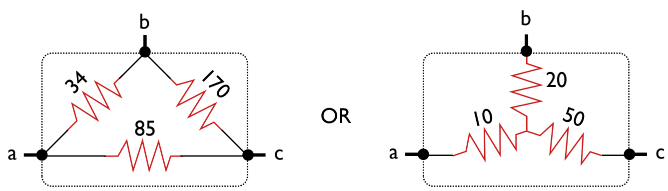
UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 102-1 / LeClair   Summer II 2008

Exam II Solutions

1 dc Circuits - solve 2/4

1. A black box with three terminals, a, b, and c, contains nothing but three resistors and connecting wire. Measuring the resistance between pairs of terminals, you measure $R_{ab} = 30\,\Omega$, $R_{ac} = 60\,\Omega$, and $R_{bc} = 70\,\Omega$. Show that the box could be either of those below.



First, consider the box on the left side. Measuring between points a b (with point c unconnected), we would find a $34\,\Omega$ resistor in parallel with a series combination of $85\,\Omega$ and $170\,\Omega$. The series combination of $85\,\Omega$ and $170\,\Omega$ just gives $255\,\Omega$, and that in parallel with $34\,\Omega$ gives

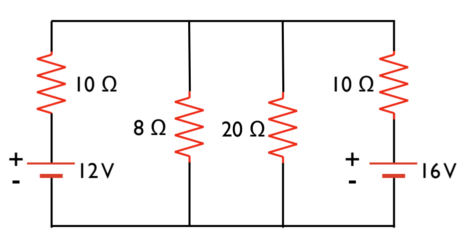
$$R_{ab} = \frac{(34\,\Omega)\,(255\,\Omega)}{34\,\Omega + 255\,\Omega} = 30\,\Omega$$

Similarly, we can find $R_{bc} = 70\,\Omega$ and $R_{ac} = 60\,\Omega$.

For the box on the right, if we connect only points a and b then the $50\,\Omega$ resistor does nothing - it has one end disconnected. Thus, $R_{ab} = 30\,\Omega$, and similarly $R_{bc} = 70\,\Omega$, $R_{ac} = 60\,\Omega$. Since a measurement of the resistance between any two terminals yields the same result, the two boxes are indistinguishable.

Establishing the equivalence of these two configurations is more generally known as a "Y-Δ" transformation:
`http://en.wikipedia.org/wiki/Y-%CE%94_transform`

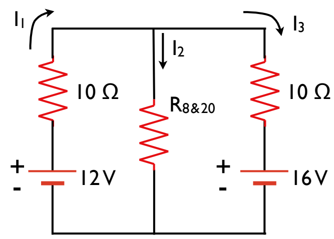
2. Find the current in the two batteries below, and find the power they deliver.



The 8 and $20\,\Omega$ resistors in the center of the circuit are purely in parallel, and can be combined immediately. Their equivalent is:

$$R_{8\&20} = \frac{1}{\frac{1}{8\,\Omega} + \frac{1}{20\,\Omega}} = \frac{40}{7}\,\Omega \approx 5.71\,\Omega \tag{1}$$

That simplifies matters greatly, and the new circuit looks like this:



Now we have a much simpler two-loop circuit. In order to solve the circuit we choose – arbitrarily – currents I_1, I_2 and I_3, labelled in the diagram above. Given a choice for the current direction in each branch, we can impose conservation of energy around the left- and right-side loops, and conservation of charge at either of the junctions to solve the circuit. This gives us three equations and three unknowns:

$$12 - 10I_1 - 5.71I_2 = 0 \tag{2}$$
$$5.71I_2 - 10I_3 - 16 = 0 \tag{3}$$
$$I_2 + I_3 = I_1 \tag{4}$$
$$\tag{5}$$

One can solve these equations by various means, which yields the current in both batteries:

$$I_1 = 0.453\,\text{A} \tag{6}$$
$$I_3 = -0.853\,\text{A} \tag{7}$$
$$\tag{8}$$

The minus sign for I_3 merely indicates that our initial direction was chosen backwards; the current actually flows upward in the right-most branch of the circuit, not downward. Given the power in each battery, the power dissipated is:

$$P_1 = I_1 \Delta V_1 = 5.44\,\text{W} \tag{9}$$
$$P_3 = I_3 \Delta V_3 = 13.7\,\text{W} \tag{10}$$

3. Two heating coils have resistances of $12.0\,\Omega$ and $6.0\,\Omega$, respectively. **(a)** What is the total power dissipated if the coils are connected in parallel to a $115\,\text{V}$ voltage source? **(b)** What if they are connected in series?

