Name ______________________________

FINAL EXAM SOLUTIONS

1. A spherical surface surrounds a point charge q. Describe what happens to the total flux through the surface if the volume of the sphere is doubled.

- □ The flux is tripled.
- □ The flux decreases by 1/3.
- ■ The flux remains constant.
- □ The flux goes to zero.

If the volume doubles, nothing happens to the flux. Let's say the initial radius of the sphere is r. Then the flux is:

$$\Phi_E = EA = \frac{k_e q}{r^2} \cdot 4\pi r^2 = 4\pi k_e q$$

The result is independent of the radius of the sphere. If we double the volume, the radius must increase by $\sqrt[3]{2}$, but it doesn't matter. Surface area increases as r^2 and electric field decreases as r^{-2}, the two dependencies cancel each other out.

2. A test charge of $+3\,\mu\text{C}$ is at a point P where the electric field due to the other charges is directed to the right and has a magnitude of 4×10^6 N/C. If the test charge is replaced with a charge of $-3\,\mu\text{C}$, what happens to the electric field at P?

- □ The field increases in magnitude and changes direction.
- □ The field decreases in magnitude and changes direction.
- □ The field has the same magnitude as before, but changes direction.
- ■ The field remains the same.

The magnitude of the electric field depends only on the charge sourcing it. It does not depend on the magnitude of the (fictitious) test charge.

3. If you are given three different capacitors, C_1, C_2, C_3, how many different combinations of capacitance can you produce, **using all the capacitors in each of your circuits**?

- □ 17
- ■ 8
- □ 12
- □ 14

If you have to use all three at once, there are only 8 ways. All three can be in series, or all three can be in parallel giving two unique combinations. There are three ways to have 2 in series with one in parallel with that combination, and three ways to have two in parallel with one in series with that combination.

4. A wire carries a current of 1.6 A. Roughly how many electrons per second pass a given point in the wire?

- ■ 10^{19}
- □ 10^{17}
- □ 10^{20}
- □ 10^{21}

We just need the definition of current for this:

$$I = \frac{\Delta Q}{\Delta t} = 1.6\,\text{A} = 1.6\,\text{C/s}$$

$$1.6\,\text{C/s}\left(\frac{1\,e^-}{1.6 \times 10^{-19}\,\text{C}}\right) \approx 1 \times 10^{19}\,e^-/\text{s}$$

Name ____

5. Four point charges are positioned on the rim of a circle. The charge on each of the four is $+0.5\,\mu$C, $+1.5\,\mu$C, $-1.0\,\mu$C, and $-0.5\,\mu$C. If the electrical potential at the center of the circle due to the $+0.5\,\mu$C charge alone is 4.5×10^4 V, what is the total electric potential at the center due to the four charges?

- ▫ 9.0×10^4 V
- ▫ 0
- ▫ -4.5×10^4 V
- ▫ 18.0×10^4 V
- ■ 4.5×10^4 V

The potential at the center of the circle can be found by superposition. Electric potential is a scalar (only magnitude, no direction), so the potentials from each charge individually will just add together. Since all charges are the same distance away, positive and negative charges of equal magnitude will cancel each other out. All we need to do is find out the net amount of charge on the ring, which is $(0.5+1.5-1-0.5)\,\mu\text{C}=+0.5\,\mu$C. We are told the potential due to a single charge of this magnitude, the potential due to a net charge of this magnitude is the same.

6. A hanging Slinky® toy is attached to a powerful battery and a switch. When the switch is closed so that the toy now carries current, does the Slinky compress or expand?

- ■ It will compress.
- ▫ It will expand.
- ▫ It will neither compress nor expand, but will heat up.
- ▫ It will not be affected.

If you are not familiar, a Slinky® is basically just a spring. If a battery is attached, current will run through the coils of the Slinky®. The currents between neighboring coils will be parallel, and thus have an attractive force between them. As a result, the Slinky® will compress.

7. Which of the following statements are true about light waves? Mark all that apply.

- ▫ The higher the frequency, the longer the wavelength.
- ▫ Higher frequency light travels faster than lower frequency light.
- ▫ The lower the frequency, the shorter the wavelength.
- ■ The lower the frequency, the longer the wavelength.
- ■ The shorter the wavelength, the higher the frequency.

Remember that for any light wave, $\lambda f = c$. If frequency decreases, wavelength must increase and *vice versa.* The speed is always the same (in a given medium).

8. Myopia, also called near- or short-sightedness, is a refractive defect of the eye in which collimated light produces image focus in front of the retina when accommodation is relaxed, rather than directly on the retina. What sort of lens(es) could be used to correct this condition?

- ▫ convex
- ▫ it depends on the degree of myopia
- ■ concave

From the wikipedia: "With myopia, the eyeball is too long, or the cornea is too steep, so images are focused in the vitreous inside the eye rather than on the retina at the back of the eye." Thus, we want to push the focal point back farther to the retina - we want to diverge the rays just a little bit to push the focal length back farther. For this we want a diverging lens, and a concave does nicely. See. `http://en.wikipedia.org/wiki/Myopia` for more information.

9. If the current carried by a conductor is doubled, what happens to the average time between collisions?

- ■ Nothing.
- □ It doubles.
- □ It decreases by two times.
- □ It increases by 4 times.
- □ It decreases by 4 times.

The average time between collisions is governed primarily by temperature and the resulting thermal velocity, which is many orders of magnitude larger than the drift velocity induced by the applied voltage. Changing the current does not appreciably change the collision time.

10. In semiconductors such as Si, the number of carriers is not fixed, it depends on *e.g.,* temperature. For a certain sample of Si, the number of carriers doubles but their drift velocity decreases by 10 times. By how much does the sample's resistance change?

