# University of Alabama <br> Department of Physics and Astronomy 

## Exam II

## Instructions

I. Answer 2 out of 4 questions in each section below. All problems have equal weight.
2. Clearly mark the answer you choose using the tick boxes.
3. Show your work for full credit. Significant partial credit will be given.
4. You are allowed 2 sides of a standard $8.5 \times 1 \mathrm{I}$ in piece of paper with notes/formulas and a calculator.

## I dc Circuits - solve $2 / 4$

$\square$ I. A black box with three terminals, $a, b$, and $c$, contains nothing but three resistors and connecting wire. Measuring the resistance between pairs of terminals, you measure $R_{a b}=30 \Omega, R_{a c}=60 \Omega$, and $R_{b c}=70 \Omega$. Show that the box could be either of those below.

$\square 2$. Find the current in the two batteries below, and find the power they deliver.


- 3. Two heating coils have resistances of $12.0 \Omega$ and $6.0 \Omega$, respectively. (a) What is the total power dissipated if the coils are connected in parallel to a 115 V voltage source? (b) What if they are connected in series?
-4. A copper wire 1 km long is connected across a 6 V battery. The resistivity of the copper is $1.7 \times 10^{-8} \Omega \mathrm{~m}$, and the number of conduction electrons per cubic meter is $8 \times 10^{28}$. (a) What is the drift velocity of the conduction electrons under these circumstances? (b) How long does it take an electron to drift once around the circuit?


## 2 Magnetism - solve 2/4

$\square \mathrm{I}$. In a motorboat, the compass is mounted at a distance of 0.80 m from a cable carrying a current of 20 A from an electric generator to a battery. (a) What magnetic field does this current produce at the location of the compass? Assume the cable is a long, straight wire. (b) The horizontal (north) component of the Earth's magnetic field is $1.8 \times 10^{-5} \mathrm{~T}$. Since the compass points in the direction of the net horizontal magnetic field, the current will cause a deviation of the compass needle. Assume that the magnetic field of the current is horizontal and at a right angle to the horizontal component of the earth's magnetic field. Under these circumstances, by
how many degrees will the compass deviate from true north?

- 2. Three long parallel wires pass through the corners of an equilateral triangle of side 0.1 m and are perpendicular to the plane of the triangle. Each wire carries a current of 15 A , the current being into the page for wires $B$ and $C$, and out of the page for $A$. (a) Find the force per unit length acting on the wire $A$. (b) Sketch the direction of the forces and their resultant.

- 3. The rate of flow of a conducting liquid can be measured with an electromagnetic flowmeter that detects the voltage induced by the motion of the liquid in a magnetic field. Suppose that a plastic pipe of diameter 0.10 m carries beer with a speed of $1.5 \mathrm{~m} / \mathrm{s}$. The pipe is in a transverse magnetic field (i.e., perpendicular to the pipe axis) of about $1.5 \times 10^{-2} \mathrm{~T}$. (a) Presume the beer is an ideal conductor. What voltage will be induced between the opposite sides of the column of liquid? (b) Does it matter whether the conductivity of the beer is due to mobile positive or negative charges?
- 4. A wire having a mass per unit length of $0.50 \mathrm{~g} / \mathrm{cm}$ carries a 2.0 A current horizontally to the right. What are the direction and magnitude of the minimum magnetic field needed to lift this wire vertically upward?


## 3 Induction - solve 2/4

$\square$ I. A technician wearing a conducting bracelet enclosing an area $0.005 \mathrm{~m}^{2}$ places her hand in a solenoid whose magnetic field is 5.0 T directed perpendicular to the plane of the bracelet. The resistance around the circumference of the bracelet is $0.02 \Omega$. A power failure causes the field to drop to 1.50 T in a time of 20.0 ms . Find (a) the current in the bracelet, and (b) the power delivered to the bracelet..

- 2. A conducting rectangular loop of mass $M$, resistance $R$, and dimensions $w$ by $l$ falls from rest into a magnetic field $\overrightarrow{\mathbf{B}}$, as shown at right. At some point before the top edge of the loop reaches the magnetic field, the loop attains a constant terminal velocity $v_{T}$. Show that the terminal velocity is:

$$
v_{T}=\frac{M g R}{B^{2} w^{2}}
$$

Hint: what must be true for the velocity to be constant?