In parallel, the heating elements have an equivalent resistance of

$$R_{\text{eq}} = \frac{1}{\frac{1}{12\,\Omega} + \frac{1}{6\,\Omega}} = 4\,\Omega \tag{11}$$

The battery applies $115\,\text{V}$ across that equivalent resistance, so the power is

$$P = \frac{\Delta V^2}{R_{\text{eq}}} = 3300\,\text{W} \tag{12}$$

In series, the two heating elements have an equivalent resistance of $18\,\Omega$. The battery applies $115\,\text{V}$ across that equivalent resistance, so the power is

$$P = \frac{\Delta V^2}{R_{\text{eq}}} = 735\,\text{W} \tag{13}$$

4. A copper wire $1\,\text{km}$ long is connected across a $6\,\text{V}$ battery. The resistivity of the copper is $1.7 \times 10^{-8}\,\Omega\,\text{m}$, and the number of conduction electrons per cubic meter is 8×10^{28}. **(a)** What is the drift velocity of the conduction electrons under these circumstances? **(b)** How long does it take an electron to drift once around the circuit?

The drift velocity v_d is given through the relationship $I = nqv_dA$, where n is the number of charges per unit volume, q the charge per electron, and A the cross-sectional area. The current can be found from the applied voltage and the resistance, the latter of which is determined by the wire's length l, cross-sectional area A, and resistivity ρ. First, the resistance is:

$$R = \frac{\rho l}{A} = \frac{\Delta V}{I} \tag{14}$$

Using this relationship in our equation for drift velocity (and remembering to convert from km to m),

$$v_d = \frac{I}{nqA} = \frac{A\Delta V}{nq\rho l} \approx 2.76 \times 10^{-5}\,\text{m/s} \tag{15}$$

At this velocity, in order to travel a distance of $1\,\text{km}$, a given charge requires a time

$$\Delta t = \frac{l}{v_d} \approx 3.6 \times 10^7\,\text{s} \sim 1\,\text{yr} \tag{16}$$

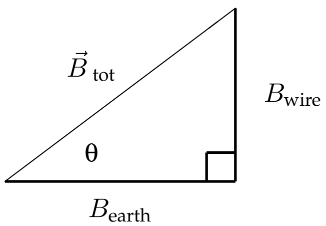
2 Magnetism - solve 2/4

1. In a motorboat, the compass is mounted at a distance of $0.80\,\text{m}$ from a cable carrying a current of $20\,\text{A}$ from an electric generator to a battery. **(a)** What magnetic field does this current produce at the location of the compass? Assume the cable is a long, straight wire. **(b)** The horizontal (north) component of the Earth's magnetic field is $1.8 \times 10^{-5}\,\text{T}$. Since the compass points in the direction of the *net* horizontal magnetic field, the current will cause a deviation of the compass needle. Assume that the magnetic field of the current is horizontal and at a right angle to the horizontal component of the earth's magnetic field. Under these circumstances, by how many degrees will the compass deviate from true north?

At a distance of $d = 0.80\,\mathrm{m}$, a wire carrying a current of $I = 20\,\mathrm{A}$ produces a magnetic field of

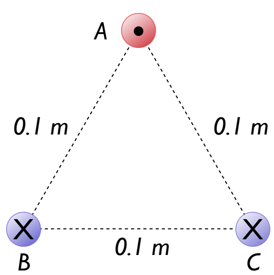
$$B_{\text{wire}} = \frac{\mu_o I}{2\pi d} \approx 5\,\mu\mathrm{T} \tag{17}$$

Without the field of the wire, the earth's magnetic field defines true north. Adding the wire's magnetic field to that, the compass will point along the resulting magnetic vector. Thus, we need to find the direction of the total magnetic field relative to the earth's field:

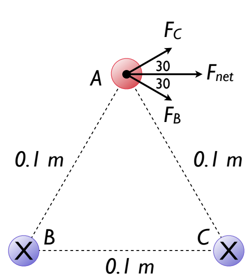


$$\begin{aligned} B_{\text{wire}} &= 5\,\mu\mathrm{T} \\ B_{\text{earth}} &= 18\,\mu\mathrm{T} \\ \tan\theta &= B_{\text{wire}}/B_{\text{earth}} \approx 0.278 \\ \theta &\approx 15.5^\circ \end{aligned}$$

2. Three long parallel wires pass through the corners of an equilateral triangle of side $0.1\,\mathrm{m}$ and are perpendicular to the plane of the triangle. Each wire carries a current of $15\,\mathrm{A}$, the current being into the page for wires B and C, and out of the page for A. **(a)** Find the force per unit length acting on the wire A. **(b)** Sketch the direction of the forces and their resultant.



