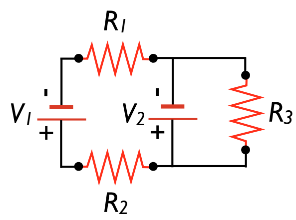
Name & ID ________________________________

PH102 Exam 2

Instructions

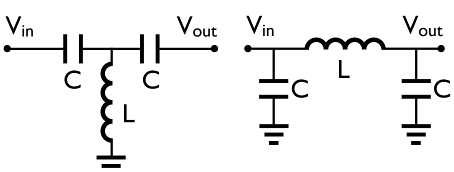
1. **Answer six of the eight questions below.** All problems have equal weight.
2. Clearly mark your which problems you have chosen using the tick box.
3. You are allowed 2 sides of a standard 8.5x11 in piece of paper and a calculator.

□ **1.** Find the current in resistors R_1 and R_3 in the circuit below if $V_1 = 10$ [V], $V_2 = 6$ [V], $R_1 = 17$ [Ω], $R_2 = R_3 = 8.3$ [Ω].



□ **2.** Refer to the figure below. The filter on the left is known as a 'T' filter, while the filter on the right is known as a 'π' filter, named for their resemblance to the respective symbols. Advantages of the T and π geometry include their suitability for insertion into a transmission line. One of these circuits is a low-pass filter, the other a high-pass filter.

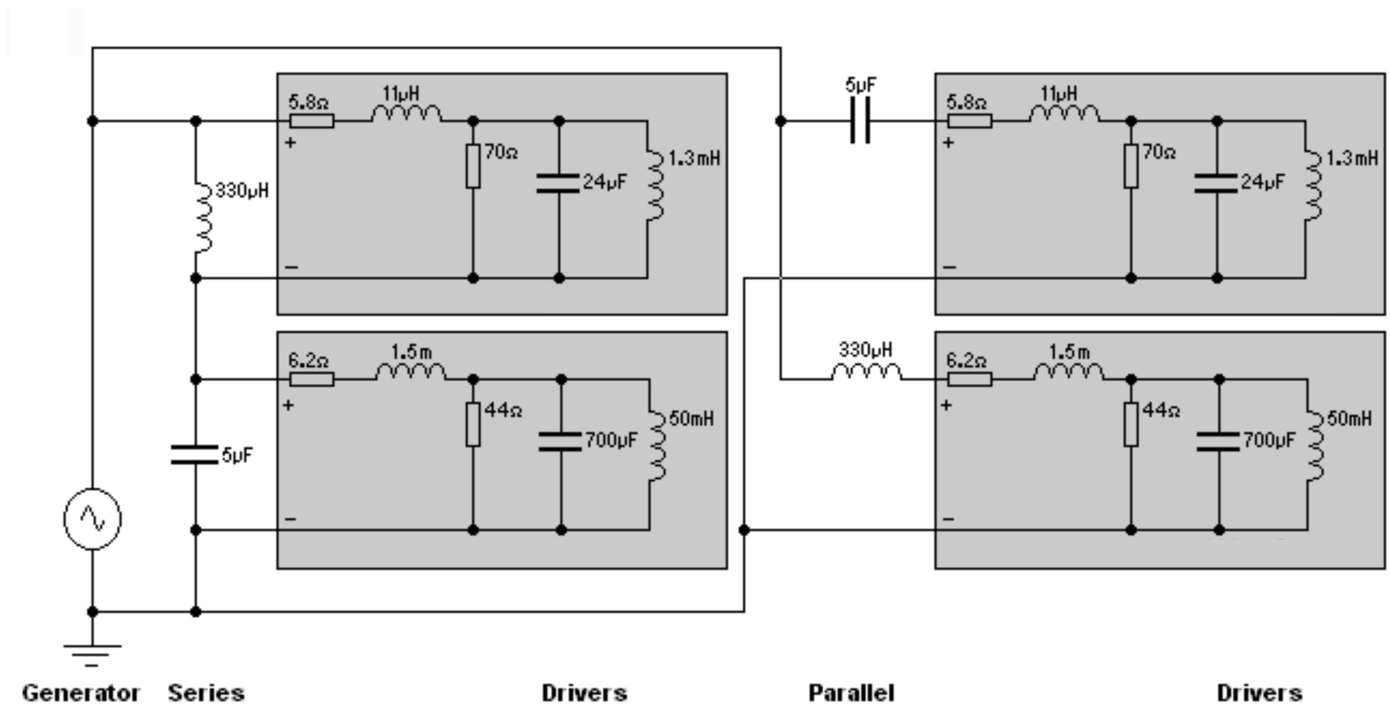
(a) Which filter is the high-pass and which is the low-pass filter?
(b) Construct a corresponding filter for each circuit using the same basic topology (π or T). In other words, for the low-pass filter, draw a corresponding high-pass filter and vice versa. Including the given diagrams, you should then have a low- and high-pass filter in both π and T layouts.



