohm's law gives us I:

$$\mathcal{E} = Blv$$
$$I = \frac{\mathcal{E}}{R} = 2.25\,\text{A}$$

The magnitude of the force is now found readily - the axle is just a wire carrying a current I in a magnetic field B:

$$|\vec{\mathbf{F}}| = BIl = 0.51\,\text{N}$$

What is the direction? For one, if the axle is traveling at constant velocity, then the external force must balance the magnetic force to give zero net force. The magnetic force must be pointing to the right (using the right hand rule), so the external force must be pointing to the left.

Alternatively, we note that the external force is what is pushing the axle in the first place! So it has to be in the same direction as $\vec{\mathbf{v}}$, namely, to the left.

○ **7.** A conducting rod of length l moves on two (frictionless) horizontal rails, as shown to the right. A constant force of magnitude $|\vec{\mathbf{F}}_{\text{app}}| = 1.0\,\text{N}$ moves the bar at a uniform speed of $|\vec{\mathbf{v}}| = 2.0\,\text{m/s}$ through a magnetic field $\vec{\mathbf{B}}$ directed into the page. The resistor has a value $R = 8.0\,\Omega$.

(a) What is the current through the resistor R?
(b) What is the mechanical power delivered by the constant force?
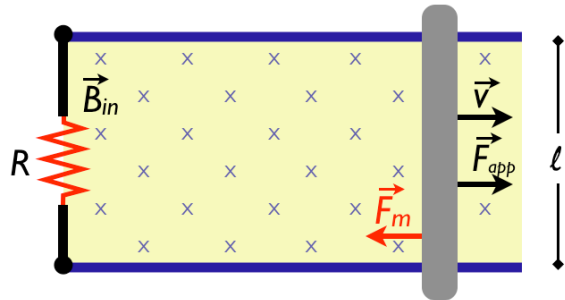
The first part is exactly like the previous problem.

$$\mathcal{E} = Blv = IR$$

The problem is now that we don't know B. We do know that the external and magnetic forces must balance for the rod to have a constant velocity.

$$F_m = BIl = F_{\text{app}} \qquad \Rightarrow \qquad B = \frac{F_{\text{app}}}{Il}$$

Plug that into the first equation:

$$I = \frac{\mathcal{E}}{R} = \frac{Blv}{R} = \frac{F_{\text{app}}lv}{IlR} = \frac{F_{\text{app}}v}{IR} \qquad \Rightarrow \qquad I^2 = \frac{Fv}{lR}$$

You should get $I = 0.5\,\text{A}$.

What about the power? The mechanical power delivered must be the same as the power dissipated in the resistor, $\mathscr{P}_F = \mathscr{P}_R = I^2 R$. You should get $2\,\text{W}$.

Alternatively, you note that power delivered by a force is $\mathscr{P}_F = F v \cos\theta$, where θ is the angle between the force and velocity. In this case, $\theta = 0$, so $\mathscr{P}_F = Fv = 3\,\text{W}$.

Super sneaky way to do everything at once: recall in the first place that the power supplied by the force must equal the power dissipated in the resistor: $\mathscr{P}_F = I^2 R = Fv$. You know F, and v, so you can calculate the power, and you also know R, so just solve for I.
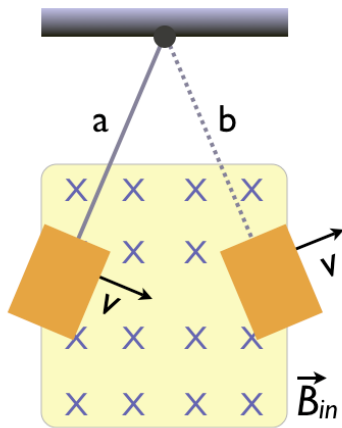
$\bigcirc$ **8.** Consider the flat metal plate swinging at the end of a bar as a pendulum, as shown at left. At position **a**, the pendulum is moving from a region where there is no magnetic field into a region where the field in is directed into the paper.

Find the direction of circulation of the eddy current and the direction of the force (relative to $\vec{v}$) at **both** positions **a** and **b**.

In position **a**, as the pendulum swings into the region of magnetic field, the flux is *increasing*. Therefore, the induced currents will want to *decrease* the flux, and will circulate in such a way to act against the external field. This means the B from the induced current will be out of the page, which implies a **counter-clockwise** current.