Wires B and C will produce magnetic fields at the position of wire A, and this will lead to a net force on the current flowing in wire A. First, we must find the magnetic field due to wires B and C alone. Their vector sum will give the net field, which will let us calculate the force. The field from both wires B and C will be at right angles to a line connecting each wire to wire A, as shown below:



Clearly, the vertical components of the field from wires B and C will cancel, and the net field will be purely in the horizontal direction. The net field from either wire at a distance of $d\!=\!0.1\,\mathrm{m}$ is, in magnitude,

$$|\vec{F}_C| = |\vec{F}_B| = \frac{\mu_o I_A I_C}{2\pi d} \tag{18}$$

The net field is then twice the horizontal component:

$$F_{\text{net}} = 2|\vec{F}_C|\cos 30 = \frac{2\mu_o I_A I_C}{2\pi d}\frac{\sqrt{3}}{2} = \frac{\sqrt{3}\mu_o I_A I_C}{2\pi d} \approx 7.8\times 10^{-4}\,\mathrm{N} \tag{19}$$

3. The rate of flow of a conducting liquid can be measured with an electromagnetic flowmeter that detects the voltage induced by the motion of the liquid in a magnetic field. Suppose that a plastic pipe of diameter $0.10\,\mathrm{m}$ carries beer with a speed of $1.5\,\mathrm{m/s}$. The pipe is in a transverse magnetic field (*i.e.*, perpendicular to the pipe axis) of about $1.5\times 10^{-2}\,\mathrm{T}$. **(a)** Presume the beer is an ideal conductor. What voltage will be induced between the opposite sides of the column of liquid? **(b)** Does it matter whether the conductivity of the beer is due to mobile positive or negative charges?

This is in the end just a motion-induced voltage problem. The presence of a magnetic field perpendicular to the movement of ions in the beer means that they experience a magnetic force.

$$F_m = qvB$$

If the flow of beer is from left to right, and the magnetic field is pointing into the page, then the force for positive ions is pointing up, and for negative ions it is pointing down. This serves to separate spatially the positive and negative charges along the diameter of the pipe, the same way that positive and negative charges were separated in a conducting rod moving perpendicularly to a magnetic field. This continues until the ions reach the surface of the pipe, at which point they are separated by its diameter d.

If the positive and negative charges are separated spatially by a distance d, then this gives rise to a (uniform) electric field E, and a potential difference $\Delta V = Ed$. At equilibrium, the electric and magnetic forces are balanced - the magnetic force pulls the charges apart, and it is perfectly counterbalanced by the resulting electric force. Set the magnetic and electric fields equal to one another, use the expression for ΔV, and take care with units:

$$\begin{aligned} F_m &= qvB = F_e = qE = q\left(\frac{\Delta V}{d}\right) \\ \Rightarrow \Delta V &= Bvd \approx 2.25\,\mathrm{mV} \end{aligned}$$

(b) Does it matter whether the conductivity of the beer is due to mobile positive or negative charges?

You can verify that the sign of the potential difference does *not* depend on whether the ions are mostly positive or negative, since the force is in different directions for each ion. No matter what, positive ions go up, negative ions go down, and the same potential difference results.

4. A wire having a mass per unit length of $0.50\,\mathrm{g/cm}$ carries a $2.0\,\mathrm{A}$ current horizontally to the right. What are the direction and magnitude of the minimum magnetic field needed to lift this wire vertically upward?

Clearly, we need the magnetic force to counteract the gravitational force pulling the wire down for the wire to be lifted. The gravitational force will act downward, and thus we need the magnetic force acting upward. If the wire runs horizontally with current flowing to the right, we require a magnetic field into the page in order to have a magnetic force acting upward.

The gravitational force on a wire of length l is just the mass per unit length $\lambda = 0.5\,\mathrm{g/cm}$ times the length times the gravitational acceleration g: $F_g = \lambda g l$. The magnetic force for a wire of length l carrying a current I in a magnetic field B is $F_B = BIl$. Equating the two,

$$F_B = F_G \tag{20}$$

$$BIl = \lambda g l \tag{21}$$

$$B = \frac{\lambda g}{I} \approx 0.245\,\mathrm{T} \tag{22}$$

Note that the linear density λ must first be converted to kg/m before evaluating the expression numerically ...