- □ 2 times lower
- □ 5 times lower
- ■ 5 times higher
- □ 2 times higher

First, we recall the relation between *current* and drift velocity:

$$I = nqAv_d$$

What we are really after is the resistance, however, which we can find with Ohm's law:

$$R = \frac{\Delta V}{I} = \frac{\Delta V}{nqAv_d} \propto \frac{1}{nv_d}$$

So the resistance is inversely proportional to the carrier density and drift velocity. Let's say the initial resistance is R_0, and the resistance after changing n and v_d is just R. If we increase the number of carriers by 2 times, the resistance goes *down* by 2 times. If we decrease the drift velocity by 10 times, the resistance goes *up* by 10 times.

$$\begin{aligned} R_o &\propto \frac{1}{nv_d} \\ R &\propto \frac{1}{(2n)\left(\frac{v_d}{10}\right)} = \frac{1}{\frac{nv_d}{5}} = \frac{5}{nv_d} \\ \implies R &= 5R_o \end{aligned}$$

Even though we don't know what the actual resistance R_0 is, we can say that R is five times more. The one tricky step here is to write down the proper relationship between *resistance* and the given quantities, not just the relationship between *current* and the given quantities.

11. A "free" electron and a "free" proton are placed in an identical electric field. Which of the following statements are true? *Check all that apply.*

- □ Each particle is acted on by the same electric force and has the same acceleration.
- □ The electric force on the proton is greater in magnitude than the force on the electron, but in the opposite direction.
- ■ The electric force on the proton is equal in magnitude to the force on the electron, but in the opposite direction.
- ■ The magnitude of the acceleration of the electron is greater than that of the proton.
- □ Both particles have the same acceleration.

Name

The electric force is the same in magnitude because both the proton and electron have the same magnitude of charge. Since they have different signs, though, the forces are in opposite directions. For the same force, the electron experiences a larger acceleration because it is much lighter than the proton.

12. Which one of these things can two observers in different frames **not** agree on?

- □ Their relative speed of motion with respect to each other.
- □ The speed of light c.
- □ The simultaneity of two events taking place at the same position and same time in some frame.
- ■ The distance between two points that remain fixed in one of their frames.

Let's go through the choices one by one. First, the relative speed of motion is the same from either reference frame, so long as no one is accelerating. Both will agree on this. They will also agree on the speed of light - it is an invariant constant, independent of reference frame.

The third choice is trickier: observers in different frames cannot *generally* agree on simultaneity in an arbitrary sense. The one case in which observers in motion can agree on this is when the two events are not spatially separated - *i.e.,* the events take place at the same position in one frame. If the events take place in the same position in one frame, this will be true in all frames. You can see this from the formula for elapsed time:

$$\Delta t' = \gamma \left(\Delta t - \frac{v\Delta x}{c^2} \right)$$

For two events to be simultaneous for both observers, we need $\Delta t = \Delta t' = 0$. This says both observers see the events happen at the same time, there is no time interval between them. For this to be true, based on the equation above, we must also have $\Delta x = 0$, *i.e.,* the events happen at the same location according to one observer. If $\Delta x = 0$ in one frame, it is true in the other as well - length contracting zero just gives you zero again. Thus, as stated, the third choice is valid for both observers.

Finally, for the last choice, the distance between two points fixed with respect to one observer is just a *proper length* measured by that observer. An observer in relative motion cannot agree on this length, they would see a contracted length. If nothing else, by elimination, the third choice must be correct.

13. An electron in a television picture tube moves with $v = 0.250c$. What is its kinetic energy in electron volts? Note that the rest energy of an electron is $m_e c^2 = 0.511\,\text{MeV}$

- □ $0.528\,\text{MeV}$
- □ $0.511\,\text{MeV}$
- ■ $0.017\,\text{MeV}$
- □ $0.253\,\text{MeV}$

Recall that the kinetic energy of a relativistic particle is

$$KE = (\gamma - 1)\, mc^2 = (\gamma - 1)\, E_R$$

Thus, we just need the γ factor and the given rest energy:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 0.25^2}} \approx 1.033$$

$$\Longrightarrow \quad KE = (1.033 - 1)\,(0.511\,\text{MeV}) \approx 0.017\,\text{MeV}$$

Name

14. Can the electron in the ground state of hydrogen absorb a photon of energy less than 13.6 eV? Can it absorb a photon of energy greater than 13.6 eV?

- ■ yes; yes
- □ yes; no
- □ no; yes
- □ no; no

A photon of energy greater than 13.6 eV will simply cause the electron to leave the hydrogen atom - ionization. A photon of less energy will excite a transition of an electron between two different energy levels.

15. After a plant or animal dies, its ^{14}C content decreases with a half-life of 5730 years. If an archaeologist finds an ancient firepit containing partially consumed firewood, and the ^{14}C content of the wood is only 10.0% that of an equal carbon sample from a present-day tree, what is the age of the ancient site?

- □ 24000 years
- ■ 19000 years
- □ 12000 years
- □ 7500 years
- □ 3700 years

The half life equation reads thusly:

$$N(t) = N_0 \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

If the ^{14}C content is only 10% of what it initially was, then $N(t)/N_0 = 0.1$. Using this and the given half life:

$$\begin{aligned} 0.1 &= \left(\frac{1}{2}\right)^{t/5730} \\ \ln 0.1 &= \frac{t}{5730} \ln\frac{1}{2} \\ t &= 5730 \left(\frac{\ln 0.1}{\ln\frac{1}{2}}\right) \approx 19000\,\text{years} \end{aligned}$$

16. Identify the unknown species in the following reaction: $\mathrm{X} +^4_2 \mathrm{He} \longrightarrow^{24}_{12} \mathrm{Mg} +^1_0 \mathrm{n}$

- □ $^{23}_{11}$Na
- □ $^{18}_{8}$O
- □ $^{20}_{9}$F
- ■ $^{21}_{10}$Ne

We just need to balance the atomic number Z (bottom numbers) and mass number A (top number) on either side:

$$\begin{aligned} Z: &\quad 2+? \longrightarrow 12+1 \quad \Longrightarrow ? = 10 \\ A: &\quad 4+? \longrightarrow 24+1 \quad \Longrightarrow ? = 21 \end{aligned}$$

Thus, the unknown element must have atomic number 10, meaning Ne, and mass 21: $^{21}_{10}$Ne

17. The top of a swimming pool is at ground level. If the pool is 2.3 m deep, how far below ground level does the bottom of the pool appear to be located when the pool is completely filled with water? Presume you are viewing the water at normal incidence. The index of refraction of water is $n = 1.333$.