By the same logic, induction will want to slow the pendulum, to decrease the rate at which the flux is increasing. So the force will be acting in the direction opposite the velocity.

In position **b**, as the pendulum swings out of the region of magnetic field, the flux is *decreasing*. The induced currents will try to increase the flux, and will create a field acting in the same direction as the external field. This implies a **clock-wise** current.

Again, the force will act in the direction opposite the velocity, to try to slow the change in flux.

○ **9**. The index of refraction for **violet** light in silica flint glass is $n_{\text{violet}} = 1.66$, and for **red** light it is $n_{\text{red}} = 1.62$. In air, $n = 1$ for both colors of light.

What is the **angular dispersion** of visible light (the angle between red and violet) passing through an equilateral triangle prism of silica flint glass, if the angle of incidence is 50°? Recall that all angles in an equilateral triangle are 60°.
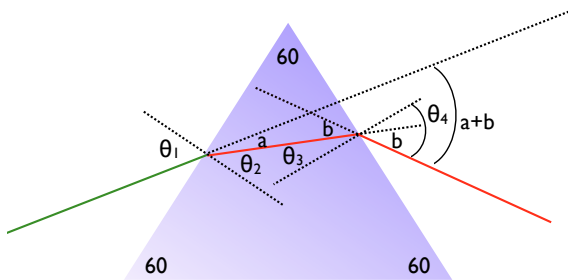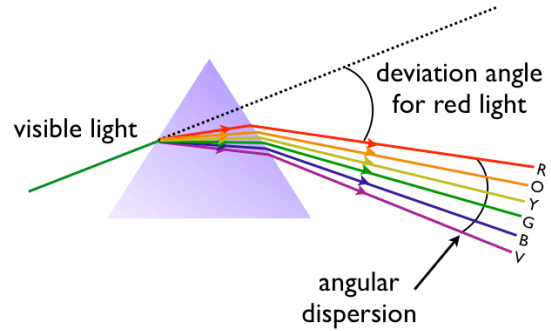
One ray at a time. If we calculate the angle of deviation for red light, and then for violet, we can just subtract those two extremal angles to find the angular dispersion. First, we will need quite a bit of plane geometry. Reference the figure at left.

The first deviation the light ray experiences is the angle a on entering the prism, and then the angle b on exiting. The total deviation is then $a + b$. Now look at the triangle formed by the red line inside the prism and the top part of the prism. For this triangle, the angles are $90 - \theta_2$, $90 - \theta_3$, and 60°. All the angles in a triangle must sum to 180°:

$$90 - \theta_2 + 90 - \theta_3 + 60 = 180 \qquad \Rightarrow \qquad \theta_2 + \theta_3 = 60$$

Now, note that $\theta_1 = a + \theta_2$, and $\theta_4 = b + \theta_3$. We can now combine all our relationships and write down the angular deviation for a single ray:

$$\text{deviation} = a + b = \theta_1 - \theta_2 + \theta_4 - \theta_3 = \theta_1 + \theta_4 - 60$$

We are given θ_1. We can find the other θ's with Snell's law at the two air-prism interfaces. Let $n_1 = 1$ be the air, and n_2 be the index of refraction for either red or violet light in the prism:

$$
\begin{aligned}
n_1 \sin \theta_1 &= n_2 \sin \theta_2 \\
n_2 \sin \theta_3 &= n_1 \sin \theta_4 \\
\Rightarrow \theta_2 &= \sin^{-1}\left[\frac{n_1}{n_2} \sin \theta_1\right] \\
\theta_3 &= 60 - \theta_2
\end{aligned}
$$

Now a bit more algebra gives us θ_4. We don't want to plug in numbers until the last minute, since we have to do this calculation *twice* - once for red light and once for violet. So we will keep everything in symbols until until the last minute.

$$\sin\theta_4 \;=\; \frac{n_2}{n_1}\sin\theta_3 = \frac{n_2}{n_1}\sin 60 - \theta_2$$

$$\Rightarrow \sin\theta_4 \;=\; \frac{n_2}{n_1}\sin\left[60 - \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right)\right]$$