3 Induction - solve 2/4

1. A technician wearing a conducting bracelet enclosing an area $0.005\,\mathrm{m}^2$ places her hand in a solenoid whose magnetic field is $5.0\,\mathrm{T}$ directed perpendicular to the plane of the bracelet. The resistance around the circumference of the bracelet is $0.02\,\Omega$. A power failure causes the field to drop to $1.50\,\mathrm{T}$ in a time of $20.0\,\mathrm{ms}$. Find **(a)** the current in the bracelet, and **(b)** the power delivered to the bracelet..

Once the power goes out, the magnetic field drops by an amount $\Delta B = 5 - 1.5 = 3.5\,\mathrm{T}$ in a time $\Delta t - 0.020\,\mathrm{s}$. This means, given the fixed area of the bracelet, that the magnetic flux is changing through the bracelet:

$$\frac{\Delta \Phi_B}{\Delta t} = A\frac{\Delta B}{\Delta t} = \left(0.005\,\mathrm{m}^2\right)\left(\frac{3.5\,\mathrm{T}}{0.020\,\mathrm{s}}\right) \approx 0.875\,\mathrm{Tm}^2/\mathrm{s} = 0.875\,\mathrm{V}$$

The change in flux leads to an induced voltage across the bracelet. Since the bracelet has a single loop of wire, the induced voltage is just

$$\Delta V = -\frac{\Delta \Phi_B}{\Delta t} = -0.875\,\mathrm{V}$$

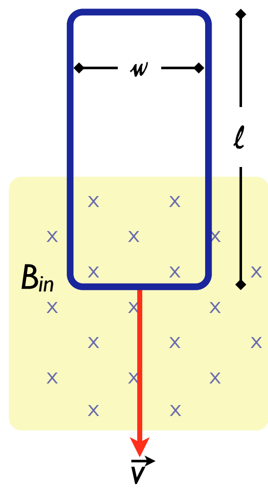
The minus sign here is not important, since the ultimate direction of current flow is not important. Given the bracelet's resistance, we can find the induced current:

$$I = \frac{\Delta V}{R} \approx 43.75\,\mathrm{A}$$

Finally, we can find the power from the current and voltage, or either one and the resistance:

$$\mathscr{P} = I^2 R = I\Delta V = \frac{\Delta V^2}{R} \approx 38.28\,\mathrm{W}$$

Conducting loop falling into a magnetic field.



This is an unreasonably large amount of power to be dumped into a bracelet on one's wrist, leading to an unreasonable generation of heat . . . for this reason, you will not see personnel working regularly with MRI machines wearing much jewelry. At least, they shouldn't be.

2. A conducting rectangular loop of mass M, resistance R, and dimensions w by l falls from rest into a magnetic field $\vec{B}$, as shown at right. At some point before the top edge of the loop reaches the magnetic field, the loop attains a constant terminal velocity v_T. Show that the terminal velocity is:

$$v_T = \frac{MgR}{B^2w^2}$$

Hint: what must be true for the velocity to be constant?

First, let us analyze the situation qualitatively. As the loop falls into the region of magnetic field, more of its area is exposed to the field, which increases the total flux through the loop. This increase in magnetic flux will cause an induced potential difference around the loop, via Faraday's law, which will create a current that tries to counteract this change in magnetic flux. Since the flux is increasing, the induced current in the loop will try to act *against* the existing field to reduce the change in flux, which means the current will circulate counterclockwise to create a field out of the page.

Once there is a current flowing in the loop, each current-carrying segment will feel a magnetic force. The left and right segments of the loop will have equal and opposite forces, leading to no net effect, but the current flowing (to the right) in the bottom segment will lead to a force $F_B = BIw$ upward. Again, this is consistent with Faraday's (and Lenz's) law - any magnetic force on the loop must act in such a way to reduce the rate at which the flux changes, which in this case clearly means slowing down the loop. The upward force on the loop will serve to counteract the gravitational force, which is ultimately responsible for the flux change in this case anyway. The faster the loop falls, the larger the upward force it experiences, and at some point the magnetic force will balance the gravitational force perfectly, leading to no net acceleration, and hence constant velocity. This is the "terminal velocity." Of course, once the whole loop is inside the magnetic field, the flux is again constant, and the loop just starts to fall normally again.[i]

[i]We would still have eddy currents, which would provide some retarding force, but for thin wires eddy current forces are probably going to