Name

- ■ 1.73 m below ground level
- □ 2.01 m below ground level
- □ 1.55 m below ground level
- □ 1.27 m below ground level

Normal incidence means looking straight down, perpendicularly to the surface of the water. For normal incidence on a flat refracting surface,

$$q = -\frac{n_2}{n_1} p$$

Our "object" would be the depth of the pool p, and the "image" would be the apparent depth q. In this case, we would consider n_1 to be the water, since that is where the "object" resides, and n_1 the air, since that is where the observer resides.

$$q = -\frac{1}{1.33}(2.3) \approx -1.73\,\text{m}$$

Here the negative sign just indicates that the apparent bottom of the pool is on the same side of the water-air interface that the real bottom is.

18. An unstable particle at rest breaks up into two fragments of unequal mass. The mass of the lighter fragment is 2.10×10^{-28} kg and that of the heavier fragment is 1.64×10^{-27} kg. If the lighter fragment has a speed of $0.893c$ after the breakup, what is the speed of the heavier fragment?

- □ $0.192c$
- □ $0.781c$
- ■ $0.246c$
- □ $0.531c$

The unstable particle at rest starts out with zero momentum in its own reference frame. Conservation of momentum dictates that the total momentum of the two decay products must also be zero, and therefore the two decay products must have the same momentum. Since the speed of the lighter particle is close to c, we will need relativity. Call the lighter particle 1, and the heavier 2.

$$\begin{aligned}
\gamma_1 m_1 v_1 &= \gamma_2 m_2 v_2 \\
\gamma_1 v_1 &= \frac{\gamma_2 m_2 v_2}{m_1} \approx 0.254c \\
\frac{v_1}{\sqrt{1 - v_1^2/c^2}} &\approx 0.254c \\
v_1 &\approx 0.254c\sqrt{1 - v_1^2/c^2} \\
v_1^2 &\approx (0.254c)^2 \left(1 - v_1^2/c^2\right) \\
v_1^2 &\approx (0.254c)^2 - 0.254^2 v_1^2 \\
v_1 &\approx \sqrt{\frac{0.254^2}{1 + 0.254^2}}\, c \approx 0.246c
\end{aligned}$$

Name ______________________________

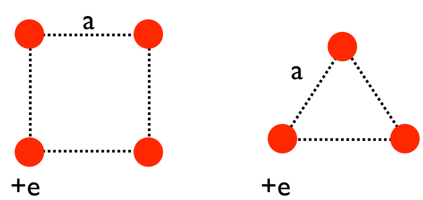
Part II: Problems (50%).

Instructions:

- Answer the indicated number of problems in each section. All problems have equal weight.
- Indicate which problems you have attempted by filling in the adjacent box.
- Show your work for full credit. Significant partial credit will be given.

Electric forces, fields, and energy: **solve 2 of 5.**

□ **1.** A single atomic layer of singly-charged ions (charge $+e$) can be arranged on a neutral insulating surface in one of two ways: either as a square or a triangular lattice. For four ions in the former configuration, and three in the latter, calculate the potential energy *per unit charge.* Which lattice is more stable?



The potential energy of the whole lattice is found by summing up the potential energies of every pair of charges in the system. For the square lattice, we have six unique pairs: four between adjacent corners (distance a), and two between opposite corners (distance $a\sqrt{2}$). Number the charges 1-4 clockwise around the square.

$$\begin{aligned}
PE_{\square} &= \sum_{\substack{\text{unique pairs}\\ ij}} \frac{k_e q_i q_j}{r_{ij}} \\
&= \frac{k_e q_1 q_2}{a} + \frac{k_e q_2 q_3}{a} + \frac{k_e q_3 q_4}{a} + \frac{k_e q_4 q_1}{a} + \frac{k_e q_1 q_3}{a\sqrt{2}} + \frac{k_e q_2 q_4}{a\sqrt{2}} \\
&= 4 \cdot \frac{k_e e^2}{a} + 2 \cdot \frac{k_e e}{a\sqrt{2}} \\
&= \frac{k_e e^2}{a}\left[4 + \frac{2}{\sqrt{2}}\right] \\
&= \frac{k_e e^2}{a}\left[4 + \sqrt{2}\right]
\end{aligned}$$

Since there are four charges, the potential energy per unit charge is

$$PE_{\square}/\text{charge} = \frac{k_e e^2}{a}\left[1 + \frac{\sqrt{2}}{4}\right] \approx 1.35 \frac{k_e e^2}{a}$$

For the triangular lattice, we proceed in the same way, but now there are only three possible pairings of charges (1-2, 2-3, 1-3), and they are all the same distance apart:

$$\begin{aligned}
PE_{\triangle} &= \sum_{\substack{\text{unique pairs}\\ ij}} \frac{k_e q_i q_j}{r_{ij}} \\
&= \frac{k_e q_1 q_2}{a} + \frac{k_e q_2 q_3}{a} + \frac{k_e q_3 q_1}{a} \\
&= 3 \frac{k_e e^2}{a}
\end{aligned}$$

Since there are three charges, the potential energy per unit charge is just

Name ____________________

$$PE_{\triangle}/\text{charge} = \frac{k_e e^2}{a}$$

This energy is smaller than that of the square lattice, which means that the triangular lattice is the more stable one. Neither is truly *stable* though - since the energy is positive, it is more favorable to not have the lattice at all, but separate the charges. This is not unexpected, however, since all charges are positive.

□ **2.** Two capacitors, one charged and the other uncharged, are connected in parallel. **(a)** Prove that when equilibrium is reached, each carries a fraction of the initial charge equal to the ratio of its capacitance to the sum of the two capacitances. **(b)** Show that the final energy is less than the initial energy, and derive a formula for the difference in terms of the initial charge and the two capacitances.

