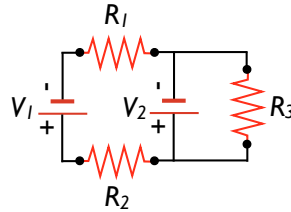# PH102 Exam 2

Instructions

1. Answer six of the eight questions below. All problems have equal weight.
2. Clearly mark your which problems you have chosen using the tick box.
3. You are allowed 2 sides of a standard 8.5x11 in piece of paper and a calculator.

☐ 1. Find the current in resistors R_1 and R_3 in the circuit below if $V_1=10$ [V], $V_2=6$ [V], $R_1=17$ [Ω], $R_2=R_3=8.3$ [Ω].

Apply Kirchhoff's rules. Let the current in V_1, R_1, and R_2 be I_1; let the current in V_2 be I_2; let the current in R_3 be I_3. Using the leftmost inner loop,

$$V_1 - I_1 R_2 - V_2 - I_1 R_1 = 0 \qquad \Longrightarrow \qquad I_1 = \frac{V_1 - V_2}{R_1 + R_2} \approx 0.158\,\text{A}$$

Using the rightmost inner loop,

$$V_3 - I_3 R_3 = 0 \qquad \Longrightarrow \qquad I_3 = \frac{V_2}{R_3} \approx 0.723\,\text{A}$$

☐ 2. Refer to the figure below. The filter on the left is known as a 'T' filter, while the filter on the right is known as a 'π' filter, named for their resemblance to the respective symbols. Advantages of the T and π geometry include their suitability for insertion into a transmission line. One of these circuits is a low-pass filter, the other a high-pass filter.

(a) Which filter is the high-pass and which is the low-pass filter?
(b) Construct a corresponding filter for each circuit using the same basic topology (π or T). In other words, for the low-pass filter, draw a corresponding high-pass filter and vice versa. Including the given diagrams, you should then have a low- and high-pass filter in both π and T layouts.
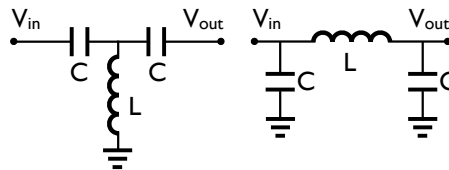
The "T" filter is a high-pass, since signals have to pass through a capacitor. The low-frequency signals will be trapped to ground by the inductor. The "π" filter is a low pass, since signals have to pass through an inductor. High frequencies are trapped by the capacitors.

Name & ID

Switching the high to a low-pass filter means making both capacitors and inductor and the inductor a capacitor. Making the low into a high-pass filter one does the same thing – the capacitors become inductors, the inductor becomes a capacitor.

▫ 3. The figure below shows a multi-stage series-parallel audio crossover, designed to deliver low frequencies to two speakers ("woofers") and high frequencies to two others ("tweeters"). The components inside the gray boxes represent the speakers themselves, which can be reasonably-well modeled by a parallel RLC circuit with a series resistance and inductance.

(a) Which two boxes have the low-frequency speakers, and which two have the high-frequency speakers? Explain your reasoning briefly.
(b) What is the characteristic ("cutoff") frequency of this crossover? This is the frequency at which half of the power goes to each type of speaker.
(bonus) Can you tell which type of speaker is which only by looking at the relative values of the components inside the gray boxes? Explain. (Worth +10% on this question).