Quantitatively, we must first find the induced voltage around the loop, which will give us the current. The current will give us the force, which will finally give us the acceleration. As the loop falls into the magnetic field, at some instant t we will say that a length x of the loop has moved into the field, out of the total length l. At this time, the total flux through the loop is then:

$$\Phi_B = \vec{B} \cdot \vec{A} = BA = Bwx$$

From the flux, we can easily find the induced voltage from Faraday's law.

$$\Delta V = -\frac{\Delta \Phi_B}{\Delta t} = -Bw\frac{\Delta x}{\Delta t} = -Bwv$$

Here we made use of the fact that the rate at which the length of the loop exposed to the magnetic field changes is simply the instantaneous velocity, $\Delta x/\Delta t = v$. Once we have the induced voltage, given the resistance of the loop R, we know the current via Ohm's law:

$$I = \frac{\Delta V}{R} = -\frac{Bwv}{R}$$

From Lenz's law we know the current circulates counterclockwise. In the right-most segment of the loop, the current is flowing up, and the magnetic field into the page. The right-hand rule then dictates that the force on this current-carrying segment must be to the left. The left-most segment of the loop has a force equal in magnitude, since the current I, the length of wire, and the magnetic field are the same, but the force is in the opposite direction. Thus, taken together, the left and right segments of the loop contribute no net force. The bottom segment, however, experiences an upward force, since the current is to the right. For a constant magnetic field and constant current (true at least instantaneously), the force is easily found:

$$F_B = BIw$$

We can substitute our expression for I above:

$$F_B = BIw = -\frac{B^2w^2v}{R}$$

At the terminal velocity v_T, this upward force will exactly balance the downward gravitational force:

$$\sum F = mg - \frac{B^2w^2v_T}{R} = 0$$
$$\Longrightarrow \quad v_T = \frac{mgR}{B^2w^2}$$

3. Very large magnetic fields can be produced using a procedure called *flux compression*. A metallic cylindrical tube of radius R is placed coaxially in a long solenoid of somewhat larger radius. The space between the tube and the solenoid is filled with a highly explosive material. When the explosive is set off, it collapses the tube to a cylinder

be negligible. This is basically what we demonstrated with our conducting pendulums swinging through a magnetic field. The pendulums that had only thin segments of conductor (it looked like a fork) experienced very little damping compared to a plain flat plate.

of radius $r < R$. If the collapse happens very rapidly, induced current in the tube maintains the magnetic flux nearly constant inside the tube, even though the area shrinks. If the initial magnetic field in the solenoid is $2.50\,\mathrm{T}$, and $R/r = 12.0$, what is the maximum field that can be reached?

The basic idea here is that the *flux* through the tube is the same before and after the explosion. Since after the explosion the cross-sectional area is severely reduced, the field must be much larger in order to make the flux the same. Here's a bit from the Wikipedia about flux compression, explaining things in more detail:

> Magneto-explosive generators use a technique called "magnetic flux compression", which will be described in detail later. The technique is made possible when the time scales over which the device operates are sufficiently brief that resistive current loss is negligible, and the magnetic flux on any surface surrounded by a conductor (copper wire, for example) remains constant, even though the size and shape of the surface may change.
>
> This flux conservation can be demonstrated from Maxwell's equations. The most intuitive explanation of this conservation of enclosed flux follows from the principle that any change in an electromagnetic system provokes an effect in order to oppose the change. For this reason, reducing the area of the surface enclosed by a conductor, which would reduce the magnetic flux, results in the induction of current in the electrical conductor, which tends to return the enclosed flux to its original value. In magneto-explosive generators, this phenomenon is obtained by various techniques which depend on powerful explosives. The compression process allows the chemical energy of the explosives to be (partially) transformed into the energy of an intense magnetic field surrounded by a correspondingly large electric current.
>
> – Wikipedia, "Flux Compression"

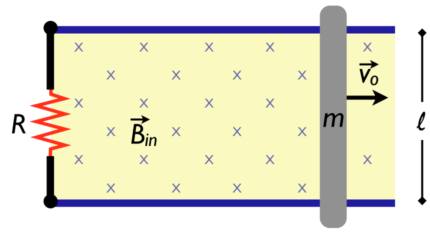
So: all we need to do is calculate the flux before and after the explosion, set them equal to each other, and solve for the field after the explosion. Quantities with 'i' subscripts refer to before the explosion, those with 'f' after the explosion, and all symbols have their usual meanings.