□ **3.** The figure below shows a multi-stage series-parallel audio crossover, designed to deliver low frequencies to two speakers ("woofers") and high frequencies to two others ("tweeters"). The components inside the gray boxes represent the speakers themselves, which can be reasonably-well modeled by a parallel RLC circuit with a series resistance and inductance.

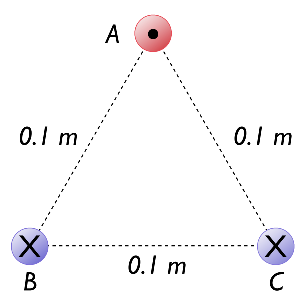
(a) Which two boxes have the low-frequency speakers, and which two have the high-frequency speakers? Explain your reasoning briefly.
(b) What is the characteristic ("cutoff") frequency of this crossover? This is the frequency at which half of the power goes to each type of speaker.
(bonus) Can you tell which type of speaker is which *only* by looking at the relative values of the components inside the gray boxes? Explain. (Worth +10% on this question).

Name & ID ____________________



□ **4.** Three long parallel wires pass through the corners of an equilateral triangle of side 0.1 m and are perpendicular to the plane of the triangle. Each wire carries a current of 15 A, the current being into the page for wires B and C, and out of the page for A.

(a) Find the total magnetic field at the center of the triangle (magnitude only).
(b) Find the magnetic field halfway between wires B and C along the line connecting those two wires.

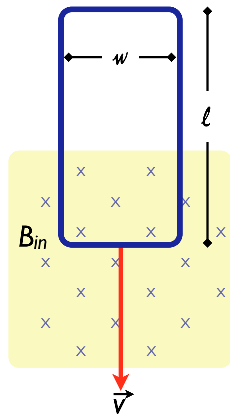


□ **5.** A conducting rectangular loop of mass M, resistance R, and dimensions w by l falls from rest into a magnetic field $\vec{B}$, as shown at right. At some point before the top edge of the loop reaches the magnetic field, the loop attains a constant terminal velocity v_T. Show that the terminal velocity is:

$$v_T = \frac{MgR}{B^2 w^2}$$

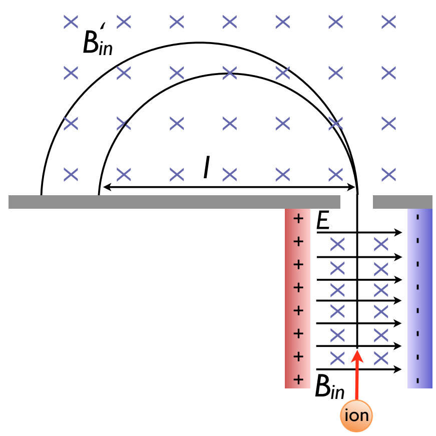
NB – terminal velocity is reached when the net acceleration is zero. See the schematic figure on the next page.

Name & ID



□ **6.** In a mass spectrometer, a beam of ions is first made to pass through a *velocity selector* with perpendicular $\vec{E}$ and $\vec{B}$ fields. Here, the electric field $\vec{E}$ is to the right, between parallel charged plates, and the magnetic field $\vec{B}$ in the same region is into the page. The selected ions are then made to enter a region of different magnetic field $\vec{B}'$, where they move in arcs of circles. The radii of these circles depend on the masses of the ions. Assume that each ion has a single charge e. Show that in terms of the given field values and the impact distance l the mass of the ion is

$$m = \frac{eBB'l}{2E}$$



□ **7.** An ocean current flows at a speed of 1 m/s in a region where the vertical component of the earth's magnetic field is 3.5×10^{-5} T. The resistivity of seawater in that region is about $\rho = 0.25\,\Omega\,\text{m}$. If there are no *external* electric fields present, what is the horizontal current density J in A/m^2? *NB – recall the general version of Ohm's law, viz.* $E = \rho J$.

□ **8.** A hair dryer intended for travelers operates at 115 V and also at 230 V. A switch on the dryer adjusts the dryer for the voltage in use. At each voltage, the dryer delivers 1000 W of heat.

(a) What must the resistance of the heating coils be for each voltage?
(b) For such a dryer, sketch a circuit consisting of two identical heating coils connected to a switch and the power outlet. Opening and closing the switch should give the proper resistance for each voltage.
(c) What is the current in the heating elements at each voltage?

