# University of Alabama <br> Department of Physics and Astronomy 

## Exam 3

## Instructions

1. Solve three of the five problems below.
2. All problems have equal weight. Do your work on separate sheets.
3. You are allowed 1 sheet of standard $8.5 \times 11$ in paper and a calculator.
$\square$ 1. Two parallel rails with negligible resistance are a distance $d$ apart and connected by a resistor $R_{m}$. The circuit also contains two metal rods, of resistance $R_{l}$ and $R_{r}$, begin pulled away from the resistor at speeds of $v_{1}$ and $v_{2}$, respectively. A uniform magnetic field $B$ is applied into the plane of the figure. Determine the current in the central resistor $R_{m}$.

$\square$ 2. Find the force on a square loop (side $a$ ) placed as shown below, near an infinite straight wire. Both loop and wire carry a steady current $I$.

$\square$ 3. In a mass spectrometer, a beam of ions is first made to pass through a velocity selector with perpendicular $\overrightarrow{\mathbf{E}}$ and $\overrightarrow{\mathbf{B}}$ fields. Here, the electric field $\overrightarrow{\mathbf{E}}$ is to the right, between parallel charged
plates, and the magnetic field $\overrightarrow{\mathbf{B}}$ in the same region is into the page. The selected ions are then made to enter a region of different magnetic field $\overrightarrow{\mathbf{B}}^{\prime}$, where they move in arcs of circles. The radii of these circles depend on the masses of the ions. Assume that each ion has a single charge $e$. Show that in terms of the given field values and the impact distance $l$ the mass of the ion is

$$
m=\frac{e B B^{\prime} l}{2 E}
$$


4. A technician wearing a brass bracelet enclosing an area $0.00500 \mathrm{~m}^{2}$ places her hand in a solenoid whose magnetic field is 5.00 T directed perpendicular to the plane of the bracelet. The electrical resistance around the circumference of the bracelet is $0.0200 \Omega$. A power failure causes the field to drop to 1.50 T in a time of 20.0 ms . Find (a) the current in the bracelet, and (b) the power delivered to the bracelet. Hint: don't wear metal jewelry when working with strong magnetic fields.
$\square$ 5. Find the magnetic field at point $P$ due to the current distribution shown below. Hint: Break the loop into segments, and use superposition.


Constants:

$$
\begin{aligned}
g & \approx 9.81 \mathrm{~m} / \mathrm{s} \\
N_{A} & =6.022 \times 10^{23} \text { things } / \mathrm{mol} \\
k_{e} & =\frac{1}{4 \pi \epsilon_{o}}=8.98755 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} \cdot \mathrm{C}^{-2} \\
\mu_{o} & \equiv 4 \pi \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A} \\
\epsilon_{o} & =\frac{1}{4 \pi k_{e}}=8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2} \\
e & =1.60218 \times 10^{-19} \mathrm{C} \\
c & =\frac{1}{\sqrt{\mu_{0} \epsilon_{0}}}=2.99792 \times 10^{8} \mathrm{~m} / \mathrm{s} \\
m_{e}- & =9.10938 \times 10^{-31} \mathrm{~kg} \\
m_{p}+ & =1.67262 \times 10^{-27} \mathrm{~kg} \\
h c & =1239.84 \mathrm{eV} \cdot \mathrm{~nm}
\end{aligned}
$$

## Basic Equations:

$$
\begin{aligned}
\overrightarrow{\mathbf{F}}_{\text {net }} & =\frac{\Delta \overrightarrow{\mathbf{p}}}{\Delta t}=m \overrightarrow{\mathbf{a}} \quad \text { Newton's Second Law } \\
\overrightarrow{\mathbf{F}}_{\text {centr }} & =-\frac{m v^{2}}{r} \hat{\mathbf{r}} \quad \text { Centripetal } \\
|\overrightarrow{\mathbf{F}}| & =\sqrt{F_{x}^{2}+F_{y}^{2}} \quad \mathrm{mag} \quad \theta=\tan ^{-1}\left[\frac{F_{y}}{F_{x}}\right] \quad \mathrm{dir} \\
0 & =a x^{2}+b x^{2}+c \Longrightarrow x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
\end{aligned}
$$