$$\begin{aligned}
\Phi_{B,i} &= B_i A_i = B_i \cdot \pi R^2 \\
\Phi_{B,f} &= B_f A_f = B_f \cdot \pi r^2 \\
\Phi_{B,i} &= \Phi_{B,f} \\
\implies \quad B_i \pi R^2 &= B_f \pi r^2 \\
B_f &= \left(\frac{R}{r}\right)^2 B_i = (12.0)^2 \cdot 2.50\,\mathrm{T} = 360\,\mathrm{T}
\end{aligned}$$

4. A metal bar of mass m slides without friction on two long parallel conducting rails a distance l apart. A resistor R is connected across the rails at one end; the resistance of the bar and rails is negligible. There is a constant uniform magnetic field $\vec{B}$ perpendicular to the page. At a time $t = 0$, the crossbar is given a velocity v_0 toward the right. **(a)** What force must be supplied to maintain constant velocity after the field is switched on? **(b)** How much power is dissipated in the resistor?

The motion of the bar will increase the size of the loop formed by the bar, the rails, and the resistor. This amounts to an increase in magnetic flux through the loop, which will result in an induced voltage, which will generate a current through the loop. Put another way, the movement of the conducting bar produces a voltage between its end points, which will generate a current through the bar and resistor.

Metal bar sliding on conducting rails.



The induced voltage is determined by the rate of change of magnetic flux. The flux changes because the loop area does. The loop area is its length times its width; the latter is fixed, but the former is the product of the elapsed time and the velocity of the bar. Thus,

$$\Delta V = \frac{\Delta\Phi}{\Delta t} = \frac{B\Delta A}{\Delta t} = \frac{Blv_0\Delta t}{\Delta t} = Blv_0 \tag{23}$$

This agrees with the formula for motionally-induced voltage we derived previously for a conducting bar in a magnetic field, as it should. This voltage will produce a current I in the loop, which will be determined by the resistor R if we presume the bar and rails have negligible resistance:

$$I = \frac{\Delta V}{R} = \frac{Blv_0}{R} \tag{24}$$

Now the bar has a current I flowing through it while exposed to a magnetic field B. Since the field and current are at right angles, the bar will experience a force

$$F = BIl = \frac{B^2 l^2 v_0}{R} \tag{25}$$

This is the force that must be supplied if we desire the bar to move at constant velocity.

The power dissipated in the resistor can now be found in two ways: first, at constant velocity the mechanical power required is the product of force and velocity:

$$P = Fv = \frac{B^2 l^2 v_0^2}{R} \tag{26}$$

The resistor will also dissipate a power I^2R due to the motion of the bar. In fact, since there are no other mechanisms of power dissipation, the resistor must dissipate all of the energy supplied by the mechanical force, provided the bar moves at constant velocity. The electrical power is:

$$P = I^2 R = \frac{B^2 l^2 v_0^2}{R} \tag{27}$$

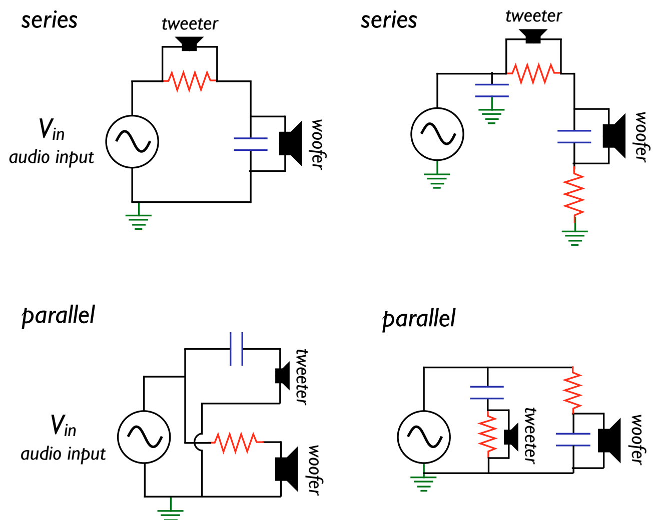
As expected, the mechanical power supplied is the same as the electrical power dissipated.

4 ac Circuits - solve 2/4

1. Using capacitors, resistors, and inductors, sketch a circuit to split an audio signal composed of many frequencies in to a low frequency part and a high frequency part, for distribution to speakers. That is, filter the incoming signal into separate low frequencies and high frequencies to send to a woofer and tweeter, respectively. Such a circuit is known as an "audio crossover." You do not need to specify the values of your components.

Even with only passive components like capacitors, inductors, and resistors, there are many ways to go about this problem. For simplicity, we will use only resistors and capacitors. We know that capacitors allow higher frequency signals through easily, while blocking lower frequency signals. A resistor, on the other hand, lets all frequencies through equally. What we can do, then, is use a capacitor to direct high frequency signals away from the woofer and toward the tweeter.

