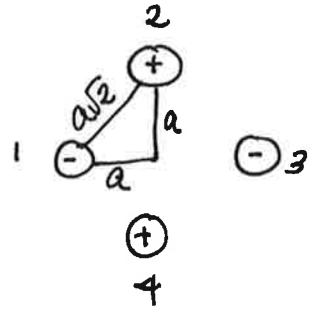
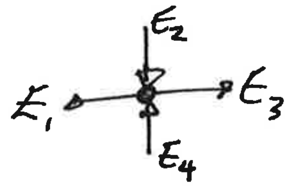
PH102 Spr 2015
LeClair Exam 1

1.



a) in center, field cancels since all charges are equal in magnitude



b) the potential is zero in the center because we have two positive and two negative charges

$$V_1 = V_3 = \frac{k(-q)}{a} \qquad V_2 = V_4 = \frac{k(+q)}{a}$$

$$V_{tot} = V_1 + V_2 + V_3 + V_4 = 0$$

c) for potential energy, sum over unique pairs of charges

12 13 14
23 24
34

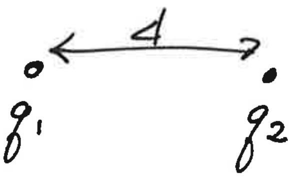
keep track of pair distances ($a\sqrt{2}$ or $2a$) and signs

$$U = kq^2\left(\frac{-1}{a\sqrt{2}} + \frac{1}{2a} - \frac{1}{a\sqrt{2}} - \frac{1}{a\sqrt{2}} + \frac{1}{2a} - \frac{1}{a\sqrt{2}}\right) = \frac{kq^2}{a}\left(1 - \frac{4}{\sqrt{2}}\right)$$

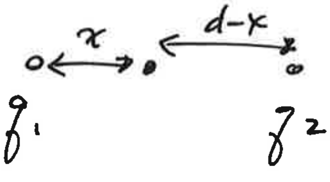
$$\boxed{U = \frac{kq^2}{a}(1 - 2\sqrt{2}) \simeq -1.83\,\frac{kq^2}{a}}$$

d) since $U < 0$, it is stable compared to having the 4 charges on their own

(2.)



a) both are (+) charges, so E can only be zero in between (to L or R fields are not in opposite directions)



pick a spot a distance x from q_1 (& $d-x$ from q_2)

$$E_1 = \frac{kq_1}{x^2} \qquad E_2 = \frac{-kq_2}{(d-x)^2} \quad (-\text{ due to direction})$$

$E_{tot} = E_1 + E_2 = 0$ at the desired spot; do this & solve for x.

$$\frac{kq_1}{x^2} - \frac{kq_2}{(d-x)^2} = 0 \Rightarrow \frac{q_2}{(d-x)^2} = \frac{q_1}{x^2} \Rightarrow q_2 x^2 = q_1 x^2 - 2q_1 dx + q_1 d^2$$

$$\Rightarrow (q_1 - q_2)x^2 - (2q_1 d)x + q_1 d = 0$$ quadratic formula ...

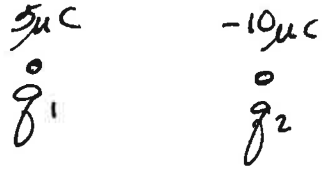
$$x = \frac{2q_1 d \pm \sqrt{4q_1^2 d^2 - 4q_1 d^2 (q_1 - q_2)}}{2(q_1 - q_2)}$$

$$x = d\left(\frac{q_1 \pm \sqrt{q_1 q_2}}{q_1 - q_2}\right) \simeq \boxed{0.04\,m},\ -0.24\,m$$

outside middle region, E can't be zero here

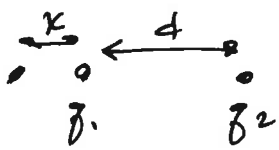
b) if both charges are (+), V isn't zero anywhere but $x = \pm\infty$

2c if $q_2 = -10\mu C$, V can be zero in the center or to the LHS of q_1



potential cancels somewhere so long as we have both ⊕ & ⊖. still have to be closer to smaller charge though, and this can only be to the LHS or in the center

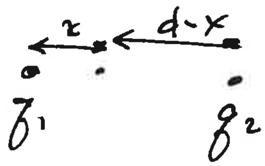
<u>LHS</u>: $V_{tot} = \frac{kq_1}{x} - \frac{kq_2}{d+x} = 0$ (proceeding as in b, with $q_2 = 10\mu C$ and sign fixed by overall − on 2^{nd} term)



$$\Rightarrow kq_2 x = kq_1(d+x)$$

$$\Rightarrow x = d\left(\frac{q_1}{q_2 - q_1}\right) \simeq \underline{\underline{0.1 m}}$$