□ **3.** A circular ring of charge of radius a has a total positive charge Q distributed uniformly around it. The ring is in the $x=0$ plane with its center at the origin. What is the electric field (both magnitude and direction) along the x axis at an arbitrary point $x=b$ due to the ring of charge? *Hint: Consider the total charge Q to be made up of many pairs of identical charges placed on opposite points on the ring.*

□ **4.** Three capacitors of 2, 4, and 6 μF, respectively, are connected in series, and a potential difference of 200 V is established across the whole combination by connecting the free terminals to the battery. **(a)** Calculate the charge on each capacitor. **(b)** Find the potential difference across each capacitor.

□ **5.** Three equal positive charges, each of magnitude Q, are held fixed at the corners of a square of side a. **(a)** Find the magnitude and direction of the electric field at the fourth corner. **(b)** Find the potential at the fourth corner. **(c)** How much work would be done in moving a fourth charge q to the fourth corner. *Hint: What is the change in the energy of the system?*

Current, resistance, and dc circuits: **solve 2 of 4.**

□ **1.** You are given two batteries, one of 9 V and internal resistance 0.50 Ω, and another of 3 V and internal resistance 0.40 Ω. How must these batteries be connected to give the largest possible current through an external 0.30 Ω resistor? What is this current?

□ **2.** You are given a voltage source, several very precise resistors, and two very accurate voltmeters. The voltmeter has an internal resistance of 100 MΩ. Design a circuit to determine accurately the resistance of an unknown specimen, and estimate an upper limit on the specimen's resistance which will maintain 5% accuracy.

□ **3.** If the voltage at the terminals of an automobile battery drops from 12.3 to 9.8 V when a 0.5 Ω resistor is connected across the battery, what is the internal resistance of the battery?

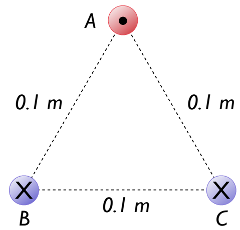
□ **4.** An aluminum wire with a cross-sectional area of $4.00 \times 10^{-6}\,\text{m}^2$ carries a current of 5.00 A. Find the drift speed of the electrons in the wire. The density of aluminum is $2.70\,\text{g/cm}^3$; assume each Al atom provides a single electron for conduction. *Hint: how many atoms per unit volume are there? Use your periodic table.*

Magnetism & induction: **solve 2 of 4.**

□ **1.** **(a)** What is the velocity of a beam of electrons which move undeflected in a region of space in which there exist both a uniform electric field $|\vec{E}|=3.4\times10^5$ V/m and a uniform magnetic field of $|\vec{B}|=2\times10^{-3}$ T? **(b)** Show the orientation of the vectors $\vec{v}$, $\vec{E}$, and $\vec{B}$ in a diagram. **(c)** What is the radius of the electron orbit when the electric field is removed, and only the magnetic field remains? You may ignore relativistic effects.

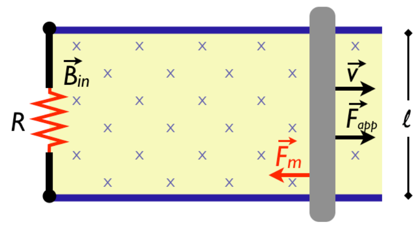
□ **2.** Three long parallel wires pass through the corners of an equilateral triangle of side 0.1 m and are perpendicular to the plane of the triangle. Each wire carries a current of 15 A, the current being into the page for wires B and C, and out of the page for A. **(a)** Find the force per unit length acting on the wire A. **(b)** Sketch the direction of the forces and their resultant.

Name ____



□ **3.** A magnetic field of 0.200 T exists within a solenoid of 500 turns and a diameter of 10 cm. How rapidly (*i.e.,* in what period of time) must the field be reduced to zero, if we want the average induced voltage within the coil during this time interval to be 10 kV? Presume that the field reduces uniformly.

□ **4.** A conducting rod of length l moves on two (frictionless) horizontal rails, as shown to the right. A constant force of magnitude $|\vec{F}_{\text{app}}| = 1.0\,\text{N}$ moves the bar at a uniform speed of $|\vec{v}| = 2.0\,\text{m/s}$ through a magnetic field $\vec{B}$ directed into the page. The resistor has a value $R = 8.0\,\Omega$.**(a)** What is the current through the resistor R? **(b)** What is the mechanical power delivered by the constant force?



ac Circuits & EM waves: **solve 1 of 3.**

□ **1.** A helium-neon laser delivers 1.05×10^{18} photons/sec in a beam diameter of 1.75 mm. Each photon has a wavelength of 601 nm. **(a)** Calculate the amplitudes of the electric and magnetic fields inside the beam. **(b)** If the beam shines perpendicularly onto a perfectly reflecting surface, what force does it exert?

□ **2.** Using resistors, capacitors, and inductors, design a two-stage filter that predominantly eliminates frequencies below 30 Hz and above 200 Hz. That is, below a lower cutoff frequency of 30 Hz and above an upper cutoff frequency of 200 Hz signals should NOT pass through the filter. Specify possible values of your components, and sketch the filter's circuit diagram and its frequency response.

□ **3.** An audio amplifier delivers to a speaker alternating voltage at audio frequencies. If the source voltage has an amplitude of 15.0 V and an internal resistance of 8.20 Ω, and the speaker can be considered equivalent to a 10.4 Ω resistor, what is the time-averaged power transferred to it? The source, its internal resistance, and the speaker are in series.

Optics: **solve 2 of 4.**

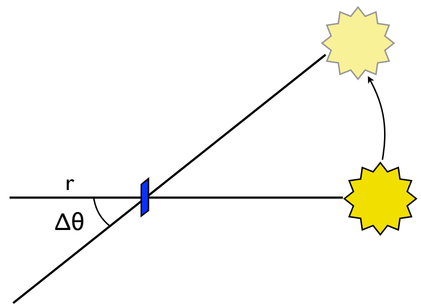
□ **1.** The walls of a prison cell are perpendicular to the four cardinal compass directions. On the first day of spring, light from the rising Sun enters a rectangular window in the eastern wall. The light traverses 2.57 m horizontally to shine perpendicularly on the wall opposite the window. A prisoner observes the patch of light moving across this western wall and for the first time forms his own understanding of the rotation of the Earth. **(a)** With what speed does the illuminated rectangle move? **(b)** The prisoner holds a small square mirror flat against the wall at one corner of the rectangle of light. The mirror reflects light back to a spot on the eastern wall close beside the window. How fast does the smaller square of light move across that wall?