The circuit in the upper left portion of the circuit below shows the simplest possible crossover. If you look closely, it is identical to the low-pass filter in the next problem! All we have done is connect the output - which will be preferentially low-frequency signals - to the woofer. The tweeter takes the full range signal developed across the resistor, which will include the high frequencies. Another way of thinking of this circuit is that the capacitor "shorts out" the high frequency signals, and keeps them away from the woofer. One serious way that this circuit is lacking is that we don't send *only* high frequencies to the tweeter.

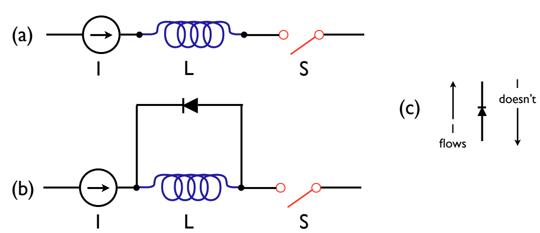


We could cure that problem by putting a low-pass and high-pass filter in series instead of just a resistor and capacitor, as shown in the upper right panel in the figure above. The low pass filter connects across the tweeter, and shorts the low frequency signals around it, such that it sees only high frequencies. The high pass filter does the reverse for the woofer, so it only sees low frequency signals. This is a perfectly workable crossover, and on paper it does just what we want. A major disadvantage (or, an advantage depending on your viewpoint) is that the two filters interact with each other - each filter is sending some signal to ground that really should be going to the other. Further, changes in any one component affect *both* high and low pass filter sections - to keep the same overall flat response, one must change all components, not just one, when one tries to tune the crossover frequency. Finally, this circuit is very sensitive to component variations and (in)accuracy. Still, the design is sufficiently simple to be appealing.

How can we do better? We can make a voltage divider to split the input signal into two, and send half to each filter and speaker. This is a *parallel* crossover. Rather than make a voltage divider out of resistors, we make one out of a pair of filters. In the lower left panel of the figure above, the audio input signal is split into two. On the branch going to the tweeter, we put a capacitor in series, which ensures that only the high-frequency part of the signal will take this path to the tweeter. The leftover low-frequency parts of the signal take the other branch to the woofer. This design is much more common, mainly because the two filter sections do not interact, which means they can be designed separately. Further, the sensitivity to component variation is far less. The lower right diagram is a re-rendering of the same circuit, adding a shorting capacitor and resistor for the woofer and tweeter. In this geometry, it is (perhaps) easier to see that both filters receive the same signal, and thus act independently.

□ **2.** A current source I is used to drive a large inductor (say, a wound wire electromagnet) as shown below. Driving inductive loads can be problematic - what happens when you open the switch providing current to an inductor in circuit **(a)** Why does adding a diode across the inductor, circuit **(b)**, add protection? Recall diodes only allow current through in one direction, as shown in **(c)**.

A "diode snubber.".



Because inductors have the property $V = -\Delta I/\Delta t$, it is not possible to turn off the current suddenly - if $\Delta t = 0$, that would imply an infinite voltage across the inductor, a violation of numerous physical laws (and good common sense). What does happen is that the voltage across the inductor rises rapidly after the switch is opened, and keeps rising until it *forces* a current to flow - for example, by making a spark jump across the poles of the switch. This is BAD, and may let the magic smoke out. Electronic devices controlling inductive loads can be damaged in this way, since essentially some component has to "break down" to satisfy the inductor's desire for constant current.

In the second circuit shown, a diode is used to protect the magic smoke in our devices, and stop the "inductive kick" that may damage other components. When the switch is initially closed in this circuit, current flows through the inductor. Current does not take the path through the diode, since it is not conducting in the direction of current flow. Now, what happens when the switch is suddenly opened? The inductor tries to keep current flowing toward the switch, as it had been moments before, and develops a negative voltage. This means that the bottom of the inductor becomes positive relative to the top - opposite the case when steady current is flowing - and a large current tries to flow *up* through the diode to maintain continuity.

