Exam II SOLUTION

□ **1.** In a motorboat, the compass is mounted at a distance of $0.80\,\text{m}$ from a cable carrying a current of $20\,\text{A}$ from an electric generator to a battery.
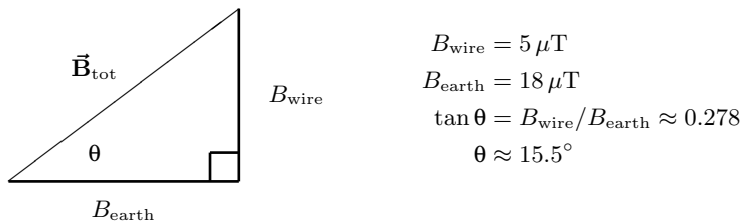
(a) What magnetic field does this current produce at the location of the compass? Assume the cable is a long, straight wire.

At a distance of $d=0.8\,\text{m}$ from the wire, the field must be:

$$
\begin{aligned}
B &= \frac{\mu_0 I}{2\pi d} \\
&= \frac{4\pi \times 10^{-7} \cdot 20}{2\pi \cdot 0.8} \\
&\approx 5\,\mu\text{T}
\end{aligned}
$$

(b) The horizontal (north) component of the Earth's magnetic field is $1.8 \times 10^{-5}\,\text{T}$. Since the compass points in the direction of the *net* horizontal magnetic field, the current will cause a deviation of the compass needle. Assume that the magnetic field of the current is horizontal and at a right angle to the horizontal component of the earth's magnetic field. Under these circumstances, by how many degrees will the compass deviate from true north?

Without the field of the wire, the earth's magnetic field defines true north. Adding the wire's magnetic field to that, the compass will point along the resulting magnetic vector. Thus, we need to find the direction of the total magnetic field relative to the earth's field:

$$
\begin{aligned}
B_{\text{wire}} &= 5\,\mu\text{T} \\
B_{\text{earth}} &= 18\,\mu\text{T} \\
\tan\theta = B_{\text{wire}}/B_{\text{earth}} &\approx 0.278 \\
\theta &\approx 15.5^\circ
\end{aligned}
$$

□ **2.** A $50\,\text{kV}$ direct-current power line consists of two conductors precisely $2.0\,\text{m}$ apart. The current travels down one conductor, to a load, and back up the other conductor.

(a) When this line is transmitting $10\,\text{MW}$, how strong is the magnetic field midway between the conductors?

Since the currents are running antiparallel to each other, they will reinforce each other (as opposed to canceling each other). You should be able to verify this by using the right hand rule and drawing the field lines around each wire - at the midpoint between the wires, the field from both wires is in the same direction, perpendicular to a line connecting the two wires. Since the currents are of the same magnitude, if we can find the field from one, the total field is just twice that. In order to find the field, we need the current, which we can get from the given power and voltage:

$$
I = \frac{\mathscr{P}}{\Delta V} = \frac{10 \times 10^6\,\text{W}}{50 \times 10^3\,\text{V}} = 200\,\text{A}
$$

We want to find the field at the midpoint between the two wires. If the distance between the wires is $d=2.0\,\text{m}$, then the midpoint is a distance of $2.0\,\text{m}/2=1.0\,\text{m}$ from either wire. The field due to one current-carrying wire at a distance of $d/2=1.0\,\text{m}$ is found as above:

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi \, (d/2)} \approx 40 \, \mu\text{T}$$

The total field due to both wires is just twice that, or $80 \, \mu\text{T}$.

(b) What is the force per unit length between the two wires? Is it attractive or repulsive?

The force between the wires is repulsive, since the currents are antiparallel. Its magnitude per unit length of wire is readily found, noting that now we want the full distance between the wires, d, and the current in both wires is I:

$$\frac{|\vec{\mathbf{F}}|}{l} = \frac{\mu_0 I^2}{2\pi d} \approx 4 \, \text{mN}$$

□ **3.** The rate of flow of a conducting liquid can be measured with an electromagnetic flowmeter that detects the voltage induced by the motion of the liquid in a magnetic field. Suppose that a plastic pipe of diameter $0.10\,\text{m}$ carries beer with a speed of $1.5\,\text{m/s}$. The pipe is in a transverse magnetic field (*i.e.,* perpendicular to the pipe axis) of about $1.5 \times 10^{-2}\,\text{T}$.