The sun appears to move at an angular velocity ω, which means that it moves through an angular displacement $\Delta\theta$ in a time Δt: $\omega = \Delta\theta/\Delta t$. We know the rotation rate of the sun: it goes through a full circle of 2π radians in 24 hours:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\,\text{rad}}{86400\,\text{s}} \approx 7.27 \times 10^{-5}\,\text{rad/s}$$

Name

The light streaming through the prison window will move through an angle $\Delta\theta$ as shown below as the sun moves through the sky:



If the distance the light covers along the wall is s, then it is clear that $s=\Delta\theta r$. The rate at which the spot moves is $\Delta s/\Delta t$. Since $r=2.37\,\mathrm{m}$ is constant, the rate is just $r\Delta\theta/\Delta t$:

$$\frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta} = r\omega = (2.37\,\mathrm{m})\left(7.27\times10^{-5}\,\mathrm{rad/s}\right) \approx 0.172\,\mathrm{mm/s}$$

If the prisoner uses a mirror, the path length of the light is simply doubled, as if the room were twice as wide, so a given angular displacement $\Delta\theta$ results in twice as large a lateral displacement s, and twice the apparent speed $\Delta s/\Delta t$. Thus, for the second case, we have just $0.345\,\mathrm{mm/s}$.

□ **2.** Object O_1 is 14.8 cm to the left of a converging lens with a 9.0 cm focal length. A second lens is positioned 10.0 cm to the right of the first lens and is observed to form a final image at the position of the original object, O_1. **(a)** What is the focal length of the second lens? **(b)** What is the overall magnification of this system? **(c)** What is the nature (i.e., real or virtual, upright or inverted) of the final image?

No, your exam will not have a question this hard. $-8.51\,\mathrm{cm}$; 2.97 times; virtual and upright.

□ **3.** Prove that if two thin lenses are placed in contact, they are equivalent to a single lens of focal length

$$f_{\mathrm{equiv}} = \frac{f_1 f_2}{f_1 + f_2}$$

where f_1 and f_2 are the focal lengths of the two thin lenses. In some sense, lenses in series add like capacitors do.

The image from the first lens serves as the object of the second. For the first lens,

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1} \tag{1}$$

The second lens sees the image created by the first lens, so $p_2=|q_1|$. However, q_1 will be on the wrong side of the lens if the two are in contact! Thus, $p_2=-q_1$

$$-\frac{1}{q_1} + \frac{1}{q_2} = \frac{1}{f_2} \tag{2}$$

Noting $1/q_1=1/f_1-1/p_1$,

$$-\frac{1}{f_1} + \frac{1}{p_1} + \frac{1}{q_2} = \frac{1}{f_2} \tag{3}$$

Name

The focal length is defined as the point at which the final image would form if the object were infinitely distant. That is, let p_1 tend toward infinity, and the position of the final image, q_2, is the equivalent focal length. Thus:

$$\frac{1}{q_2} = \frac{1}{f_2} + \frac{1}{f_1} - \frac{1}{p_1} \tag{4}$$

$$\frac{1}{f_{\text{equiv}}} = \frac{1}{f_2} + \frac{1}{f_1} = \frac{f_1 + f_2}{f_1 f_2} \qquad p_1 \text{ to } \infty\text{, so } 1/p_1 \text{ to zero} \tag{5}$$

$$\implies \qquad f_{\text{equiv}} = \frac{f_1 f_2}{f_1 + f_2} \tag{6}$$

□ **4.** A dedicated sports car enthusiast polishes the inside and outside surfaces of a hubcap that is a section of a sphere. When he looks into one side of the hubcap, he sees an image of his face 28.0 cm in back of the hubcap. He then turns the hubcap over, keeping it the same distance from his face. He now sees an image of his face 10.8 cm in back of it. **(a)** How far is his face from the hubcap? **(b)** What is the radius of curvature of the hubcap?

Assume the object distance p (face to hubcap distance) is the same in both cases, regardless of orientation. We have two image positions, $-q_1$ and $-q_2$, both of which must be negative since the images form behind the mirror. We will have a single radius and object distance. The only difference between the two cases is that in the concave (converging) case the focal point (and thus radius, since $f = R/2$) is positive, while in the convex (diverging) case, the focal point must be negative. Which case is which? Examine the lens equation, $1/f = 1/p + 1/q$. If f is negative, then for a given p, q will be much larger in magnitude. The 28.0 cm image distance must therefore correspond to the convex case, and the 10.0 cm distance to the concave case. With $q_1 = 10.8$ cm and $q_2 = 28.0$ cm (we will put the negative sign in separately) we have then

$$\frac{1}{p} - \frac{1}{q_1} = \frac{1}{f} = \frac{2}{R} \tag{7}$$

$$\frac{1}{p} - \frac{1}{q_2} = -\frac{1}{f} = -\frac{2}{R} \tag{8}$$

Rearranging,

$$\frac{2}{R} = \frac{q_1 - p}{q_1 p} \tag{9}$$

$$\frac{2}{R} = \frac{p - q_2}{q_2 p} \tag{10}$$

$$\implies \qquad \frac{q_1 - p}{q_1 p} = \frac{p - q_2}{q_2 p} \tag{11}$$

Now we can solve for p:

$$\frac{q_1 - p}{q_1 p} = \frac{p - q_2}{q_2 p} \tag{12}$$

$$\frac{q_1 - p}{q_1} = \frac{p - q_2}{q_2} \tag{13}$$

$$q_2 (q_1 - p) = q_1 (p - q_2) \tag{14}$$

$$p_1 (q_1 + q_2) = 2 q_1 q_2 \tag{15}$$

$$p_1 = \frac{2 q_1 q_2}{q_1 + q_2} \approx 15.6\,\text{cm} \tag{16}$$

Given p, we can find R from either of our original equations. Solving the second for R,

$$R = \frac{2 p q_2}{q_2 - p} = 70.3\,\text{cm} \tag{17}$$

Name ____________________

Relativity: **solve 1 of 3.**

□ **1.** An interstellar space probe is moving at a constant speed relative to earth of $0.76c$ toward a distant planet. Its radioisotope generators have enough energy to keep its data transmitter active continuously for 15 years, as measured in their own reference frame. **(a)** How long do the generators last as measured from earth? **(b)** How far is the probe from earth when the generators fail, as measured from earth? **(c)** How far is the probe from earth when the generators fail, *as measured by its built-in trip odometer*?