- 3. Very large magnetic fields can be produced using a procedure called flux compression. A metallic cylindrical tube of radius $R$ is placed coaxially in a long solenoid of somewhat larger radius. The space between the tube and the solenoid is filled with a highly explosive material. When the explosive is set off, it collapses the tube to a cylinder of radius $r<R$. If the collapse happens very rapidly, induced current in the tube maintains the magnetic flux nearly constant inside the tube,
 even though the area shrinks. If the initial magnetic field in the solenoid is 2.50 T , and $R / r=12.0$, what is the maximum field that can be reached?
- 4. A metal bar of mass $m$ slides without friction on two long parallel conducting rails a distance $l$ apart. A resistor $R$ is connected across the rails at one end; the resistance of the bar and rails is negligible. There is a constant uniform magnetic field $\overrightarrow{\mathbf{B}}$ perpendicular to the page. At a time $t=0$, the crossbar is given a velocity $v_{0}$ toward the right. (a) What force must be supplied to maintain constant velocity after the field is switched on? (b) How much power is dissipated in the resistor?



## 4 ac Circuits - solve $2 / 4$

$\square$ I. Using capacitors, resistors, and inductors, sketch a circuit to split an audio signal composed of many frequencies in to a low frequency part and a high frequency part, for distribution to speakers. That is, filter the incoming signal into separate low frequencies and high frequencies to send to a woofer and tweeter, respectively. Such a circuit is known as an "audio crossover." You do not need to specify the values of your components.

- 2. A current source $I$ is used to drive a large inductor (say, a wound wire electromagnet) as shown below. Driving inductive loads can be problematic - what happens when you open the switch providing current to an inductor in circuit (a) Why does adding a diode across the inductor, circuit (b), add protection? Recall diodes only allow current through in one direction, as shown in (c).

$\square$ 3. Any two adjacent conductors can be considered as a capacitor, although the capacitance will be small unless the conductors are close together or long. This (unwanted) effect is termed "stray" or "parasitic" capacitance. Stray capacitance can allow signals to leak between otherwise isolated circuits (an effect called crosstalk), and it can be a limiting factor for proper functioning of circuits at high frequency. A stray capacitance can result when you touch or come close the wires in a circuit - your body provides a capacitive path between the circuit of interest and an adjacent noise source. (a) Explain, referencing the figure at right, why the stray capacitance allows unwanted ac signals to couple into the circuit, but does not allow dc signals. (b) Suggest a method for minimizing this effect.
- 4. The circuit below is a simple filter based on an inductor and a capacitor. (a) What sort of filter is this? Briefly explain your rationale. (b) Sketch the output of this circuit versus frequency, assuming the input contains all frequencies with equal amplitudes. (c) The cutoff frequency of a filter is the frequency at which the reactance of each component is equal - in this case, the frequency at which the inductive reactance is equal to the capacitive reactance. What is the cutoff frequency $f$ in terms of $L$ and $C$ ?



## Cheat Sheet

## Constants:

$$
\begin{aligned}
k_{e} & =8.98755 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} \cdot \mathrm{C}^{-2} \\
\epsilon_{0} & =4 \pi k_{e}=8.85 \times 10^{12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2} \\
e & =1.60218 \times 10^{-19} \mathrm{C} \\
\mu_{0} & \equiv 4 \pi \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A} \\
c & =2.99792 \times 10^{8} \mathrm{~m} / \mathrm{s} \\
m_{e^{-}} & =9.10938 \times 10^{-31} \mathrm{~kg} \\
m_{p^{+}} & =1.67262 \times 10^{-27} \mathrm{~kg}
\end{aligned}
$$

## Quadratic formula \& vectors:

$$
\begin{aligned}
0 & =a x^{2}+b x^{2}+c \Longrightarrow x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \\
|\overrightarrow{\mathbf{F}}| & =\sqrt{F_{x}^{2}+F_{y}^{2}} \text { magnitude } \\
\theta & =\tan ^{-1}\left[\frac{F_{y}}{F_{x}}\right] \quad \text { direction }
\end{aligned}
$$

## Resistors:

$$
\begin{aligned}
R_{\mathrm{eq}} & =R_{1}+R_{2} \quad \text { series } \\
\frac{1}{R_{\mathrm{eq}}} & =\frac{1}{R_{1}}+\frac{1}{R_{2}} \quad \text { parallel }
\end{aligned}
$$

## Induction:

$$
\begin{aligned}
\Phi_{B} & =B_{\perp} A=B A \cos \theta_{B A} \\
\Delta V & =-N \frac{\Delta \Phi_{B}}{\Delta t} \\
L & =N \frac{\Delta \Phi_{B}}{\Delta I}=\frac{N \Phi_{B}}{I} \\
\Delta V & =|\overrightarrow{\mathbf{v}}||\overrightarrow{\mathbf{B}}| l=|\overrightarrow{\mathbf{E}}| l \quad \text { motional voltage }
\end{aligned}
$$

## Other:

$$
\Delta V=|\overrightarrow{\mathbf{E}}| \Delta x \quad \text { const. field }
$$

| Power | Prefix | Abbreviation |
| :--- | :--- | :---: |
| $10^{-12}$ | pico | p |
| $10^{-9}$ | nano | n |
| $10^{-6}$ | micro | $\mu$ |
| $10^{-3}$ | milli | m |
| $10^{-2}$ | centi | c |
| $10^{3}$ | kilo | k |
| $10^{6}$ | mega | M |
| $10^{9}$ | giga | G |
| $10^{12}$ | tera | T |

Point your thumb on your right hand along the direction of the current. Your fingers curl in the direction of the magnetic field circulating around the wire.