(a) Presume the beer is an ideal conductor. What voltage will be induced between the opposite sides of the column of liquid?

This is in the end just a motion-induced voltage - see problem 1 in chapter 8 of the notes, it is basically the same thing. The presence of a magnetic field perpendicular to the movement of ions in the beer means that they experience a magnetic force.

$$F_m = qvB$$

If the flow of beer is from left to right, and the magnetic field is pointing into the page, then the force for positive ions is pointing up, and for negative ions it is pointing down. This serves to separate spatially the positive and negative charges along the diameter of the pipe. This continues until the ions reach the surface of the pipe, at which point they are separated by its diameter d.

If the positive and negative charges are separated spatially by a distance d, then this gives rise to a (uniform) electric field E, and a potential difference $\Delta V = Ed$. At equilibrium, the electric and magnetic forces are balanced - the magnetic force pulls the charges apart, and it is perfectly counterbalanced by the resulting electric force. Set the magnetic and electric fields equal to one another, use the expression for ΔV, and take care with units:

$$
\begin{aligned}
F_m &= qvB = F_e = qE = q\left(\frac{\Delta V}{d}\right) \\
\Rightarrow \Delta V &= Bvd \approx 2.25 \, \text{mV}
\end{aligned}
$$

(b) Does it matter whether the conductivity of the beer is due to mobile positive or negative charges?

You can verify that the sign of the potential difference does *not* depend on whether the ions are mostly positive or negative, since the force is in different directions for each ion. No matter what, positive ions go up, negative ions go down, and the same potential difference results.

□ **4.** The circuit below shows what is known as a *bandpass filter,* it may be thought of as two simple filters in series.

(a) Sketch the output of this circuit versus frequency, assuming the input contains all frequencies with equal amplitudes.

This is the same circuit as the combined filter of Homework 8, problem 2.

(b) Let $R_1 = 200\,\Omega$, $R_2 = 100\,\Omega$, $C_1 = 0.5\,\mu\text{F}$, and $C_2 = 0.01\,\mu\text{F}$. Determine the upper and lower cutoff frequencies for this filter **in Hz**, and point them out on your sketch.

The cutoff frequencies are determined by the product RC, noting that $\omega = 2\pi f = 1/\tau$:
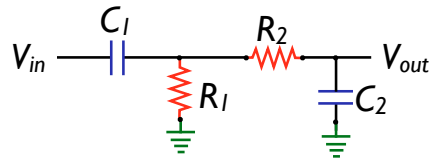
$$f_1 = \frac{1}{2\pi R_1 C_1} \approx 1590\,\text{Hz}$$

$$f_2 = \frac{1}{2\pi R_2 C_2} \approx 159\,\text{kHz}$$

(c) Could this circuit be replaced by one with a single resistor and capacitor? Explain your reasoning briefly.

No. Any circuit we could make with a single resistor and capacitor would at best form a single low- or high- pass filter with a single cutoff frequency.

□ **5.** The circuit below is a simple filter based on an inductor and a capacitor.
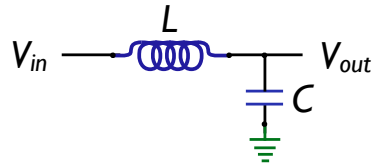
(a) What sort of filter is this? Briefly explain your rationale.

This is a low-pass filter. High frequencies will see the capacitor as an easy path to ground, while low frequencies will avoid the capacitor and reach the output.

(b) Sketch the output of this circuit versus frequency, assuming the input contains all frequencies with equal amplitudes.

It looks basically like any other low-pass filter ... *e.g.,* an RC low-pass filter.

(c) The cutoff frequency of a filter is the frequency at which the reactance of each component is equal - in this case, the frequency at which the inductive reactance is equal to the capacitive reactance. What is the cutoff frequency f in terms of L and C?

The cutoff frequency is the one at which the inductor's and the capacitor's reactances are equal:

$$X_L = X_C \qquad \text{at cutoff frequency } f_c$$

$$2\pi f_c L = \frac{1}{2\pi f_c C}$$

$$4\pi^2 f_c^2 L = \frac{1}{C}$$

$$f_c^2 = \frac{1}{4\pi^2 LC}$$

$$\implies \quad f_c = \frac{1}{2\pi\sqrt{LC}}$$

□ **6.** A scuba diver is underwater; from there, the sun appears to be at $30°$ from the vertical. At what actual angle is the sun located (with respect to directly overhead)? The index of refraction of water may be taken as $n_{\text{water}} = 1.33$, while that of the air may be taken as $n_{\text{air}} = 1.00$.