□ **2.** A Klingon space ship moves away from Earth at a speed of $0.700c$. The starship Enterprise pursues at a speed of $0.900c$ relative to Earth. Observers on Earth see the Enterprise overtaking the Klingon ship at a relative speed of $0.200c$. With what speed is the Enterprise overtaking the Klingon ship as seen by the crew of the Enterprise?

□ **3.** A muon formed high in the Earth's atmosphere travels at $v = 0.990c$ for 4.60 km before it decays into an electron, a neutrino, and an antineutrino ($\mu^- \to e^- + \nu + \overline{\nu}$). **(a)** How long does the muon live, as measured in its own reference frame? **(b)** How far does the Earth travel, as measured in the frame of the muon?

"Modern" Physics: **solve 1 of 3.**

□ **1.** Fill in the missing elements, atomic numbers, and atomic masses (denoted by question marks) in the following radioactive decay series.

$$ {}^{228}_{90}\mathrm{Th} \xrightarrow{\alpha} {}^{224}_{?}\mathrm{Ra} \xrightarrow{\alpha} {}^{?}_{86}? \xrightarrow{\alpha} {}^{216}_{84}\mathrm{Po} \xrightarrow{\alpha} {}^{212}_{?}? \xrightarrow{\beta} {}^{212}_{83}? \xrightarrow{\beta} {}^{?}_{?}\mathrm{Po} \xrightarrow{\alpha} {}^{?}_{?}? $$

□ **2.** An electron is moving at a speed of $0.01c$ on a circular orbit of radius 10^{-10} m around a proton. **(a)** What is the strength of the resulting magnetic field at the center of the orbit? (The numbers given are typical, in order of magnitude, for an electron in an atom.) **(b)** If the nucleus of the atom (at the center of the orbit) consists of a single proton, what would its precession frequency be? *Hint: from the nucleus' point of view, it orbiting the electron in a circular path.* Recall $\omega = qB/m$ and $\omega = 2\pi f$.

□ **3.** The average lifetime of a neutral pion (π^0) is about 8.4×10^{-17} s. Estimate the minimum uncertainty in the energy of a π^0 in electron volts.

Constants:

$$\begin{aligned}
N_A &= 6.022\times10^{23}\,\text{things/mol}\\
k_e &= 8.98755\times10^{9}\,\text{N}\cdot\text{m}^2\cdot\text{C}^{-2}\\
\mu_0 &\equiv 4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\\
\epsilon_0 &= 8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\\
e &= 1.60218\times10^{-19}\,\text{C}\\
h &= 6.6261\times10^{-34}\,\text{J}\cdot\text{s} = 4.1357\times10^{-15}\,\text{eV}\cdot\text{s}\\
\hbar &= \frac{h}{2\pi}\\
c &= \frac{1}{\sqrt{\mu_0\epsilon_0}} = 2.99792\times10^{8}\,\text{m/s}\\
m_{e^-} &= 9.10938\times10^{-31}\,\text{kg} = 0.510998\,\text{MeV}/c^2\\
m_{p^+} &= 1.67262\times10^{-27}\,\text{kg} = 938.272\,\text{MeV}/c^2\\
m_{n^0} &= 1.67493\times10^{-27}\,\text{kg} = 939.565\,\text{MeV}/c^2\\
1\,\text{u} &= 931.494\,\text{MeV}/c^2\\
hc &= 1239.84\,\text{eV}\cdot\text{nm}
\end{aligned}$$

Quadratic formula:

$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Basic Equations:

$$\begin{aligned}
\vec{\mathbf{F}}_{\text{net}} &= m\vec{\mathbf{a}}\ \text{ Newton's Second Law}\\
\vec{\mathbf{F}}_{\text{centr}} &= -\frac{mv^2}{r}\hat{\boldsymbol{r}}\ \text{ Centripetal}
\end{aligned}$$

Magnetism

$$\begin{aligned}
|\vec{F}_B| &= q|\vec{v}||\vec{B}|\sin\theta_{vB}\\
|\vec{F}_B| &= BIl\sin\theta\ \text{ wire}\\
|\vec{\tau}| &= BIAN\sin\theta\ \text{ torque current loop}\\
\vec{B} &= \frac{\mu_0 I}{2\pi r}\hat{\theta}\ \text{ wire}\\
\vec{B} &= \frac{\mu_0 I}{2r}\hat{\theta}\ \text{ loop}\\
\vec{B} &= \mu_0\frac{N}{L}I\,\hat{\boldsymbol{z}} \equiv \mu_0 nI\,\hat{\boldsymbol{z}}\ \text{ solenoid}\\
\frac{|\vec{F}_{12}|}{l} &= \frac{\mu_0 I_1 I_2}{2\pi d}\ \text{ 2 wires, force per length}
\end{aligned}$$

Current:

$$\begin{aligned}
I &= \frac{\Delta Q}{\Delta t} = nqAv_d\\
J &= \frac{I}{A} = nqv_d\\
v_d &= \frac{-e\tau}{m}E \quad \tau = \text{scattering time}\\
\varrho &= \frac{m}{ne^2\tau}\\
\Delta V &= \frac{\varrho l}{A}I = RI\\
R &= \frac{\Delta V}{I} = \frac{\varrho l}{A}\\
\mathscr{P} &= E\cdot\Delta t = I\Delta V = I^2R = \frac{[\Delta V]^2}{R}\ \text{ power}
\end{aligned}$$