The deviation angle we want is just $a + b = \theta_1 + \theta_4 - 60$. We are given $\theta_1 = 50°$, so we have $a + b = \theta_4 - 10$. Plug in the values of n_2 corresponding to red and violet light to find the deviation for red and violet light, noting $n_1 = 1$:

$$
\begin{aligned}
\text{dev.} &= a + b = \theta_4 - 10 \\
\theta_{4,\text{red}} &= 58.55 \\
\Rightarrow \text{dev. red} &= 48.55 \\
\theta_{4,\text{violet}} &= 63.17 \\
\Rightarrow \text{dev. violet} &= 53.17
\end{aligned}
$$

Finally, the *angular dispersion* is the difference in deviation angles between violet and red light.

$$\text{dispersion} = (\text{dev. violet} - \text{dev. red}) = 53.17 - 48.55 = 4.62°$$
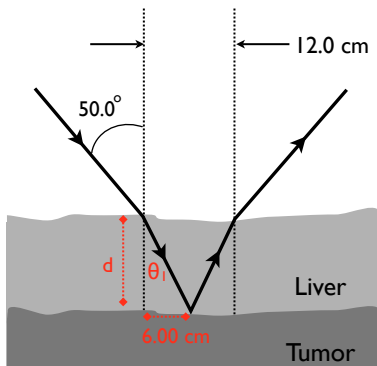
◯ **10**. A narrow beam of ultrasonic waves reflects off the liver tumor in the figure at left.

If the speed of the wave is 15.0% less in the liver than in the surrounding medium, determine the depth of the tumor.

Note: Ultrasonic waves are **NOT** *light. But! They are waves, so we can apply our optics knowledge without problem. More on this in class.*

See the modified figure. We can use Snell's law at the air-liver interface. Let n_1 be the refractive index for the surrounding medium, and n_2 be the refractive index for the liver.

$$n_1 \sin 50° = n_2 \sin\theta_1 \quad\Rightarrow\quad \sin\theta_1 = \frac{n_1}{n_2}\sin 50°$$

We can relate θ_1 and d with geometry:

$$\tan\theta_1 = \frac{6}{d} \qquad \Rightarrow \qquad d = \frac{6}{\tan\theta_1}$$

Next, we need n_1/n_2. Recall the definition of the index of refraction - it is just proportional to $1/v$, where v is the velocity in the media. Therefore, since we are told $v_2 = 0.85v_1$:
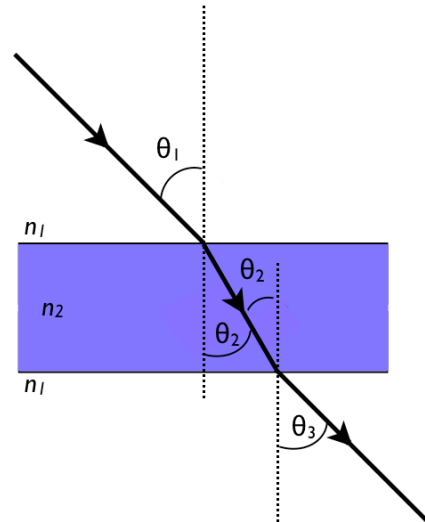
$$\frac{n_1}{n_2} = \frac{v_2}{v_1} = 0.85 \qquad \Rightarrow \qquad \theta_1 = \sin^{-1}\left[0.85\sin 50\right] \approx 40.6$$

Put what we have together ...

$$d = \frac{6}{\tan\theta_1} = \frac{6}{\tan 40.6} \approx 7\,\text{cm}$$
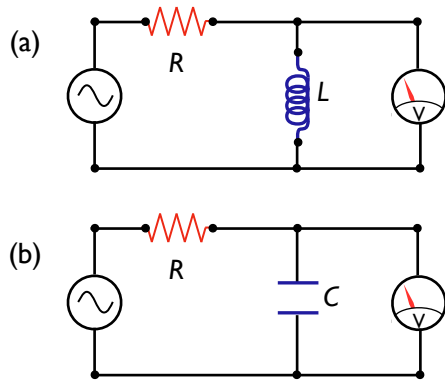
○ **11**. A light beam traveling through a transparent medium of index of refraction n_1 passes through a thick transparent slab with parallel faces and an index of refraction n_2.