Without the diode, the inductor would try to pull this large current from the switch or a nearby component, leading to a nasty spark somewhere in the circuit.[ii] When we have the diode protection, however, after the switch is thrown the back-current from the inductor can flow up through the diode - it is conducting in this direction. The back-current from the inductor will flow through the diode, creating a voltage drop from bottom to top. This voltage

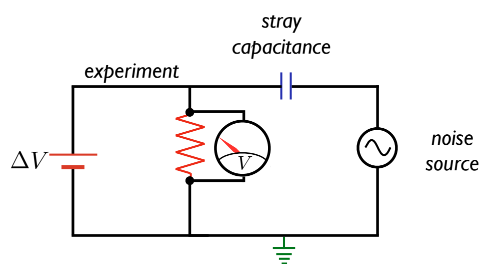
[ii] In older cars, this is more or less how an ignition coil works.

drop makes the top switch terminal at a slightly higher voltage than the supply or the top of the inductor, and no current will be "pulled" from the switch. Basically, the back-current will be short-circuited by the diode, but the forward current during normal operation will not be.

The back-voltage on an inductive load can easily be 1000 V, enough to kill nearly any solid state electronics. Problem is, inductive loads are rather common, in the form of *relays*, which are basically current-controled switches. It is virtually certain that you have used a relay at some point today, and it is equally certain that said relay had a protection diode or its RC equivalent.

`http://en.wikipedia.org/wiki/Relay`

3. Any two adjacent conductors can be considered as a capacitor, although the capacitance will be small unless the conductors are close together or long. This (unwanted) effect is termed "stray" or "parasitic" capacitance. Stray capacitance can allow signals to leak between otherwise isolated circuits (an effect called crosstalk), and it can be a limiting factor for proper functioning of circuits at high frequency. A stray capacitance can result when you touch or come close the wires in a circuit - your body provides a capacitive path between the circuit of interest and an adjacent noise source. **(a)** Explain, referencing the figure at right, why the stray capacitance allows unwanted ac signals to couple into the circuit, but does not allow dc signals. **(b)** Suggest a method for minimizing this effect.

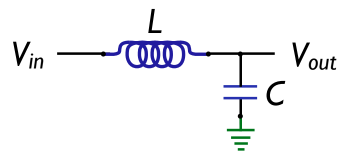


In the circuit shown, the noise source and experiment share a common ground point, which means the circuits are connected at one point. Connecting the circuits at a second point will allow signals to pass from one circuit to the other. If this connection is made by a stray capacitance, constant dc currents can pass through the ground connection, but not through the capacitance. Thus, dc signals still have only one path into the experimental circuit, there is no closed loop for them to couple into the experiment. Ac signals, on the other hand, *can* travel through the capacitor, and now have a closed loop to travel between the experiment and noise source. The higher the frequency, the easier the signals can couple into the experiment.

The solution to this problem is simply to use coaxial shielded cable. The stray capacitance arises when wires from the experiment and noise source come too close together, and the intervening region makes a capacitor out of the adjacent wires. Time-varying electric fields from one circuit can be coupled across the intervening region into the other circuit, just like ac currents can pass through a capacitor. If the experimental circuit's wires are encased in a conducting shell, the electric fields from nearby circuits outside the shell are shielded out, a result of the fact that the field inside a conductor must be zero. Of course, another solution is just to put the experiment far, far away from any potential noise sources, but this is not always practical.

4. The circuit below is a simple filter based on an inductor and a capacitor. **(a)** What sort of filter is this? Briefly explain your rationale. **(b)** Sketch the output of this circuit versus frequency, assuming the input contains all frequencies with equal amplitudes. **(c)** The cutoff frequency of a filter is the frequency at which the reactance of each component is equal - in this case, the frequency at which the inductive reactance is equal to the capacitive reactance. What is the cutoff frequency f in terms of L and C?

An LC filter.



This is a low-pass filter. High frequencies will see the capacitor as an easy path to ground, while low frequencies will avoid the capacitor and reach the output.

(b) Sketch the output of this circuit versus frequency, assuming the input contains all frequencies with equal amplitudes.

It looks basically like any other low-pass filter . . . *e.g.*, an RC low-pass filter. See your notes

(c) The cutoff frequency of a filter is the frequency at which the reactance of each component is equal - in this case, the frequency at which the inductive reactance is equal to the capacitive reactance. What is the cutoff frequency f in terms of L and C?

The cutoff frequency is the one at which the inductor's and the capacitor's reactances are equal:

$$\begin{aligned}
X_L &= X_C \qquad \text{at cutoff frequency } f_c \\
2\pi f_c L &= \frac{1}{2\pi f_c C} \\
4\pi^2 f_c^2 L &= \frac{1}{C} \\
f_c^2 &= \frac{1}{4\pi^2 LC} \\
\implies \quad f_c &= \frac{1}{2\pi\sqrt{LC}}
\end{aligned}$$