Ohm:

$$\begin{aligned}
\Delta V &= IR\\
\mathscr{P} &= E\cdot\Delta t = I\Delta V = I^2R = \frac{[\Delta V]^2}{R}\ \text{ power}
\end{aligned}$$

EM Waves:

$$\begin{aligned}
c &= \lambda f = \frac{|\vec{E}|}{|\vec{B}|}\\
\mathcal{I} &= \left[\frac{\text{photons}}{\text{time}}\right]\left[\frac{\text{energy}}{\text{photon}}\right]\left[\frac{1}{\text{Area}}\right]\\
\mathcal{I} &= \frac{\text{energy}}{\text{time}\cdot\text{area}} = \frac{E_{\max}B_{\max}}{2\mu_0} = \frac{\text{power }(\mathscr{P})}{\text{area}} = \frac{E^2_{\max}}{2\mu_0 c}
\end{aligned}$$

Electric Potential:

$$\begin{aligned}
\Delta V &= V_B - V_A = \frac{\Delta\text{PE}}{q}\\
\Delta PE &= q\Delta V = -q|\vec{E}||\Delta\vec{x}|\cos\theta = -qE_x\Delta x\\
&\quad \uparrow\ \text{constant E field}\\
V_{\text{point charge}} &= k_e\frac{q}{r}\\
PE_{\text{pair of point charges}} &= k_e\frac{q_1q_2}{r_{12}}\\
PE_{\text{system}} &= \text{sum over unique pairs of charges} = \sum_{\text{pairs } ij}\frac{k_eq_iq_j}{r_{ij}}\\
-W &= \Delta\text{PE} = q(V_B - V_A)
\end{aligned}$$

Optics:

$$\begin{aligned}
\mathscr{E} &= hf = \frac{hc}{\lambda}\\
n &= \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v}\\
\frac{\lambda_1}{\lambda_2} &= \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}\quad \text{refraction}\\
n_1\sin\theta_1 &= n_2\sin\theta_2\quad \text{Snell's refraction}\\
\lambda f &= c\\
M &= \frac{h'}{h} = -\frac{q}{p}\\
\frac{1}{f} &= \frac{1}{p}+\frac{1}{q} = \frac{2}{R}\quad \text{mirror \& lens}\\
\frac{n_1}{p}+\frac{n_2}{q} &= \frac{n_2-n_1}{R}\quad \text{spherical refracting}\\
q &= -\frac{n_2}{n_1}p\quad \text{flat refracting}\\
\frac{1}{f} &= \left(\frac{n_2-n_1}{n_1}\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]\quad \text{lensmaker's}
\end{aligned}$$

Electric Force & Field

$$\begin{aligned}
\vec{F}_{e,12} &= q\vec{E}_{12} = \frac{k_eq_1q_2}{r^2_{12}}\hat{\boldsymbol{r}}_{12}\\
\vec{E} &= k_e\frac{|q|}{r^2}\\
\Phi_E &= |\vec{E}|A\cos\theta_{EA} = \frac{Q_{\text{inside}}}{\epsilon_0}\\
\Delta PE &= -W = -q|\vec{E}||\Delta\vec{x}|\cos\theta = -qE_x\Delta x\\
&\quad \uparrow\ \text{constant E field}
\end{aligned}$$

Capacitors:

$$\begin{aligned}
Q_{\text{capacitor}} &= C\Delta V\\
C_{\text{parallel plate}} &= \frac{\epsilon_0 A}{d}\\
E_{\text{capacitor}} &= \frac{1}{2}Q\Delta V = \frac{Q^2}{2C}\\
C_{\text{eq, par}} &= C_1 + C_2\\
C_{\text{eq, series}} &= \frac{C_1C_2}{C_1+C_2}\\
C_{\text{with dielectric}} &= \kappa C_{\text{without}}
\end{aligned}$$

Cheat Sheets

Resistors:

$$I_{\text{V source}} = \frac{\Delta V}{R+r}$$
$$\Delta V_{\text{V source}} = \Delta V_{\text{rated}}\frac{R}{r+R}$$
$$I_{\text{I source}} = I_{\text{rated}}\frac{r}{r+R}$$
$$R_{\text{eq, series}} = R_1 + R_2$$
$$\frac{1}{R_{\text{eq, par}}} = \frac{1}{R_1}+\frac{1}{R_2}$$
$$R_{\text{eq, par}} = \frac{R_1R_2}{R_1+R_2}$$

RC circuits

$$Q_C(t) = Q_0\left[1-e^{-t/\tau}\right] \quad \text{charging}$$
$$Q_C(t) = Q_0 e^{-t/\tau} \quad \text{discharging}$$
$$Q(t) = C\Delta V(t)$$
$$\tau = RC$$

Vectors:

$$|\vec{F}| = \sqrt{F_x^2+F_y^2} \text{ magnitude}$$
$$\theta = \tan^{-1}\left[\frac{F_y}{F_x}\right] \text{ direction}$$

Induction:

$$\Phi_B = B_\perp A = BA\cos\theta_{BA}$$
$$\Delta V = -N\frac{\Delta\Phi_B}{\Delta t}$$
$$L = N\frac{\Delta\Phi_B}{\Delta I} = \frac{N\Phi_B}{I}$$
$$\Delta V = |\vec{v}||\vec{B}|l = |\vec{E}|l \text{ motional voltage}$$

ac Circuits

$$\tau = L/R \text{ RL circuit}$$
$$\tau = RC \text{ RC circuit}$$
$$X_C = \frac{1}{2\pi fC} \text{ “resistance” of a capacitor for ac}$$
$$X_L = 2\pi fL \text{ “resistance” of an inductor for ac}$$
$$\omega_{\text{cutoff}} = \frac{1}{\tau} = 2\pi f$$