Cheat Sheet

Constants:

$$k_e = 8.98755 \times 10^9\,\mathrm{N \cdot m^2 \cdot C^{-2}}$$
$$\epsilon_0 = 4\pi k_e = 8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}$$
$$e = 1.60218 \times 10^{-19}\,\mathrm{C}$$
$$\mu_0 \equiv 4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}$$
$$c = 2.99792 \times 10^8\,\mathrm{m/s}$$
$$m_{e^-} = 9.10938 \times 10^{-31}\,\mathrm{kg}$$
$$m_{p^+} = 1.67262 \times 10^{-27}\,\mathrm{kg}$$

Quadratic formula & vectors:

$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} \quad \text{magnitude}$$
$$\theta = \tan^{-1}\left[\frac{F_y}{F_x}\right] \quad \text{direction}$$

Resistors:

$$R_{eq} = R_1 + R_2 \quad \text{series}$$
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{parallel}$$

Induction:

$$\Phi_B = B_\perp A = BA\cos\theta_{BA}$$
$$\Delta V = -N\frac{\Delta\Phi_B}{\Delta t}$$
$$L = N\frac{\Delta\Phi_B}{\Delta I} = \frac{N\Phi_B}{I}$$
$$\Delta V = |\vec{v}||\vec{B}|l = |\vec{E}|l \quad \text{motional voltage}$$

Other:

$$\Delta V = |\vec{E}|\Delta x \qquad \text{const. field}$$

Power	Prefix	Abbreviation
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T

Right-hand rule #2:
Point your thumb on your right hand along the direction of the current (field). Your fingers curl in the direction of the circulating magnetic field (current).

Kinetmatics:

$$x(t) = x(0) + v(0)t + \frac{1}{2}at^2 \qquad \text{const. accel.}$$
$$v(t) = v(0) + at$$

Magnetic fields & forces:

$$|\vec{F}_B| = q|\vec{v}||\vec{B}|\sin\theta_{vB} \quad \text{charge } q \text{ at velocity } \vec{v}$$
$$|\vec{F}_B| = BIl\sin\theta \quad \text{current-carrying wire}$$
$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\theta} \quad \text{wire}$$
$$\vec{B} = \mu_0\frac{N}{L}I\hat{z} \equiv \mu_0 nI\hat{z} \quad \text{solenoid}$$
$$\frac{|\vec{F}_{12}|}{l} = \frac{\mu_0 I_1 I_2}{2\pi d} \quad \text{2 wires, force per length}$$

ac Circuits

$$\tau = L/R \quad \text{RL circuit} \qquad \tau = RC \quad \text{RC circuit}$$
$$\omega_{\text{cutoff}} = \frac{1}{\tau} = 2\pi f_{\text{cutoff}} \qquad \text{at cutoff, } X\text{'s are equal.}$$
$$X_C = \frac{1}{2\pi fC} \quad \text{“resistance” of a capacitor for ac}$$
$$X_L = 2\pi fL \quad \text{“resistance” of an inductor for ac}$$
$$f_{\text{resonance}} = \frac{1}{2\pi\sqrt{LC}} \qquad \text{resonant LC}$$

Current

$$I = \frac{\Delta Q}{\Delta t} = nqAv_d$$
$$J = \frac{I}{A} = nqv_d = \frac{E}{\rho}$$
$$v_d = \frac{-e\tau}{m}E \quad \tau = \text{scattering time}$$
$$\rho = \frac{m}{ne^2\tau}$$
$$\Delta V = \frac{\rho l}{A}I = RI$$
$$R = \frac{\Delta V}{I} = \frac{\rho l}{A}$$
$$\mathscr{P} = E \cdot \Delta t = I\Delta V = I^2 R = \frac{[\Delta V]^2}{R} \quad \text{power}$$

Right-hand rule #1

1. Point the fingers of your right hand along the direction of $\vec{v}$.
2. Point your thumb in the direction of $\vec{B}$.
3. The magnetic force on a + charge points out from the back of your hand.

Derived unit	Symbol	equivalent to
newton	N	$kg \cdot m/s^2$
joule	J	$kg \cdot m^2/s^2 = N \cdot m$
watt	W	$J/s = m^2 \cdot kg/s^3$
coulomb	C	$A \cdot s$
V	$W/A = m^2 \cdot kg/\cdot s^3 \cdot A$	
farad	F	$C/V = A^2 \cdot s^4/m^2 \cdot kg$
ohm	Ω	$V/A = m^2 \cdot kg/s^3 \cdot A^2$
tesla	T	$Wb/m^2 = kg/s^2 \cdot A$