<u>Center</u>:



$$V_{tot} = \frac{kq_1}{x} - \frac{kq_2}{d-x} = 0$$

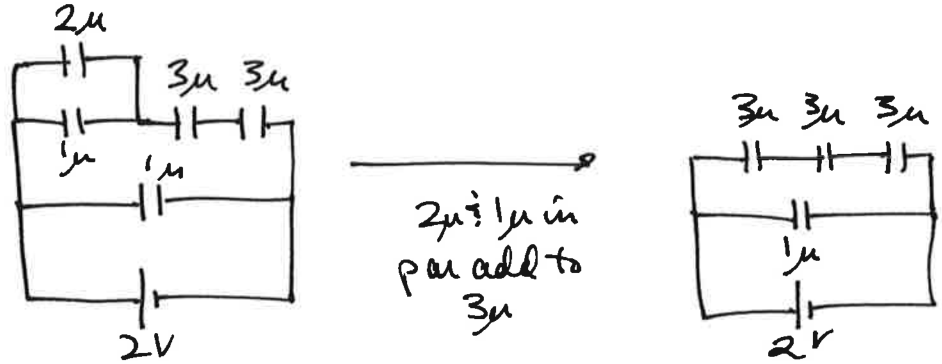
again, $q_2 = 10\mu C$ sign is OK overall

$$\Rightarrow x = d\left(\frac{q_1}{q_1 + q_2}\right) \simeq \underline{\underline{0.033 m}}$$

note $\frac{1}{3}+\frac{1}{6}=\frac{1}{2}$, so $3\mu F$ and $6\mu F$ in series give $2\mu F$

a)

starting circuit →



$3 \times 3\mu F$ in series : $\frac{1}{C_{eq}} = \frac{1}{3\mu} + \frac{1}{3\mu} + \frac{1}{3\mu} = \frac{1}{1\mu} \Rightarrow C_{eq} = 1\mu F$

⇒

$\boxed{C_{eq} = 2\mu F}$

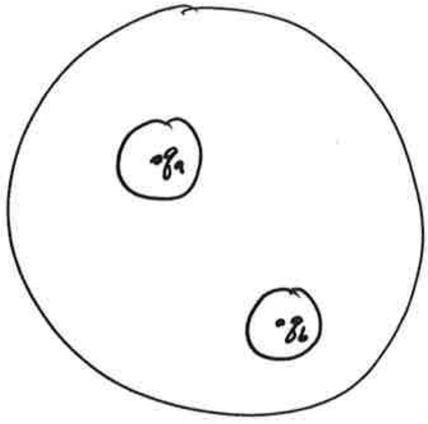
b) $Q_{tot} = C_{eq}\Delta V = 2\mu F \cdot 2V = \boxed{4\mu C}$

c) $U = \frac{1}{2} Q_{tot} \Delta V = \frac{1}{2}(4\mu C)(2V) = \boxed{4\mu J}$

d) bottom $1\mu F$ is connected directly to the 2V battery!

$\Rightarrow \quad Q = C\Delta V = (1\mu F)(2V) = \boxed{2\mu C}$

(4.)



a) from Gauss' Law, the field inside the conductor just outside the cavity must be zero. Therefore, the surface must have exactly the same charge as is inside, but with the opposite sign (so that $q_{encl} = 0$ for a surface just outside each cavity).

cavity a: $-q_a$

cavity b: $-q_b$

for the whole shell, we still need the field outside to depend only on the enclosed charge. Only q_a and q_b are enclosed - the conductor is neutral overall. If $-q_a$ & $-q_b$ built up on the smaller cavities, $+q_a + q_b$ are left in the conductor. This net charge must reside on the surface of the conductor

conductor: $+(q_a + q_b)$

b) the field inside a cavity depends only on enclosed charge.

c)

cavity a encloses q_a $\longrightarrow$ $E_a = \frac{kq_a}{r^2}$

cavity b encloses q_b $\longrightarrow$ $E_b = \frac{kq_b}{r^2}$

entire conductor encloses q_a+q_b $\longrightarrow$ $E_{out} = \frac{k(q_a+q_b)}{r^2}$

d) the net force is zero - each q is "screened" from the other by the surface charge on each cavity. Since the field inside the conductor is zero, they can't influence eachother.

e) only the surface charge distribution on the conductor could change.