Relativity

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$\Delta t'_{\text{moving}} = \gamma\Delta t_{\text{stationary}} = \gamma\Delta t_p$$
$$L'_{\text{moving}} = \frac{L_{\text{stationary}}}{\gamma} = \frac{L_p}{\gamma} x' = \gamma(x-vt)$$
$$\Delta t' = t'_1 - t'_2 = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$$
$$p = \gamma mv$$
$$v_{\text{obj}} = \frac{v+v'_{\text{obj}}}{1+\frac{vv'_{\text{obj}}}{c^2}} \qquad v'_{\text{obj}} = \frac{v_{\text{obj}}-v}{1-\frac{vv_{\text{obj}}}{c^2}}$$
$$\text{KE} = (\gamma-1)mc^2$$
$$E_{\text{rest}} = mc^2$$
$$E^2 = p^2c^2+m^2c^4$$

Nuclear

$$E^2 = p^2c^2+m^2c^4$$
$$\text{alpha particle} = {}^4_2\alpha = {}^4_2\text{He} \qquad \text{beta particle} = {}^{\;0}_{-1}\beta = e^-$$
$$\text{Binding Energy} = \left[\sum_{p^+\&n^0} mc^2\right] - m_{\text{atom}}c^2$$

Quantum & Atomic

$$\lambda_{\text{out}} - \lambda_{\text{in}} = \frac{h}{m_ec}(1-\cos\theta)$$
$$\lambda = \frac{h}{|\vec{p}|} = \frac{h}{\gamma mv} \approx \frac{h}{mv}$$
$$\Delta x\Delta p \geq \frac{h}{4\pi}$$
$$\Delta E\Delta t \geq \frac{h}{4\pi}$$
$$E_n = -13.6\,\text{eV}/n^2$$
$$E_i - E_f = -13.6\,\text{eV}\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right) = hf \text{ \textit{Hydrogen only}}$$
$$mvr = n\hbar$$
$$v^2 = \frac{n^2\hbar^2}{m_e^2r^2} = \frac{k_ee^2}{m_er}$$
$$N(t) = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$$

Right-hand rule #1

1. Point the fingers of your right hand along the direction of the velocity.
2. Point your thumb in the direction of the magnetic field $\vec{B}$.
3. The magnetic force on a positive charge points out from the back of your hand.

Right-hand rule #2:
Point your thumb on your right hand along the wire in the direction of the current. Your fingers naturally curl around the direction of the magnetic field caused by the current, which circulates around the wire.

Derived unit	Symbol	equivalent to
newton	N	kg·m/s^2
joule	J	kg·m^2/s^2 = N·m
watt	W	J/s=m^2·kg/s^3
coulomb	C	A·s
V	W/A = m^2·kg/·s^3·A	
farad	F	C/V = A^2·s^4/m^2·kg
ohm	Ω	V/A = m^2·kg/s^3·A^2
tesla	T	Wb/m^2 = kg/s^2·A
electron volt	eV	1.6×10^{-19} J
-	1 T · m/A	1 N/A^2
-	1 T · m^2	1 V · s
-	1 N/C	1 V/m

Power	Prefix	Abbreviation
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	µ
10^{-3}	milli	m
10^{-2}	centi	c
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T

Periodic Table of the Elements

1 H 1.00794																1 H 1.00794	2 He 4.002602
3 Li 6.941	4 Be 9.012182											5 B 10.811	6 C 12.0107	7 N 14.00674	8 O 15.9994	9 F 18.9984032	10 Ne 20.1797
11 Na 22.989770	12 Mg 24.3050											13 Al 26.981538	14 Si 28.0855	15 P 30.973761	16 S 32.066	17 Cl 35.4527	18 Ar 39.948
19 K 39.0983	20 Ca 40.078	21 Sc 44.955910	22 Ti 47.867	23 V 50.9415	24 Cr 51.9961	25 Mn 54.938049	26 Fe 55.845	27 Co 58.933200	28 Ni 58.6934	29 Cu 63.546	30 Zn 65.39	31 Ga 69.723	32 Ge 72.61	33 As 74.92160	34 Se 78.96	35 Br 79.904	36 Kr 83.80
37 Rb 85.4678	38 Sr 87.62	39 Y 88.90585	40 Zr 91.224	41 Nb 92.90638	42 Mo 95.94	43 Tc (98)	44 Ru 101.07	45 Rh 102.90550	46 Pd 106.42	47 Ag 107.8682	48 Cd 112.411	49 In 114.818	50 Sn 118.710	51 Sb 121.760	52 Te 127.60	53 I 126.90447	54 Xe 131.29
55 Cs 132.90545	56 Ba 137.327	57 La 138.9055	72 Hf 178.49	73 Ta 180.9479	74 W 183.84	75 Re 186.207	76 Os 190.23	77 Ir 192.217	78 Pt 195.078	79 Au 196.96655	80 Hg 200.59	81 Tl 204.3833	82 Pb 207.2	83 Bi 208.98038	84 Po (209)	85 At (210)	86 Rn (222)
87 Fr (223)	88 Ra (226)	89 Ac (227)	104 Rf (261)	105 Db (262)	106 Sg (263)	107 Bh (262)	108 Hs (265)	109 Mt (266)	110 (269)	111 (272)	112 (277)		114 (289) (287)		116 (289)		118 (293)

58 Ce 140.116	59 Pr 140.90765	60 Nd 144.24	61 Pm (145)	62 Sm 150.36	63 Eu 151.964	64 Gd 157.25	65 Tb 158.92534	66 Dy 162.50	67 Ho 164.93032	68 Er 167.26	69 Tm 168.93421	70 Yb 173.04	71 Lu 174.967
90 Th 232.0381	91 Pa 231.03588	92 U 238.0289	93 Np (237)	94 Pu (244)	95 Am (243)	96 Cm (247)	97 Bk (247)	98 Cf (251)	99 Es (252)	100 Fm (257)	101 Md (258)	102 No (259)	103 Lr (262)

S.E. Van Bramer,   7/22/99

1995 IUPAC masses and Approved Names from http://www.chem.qmw.ac.uk/iupac/AtWt/

masses for 107-111 from C&EN, March 13, 1995, P 35

112 from http://www.gsi.de/z112e.html

114 from C&EN July 19, 1999

116 and 118 from http://www.lbl.gov/Science-Articles/Archive/elements-116-118.html