The angle the scuba diver appears to see is the *refracted* angle, where the rays would appear to be coming from if they weren't refracted at the air-water interface. We can apply Snell's law to find the actual angle of the sun with respect to the surface normal of the water (which is with respect to directly overhead).

$$n_{\text{water}} \sin 30° = n_{\text{air}} \sin \theta_i$$

$$\implies \quad \theta_i \approx 42°$$

□ **7.** Two flat mirrors are arranged at an angle of $145°$ with respect to each other (see below). A ray of light strikes one of the mirrors at an angle of incidence of $60°$, it is reflected, and then strikes the other mirror. What is the angle of reflection at the second mirror, relative to a line perpendicular to the second mirror?
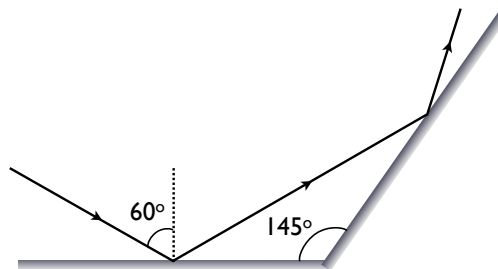
See the revised figure below. Nothing more than geometry, noting that the incident and reflected angles are always equal, and the angles of a triangle add up to $180°$.
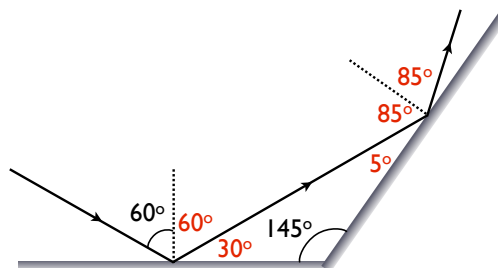
□ **8.** A point source of light is placed at a fixed distance l from a screen. A thin convex lens of focal length f is placed somewhere between the source and screen, a distance q from the screen and p from the source. The lens is moved back and forth between the source and screen, but both screen and source remain fixed, thus $p + q = l$ at all times.

What is the minimum value of l such that a focused image will be formed at two different positions of the lens? Recall our recent laboratory experiment.

What we are basically told is that $p + q = l$ at all times. We can use this along with the lens equation to come up with a set of solutions for q in terms of l and f - we will get a quadratic, and we will be able to readily see what conditions give two, one, or no real solutions.

$$
\begin{aligned}
& l = p + q \\
& \frac{1}{f} = \frac{1}{p} + \frac{1}{q} \\
\Longrightarrow \quad & \frac{1}{q} = \frac{1}{f} - \frac{1}{p} \\
& \frac{1}{q} = \frac{1}{f} - \frac{1}{l - q} \\
& \frac{1}{q} = \frac{l - q}{f\,(l - q)} - \frac{f}{f\,(l - q)} \\
& \frac{1}{q} = \frac{l - q - f}{f\,(l - q)}
\end{aligned}
$$

Now we have an equation purely in terms of l, q, and f, which we can readily solve for q. Start by cross-mulitplying.

$$
\begin{aligned}
f\,(l - q) &= q\,(l - q - f) \\
fl - fq &= ql - q^2 - qf \\
q^2 - lq + fl &= 0 \\
\Longrightarrow \quad q &= \frac{l \pm \sqrt{l^2 - 4fl}}{2}
\end{aligned}
$$

From the solution to the quadratic above, we can see that there are two real image positions when the factor under the square root is positive, when $l^2 > 4fl$ or $l > 4f$. When the length l is exactly four times the focal length, $l = 4f$, there is only one solution to the quadratic. Thus, the critical position is when $l = 4f$, which results in $q = \frac{l}{2} = p$.

BONUS: *worth 1/3 of a normal question*
During an in-class demonstration, we dropped a magnet and a non-magnet of equal weight and size through a copper tube. The non-magnet fell through the tube at the expected rate, but the non-magnet took many times longer to fall out, due to eddy current braking. Is it possible to have a magnet strong enough (or a tube conductive enough, etc) that it would actually stop inside the tube? Explain.

Chapter 8 of the notes, problem 4.