## Kinetmatics:

$$
\begin{aligned}
& x(t)=x(0)+v(0) t+\frac{1}{2} a t^{2} \quad \text { const. accel. } \\
& v(t)=v(0)+a t
\end{aligned}
$$

## Magnetic fields \& forces:

$$
\begin{aligned}
\left|\overrightarrow{\mathbf{F}}_{B}\right| & =q\left|\overrightarrow{\mathbf{v}} \||\overrightarrow{\mathbf{B}}| \sin \theta_{v B} \quad \text { charge } q\right. \\
\left|\overrightarrow{\mathbf{F}}_{B}\right| & =B I l \sin \theta \quad \text { wire } \\
|\overrightarrow{\boldsymbol{\tau}}| & =B I A N \sin \theta \quad \text { torque current loop } \\
\overrightarrow{\mathbf{B}} & =\frac{\mu_{0} I}{2 \pi r} \hat{\theta} \quad \text { wire } \\
\overrightarrow{\mathbf{B}} & =\mu_{0} \frac{N}{L} I \hat{\mathbf{z}} \equiv \mu_{0} n I \hat{\mathbf{z}} \quad \text { solenoid } \\
\frac{\left|\overrightarrow{\mathbf{F}}_{12}\right|}{l} & =\frac{\mu_{0} I_{1} I_{2}}{2 \pi d} \quad 2 \text { wires, force per length }
\end{aligned}
$$

ac Circuits

$$
\begin{aligned}
\tau & =L / R \quad \text { RL circuit } \quad \tau=R C \quad \text { RC circuit } \\
\omega_{\text {cutoff }} & =\frac{1}{\tau}=2 \pi f_{\text {cutoff }} \quad \text { at cutoff, } X \text { 's are equal. } \\
X_{C} & =\frac{1}{2 \pi f C} \quad \text { "resistance" of a capacitor for ac } \\
X_{L} & =2 \pi f L \quad \text { "resistance" of an inductor for ac }
\end{aligned}
$$

## Current

$$
\begin{aligned}
I & =\frac{\Delta Q}{\Delta t}=n q A v_{d} \\
J & =\frac{I}{A}=n q v_{d} \\
v_{d} & =\frac{-e \tau}{m} E \quad \tau=\text { scattering time } \\
\varrho & =\frac{m}{n e^{2} \tau} \\
\Delta V & =\frac{\varrho l}{A} I=R I \\
R & =\frac{\Delta V}{I}=\frac{\varrho l}{A} \\
\mathscr{P} & =E \cdot \Delta t=I \Delta V=I^{2} R=\frac{[\Delta V]^{2}}{R} \text { power }
\end{aligned}
$$

## Right-hand rule \# I

I. Point the fingers of your right hand along the direction of $\overrightarrow{\mathbf{v}}$.
2. Point your thumb in the direction of $\overrightarrow{\mathbf{B}}$.
3. The magnetic force on a + charge points out from the back of your hand.

| Derived unit | Symbol | equivalent to |
| :--- | :---: | :---: |
| newton | N | $\mathrm{kg} \cdot \mathrm{m} / \mathrm{s}^{2}$ |
| joule | J | $\mathrm{kg} \cdot \mathrm{m}^{2} / \mathrm{s}^{2}=\mathrm{N} \cdot \mathrm{m}$ |
| watt | W | $\mathrm{J} / \mathrm{s}=\mathrm{m}^{2} \cdot \mathrm{~kg} / \mathrm{s}^{3}$ |
| coulomb | C | $\mathrm{A} \cdot \mathrm{s}$ |
| V | $\mathrm{W} / \mathrm{A}=\mathrm{m}^{2} \cdot \mathrm{~kg} / \cdot \mathrm{s}^{3} \cdot \mathrm{~A}$ |  |
| farad | F | $\mathrm{C} / \mathrm{V}=\mathrm{A}^{2} \cdot \mathrm{~s}^{4} / \mathrm{m}^{2} \cdot \mathrm{~kg}$ |
| ohm | $\Omega$ | $\mathrm{V} / \mathrm{A}=\mathrm{m}^{2} \cdot \mathrm{~kg} / \mathrm{s}^{3} \cdot \mathrm{~A}^{2}$ |
| tesla | T | $\mathrm{Wb} / \mathrm{m}^{2}=\mathrm{kg} / \mathrm{s}^{2} \cdot \mathrm{~A}$ |