Find the angle θ_3 in terms of (at most) θ_1, n_1, and n_2. Detailed calculation is not necessary if you have a solid physical argument.

Apply Snell's law to each interface:

$$n_1\sin\theta_1 = n_2\sin\theta_2 = n_1\sin\theta_3 \qquad \Rightarrow \qquad \sin\theta_1 = \sin\theta_3 \qquad \Rightarrow \qquad \theta_1 = \theta_3$$

(a)

(b)

○ **12**. A variable-frequency ac voltage source(circles with sine waves inside) is hooked up to **(a)** a resistor R and an inductor L, and **(b)** a resistor R and a capacitor C. The resistor is the same in both cases. A voltmeter monitors the voltage on the inductor in circuit **(a)**, and on the capacitor in circuit **(b)**.

Make a *rough* sketch of the relative voltage read by the meter as a function of the source frequency in each case (V versus f). Identify which one of these circuits the voltmeter preferentially reads low frequencies ("low-pass filter"), and which one the voltmeter preferentially reads high frequencies ("high-pass filter").

Hint: how does each component respond to high and low frequencies? Which one(s) dislike fast changes in voltage, which one(s) like it, and which one(s) don't care?

(a) is the high-pass filter. At high frequencies, the inductor represents a large resistance path, so high frequencies want to go to the voltmeter. At low frequencies, the inductor has very low resistance, so low frequencies want to go back to the source and not to the voltmeter.

(b) is the low-pass filter. A capacitor has a very high resistance at low frequencies, so low frequencies want to go to the voltmeter. At high frequencies, the capacitor has a low resistance, so the high frequencies want to go back to the source.
The figure below shows a low-pass filter sketch. The high pass filter would essentially the same, but with the frequency axis inverted (so the curve flattens at high frequencies, and "rolls off" at low frequencies.

low-pass filter
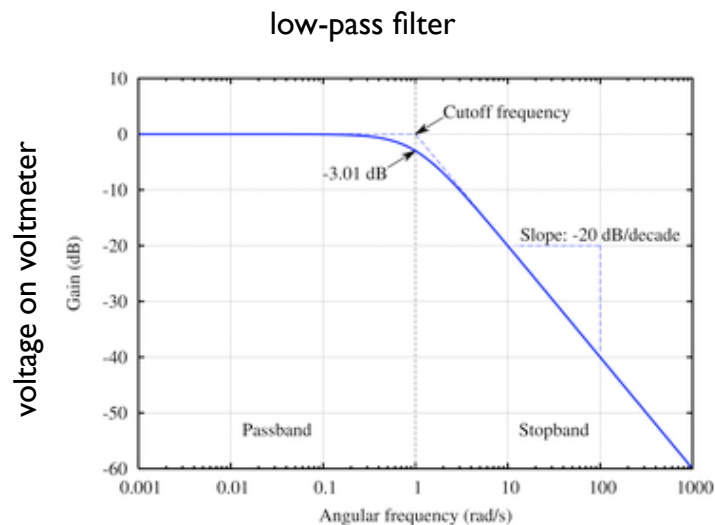
BONUS (worth $\frac{1}{2}$ a normal question): During an in-class demonstration, we dropped a magnet and a non-magnet of equal weight and size through a copper tube. The non-magnet fell through the tube at the expected rate, but the non-magnet took many times longer to fall out, due to eddy current braking.

Is it possible to have a magnet strong enough (or a tube conductive enough, *etc*) that it would actually *stop* inside the tube? Explain.

No. The eddy current braking comes from induced currents in the copper tube due to the falling magnet. The falling magnet represents a time-varying B field, which creates a time-varying flux through the copper tube. If the magnet actually stopped, there would be no eddy currents at all, and nothing to hold the magnet against gravity.

Once the magnet stops, the very force slowing it down ceases to exist. The flux in the tube is changing only because the magnet has some non-zero velocity. No emf, and therefore no eddy currents result from a stationary magnet giving a constant flux through the tube.

Putting it another way: the force is due to the relative velocity of the magnet and the charges in the copper. The magnetic force is $F = qvB$, where v is the *relative* velocity o f the tube and magnet. If $v = 0$, there is no force - so if the magnet could actually be stopped, the force holding it up would go to zero, and it would fall again! Clearly, the answer is no.