## Magnetism

$$
\begin{aligned}
\left|\overrightarrow{\mathbf{F}}_{B}\right| & =q|\overrightarrow{\mathbf{v}} \| \overrightarrow{\mathbf{B}}| \sin \theta_{v B} \\
\left|\overrightarrow{\mathbf{F}}_{B}\right| & =B I l \sin \theta \text { wire } \\
|\overrightarrow{\mathbf{B}}| & =\frac{\mu_{0} I}{2 \pi r} \text { wire } \\
|\overrightarrow{\mathbf{B}}| & =\frac{\mu_{0} I}{2 r} \text { loop } \\
|\overrightarrow{\mathbf{B}}| & =\mu_{0} \frac{N}{L} I \equiv \mu_{0} n I \hat{\mathbf{z}} \text { solenoid } \\
\frac{\left|\overrightarrow{\mathbf{F}}_{12}\right|}{l} & =\frac{\mu_{0} I_{1} I_{2}}{2 \pi d} 2 \text { wires, force per length }
\end{aligned}
$$

Current/resistors/circuits:

$$
\begin{aligned}
\Delta V & =\frac{\varrho l}{A} I=R I \\
R & =\frac{\Delta V}{I}=\frac{\varrho l}{A} \\
\mathscr{P} & =E \cdot \Delta t=I \Delta V=I^{2} R=\frac{[\Delta V]^{2}}{R} \text { power } \\
R_{\text {eq, series }} & =R_{1}+R_{2} \\
\frac{1}{R_{\text {eq, par }}} & =\frac{1}{R_{1}}+\frac{1}{R_{2}}
\end{aligned}
$$

## Induction:

$\Phi_{B}=B_{\perp} A=B A \cos \theta_{B A}$
$\Delta V=-N \frac{\Delta \Phi_{B}}{\Delta t} \quad N=$ number of turns
$\Delta V=-L \frac{\Delta I}{\Delta t}$
$\Delta V=|\overrightarrow{\mathbf{v}}||\overrightarrow{\mathbf{B}}| l=|\overrightarrow{\mathbf{E}}| l$ motional voltage

| Unit | Symbol | equivalent to |
| :--- | :---: | :---: |
| newton | N | $\mathrm{kg} \cdot \mathrm{m} / \mathrm{s}^{2}$ |
| joule | J | $\mathrm{kg} \cdot \mathrm{m}^{2} / \mathrm{s}^{2}=\mathrm{N} \cdot \mathrm{m}$ |
| watt | W | $\mathrm{J} / \mathrm{s}=\mathrm{m}^{2} \cdot \mathrm{~kg} / \mathrm{s}^{3}$ |
| coulomb | C | $\mathrm{A} \cdot \mathrm{s}$ |
| amp | A | $\mathrm{C} / \mathrm{s}$ |
| volt | V | $\mathrm{W} / \mathrm{A}=\mathrm{m}^{2} \cdot \mathrm{~kg} / \cdot \mathrm{s}^{3} \cdot \mathrm{~A}$ |
| farad | F | $\mathrm{C} / \mathrm{V}=\mathrm{A}^{2} \cdot \mathrm{~s}^{4} / \mathrm{m}^{2} \cdot \mathrm{~kg}$ |
| ohm | $\Omega$ | $\mathrm{V} / \mathrm{A}=\mathrm{m}^{2} \cdot \mathrm{~kg} / \mathrm{s}^{3} \cdot \mathrm{~A}^{2}$ |
| tesla | T | $\mathrm{Wb} / \mathrm{m}^{2}=\mathrm{kg} / \mathrm{s}^{2} \cdot \mathrm{~A}$ |
| electron volt | eV | $1.6 \times 10^{-19} \mathrm{~J}$ |
| - | $1 \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A}$ | $1 \mathrm{~N} / \mathrm{A}^{2}$ |
| - | $1 \mathrm{~T} \cdot \mathrm{~m}^{2}$ | $1 \mathrm{~V} \cdot \mathrm{~s}$ |
| - | $1 \mathrm{~N} / \mathrm{C}$ | $1 \mathrm{~V} / \mathrm{m}$ |