The upper two boxes are tweeters, the lower two boxes are woofers. The cutoff is about $4000\,\text{Hz}$, determined only by the $L = 330\,\mu\text{H}$ and $C = 5\,\mu\text{F}$ adjacent to each box (take care of the μ symbols). Setting $X_L = X_C$ gives $f = 1/2\pi\sqrt{LC} \approx 4000\,\text{Hz}$. The speakers with bigger L values inside must be low frequency speakers, since larger speakers need larger coils of wire to move up and down.
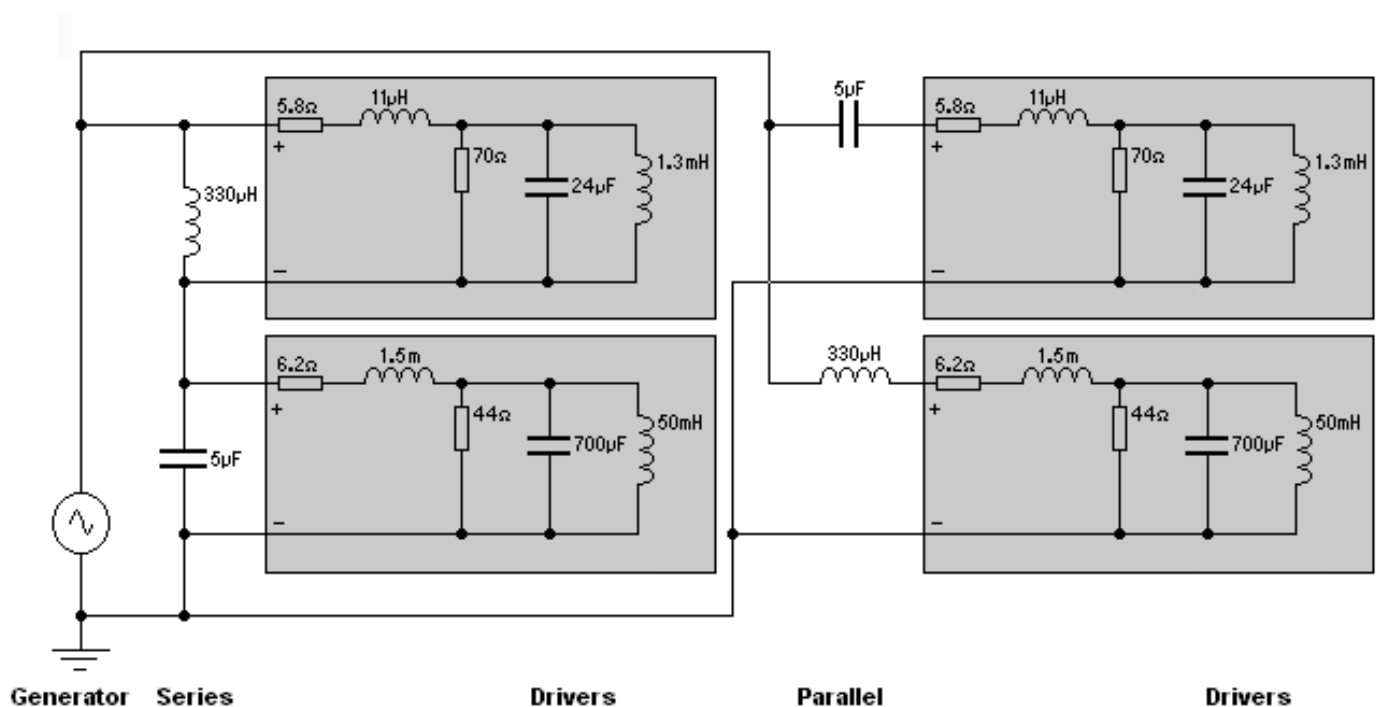
▫ 4. Three long parallel wires pass through the corners of an equilateral triangle of side $0.1\,\text{m}$ and are perpendicular to the plane of the triangle. Each wire carries a current of $15\,\text{A}$, the current being into the page for wires B and C, and out of the page for wire A.

(a) Find the total magnetic field at the center of the triangle (magnitude only).

(b) Find the magnetic field halfway between wires B and C along the line connecting those two wires.
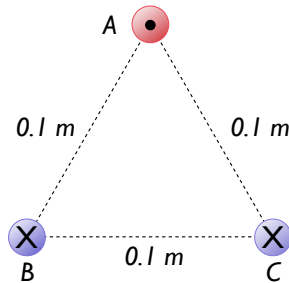
About 1.04×10^{-4} T at the center of the triangle, and about 3.5×10^{-5} T halfway between wires B and C.

(a) If an equilateral triangle has side c, you can verify that the distance to the center from any vertex is $a = c/\sqrt{3}$ (hint: law of cosines). Every wire is the same distance from the center, and they all carry the same current. Wire A will give a field that points directly to the right, and wires B and C will give a field at the center that points straight to the left. The field of wire A will cancel that of wire B, being equal in magnitude and opposite in direction, leaving only the field of wire C - a single wire a distance $0.1/\sqrt{3}$ away carrying 15 A:

$$B_{net} = B_c = \frac{\mu_o I}{2\pi a} \approx 52 \,\mu T \tag{1}$$

The original answer to this question was in error by a factor 2.

(b) Halfway between wires B and C, the field from wire B will be straight down, and the field from wire C straight up. Since the currents are the same, and we are the same distance form either wire, their fields will cancel. That leaves only the field from wire A, which will be horizontally to the right. It is a distance $a = c\sqrt{3}/2$ away if the sides have length c, so the field is

$$B_{net} = B_a = \frac{\mu_o I}{2\pi a} \approx 35 \,\mu T \tag{2}$$

□ 5. A conducting rectangular loop of mass M, resistance R, and dimensions w by l falls from rest into a magnetic field $\vec{B}$, as shown at right. At some point before the top edge of the loop reaches the magnetic field, the loop attains a constant terminal velocity v_T. Show that the terminal velocity is:

$$v_T = \frac{MgR}{B^2 w^2}$$

NB – terminal velocity is reached when the net acceleration is zero. See the schematic figure on the next page.

First, let us analyze the situation qualitatively. As the loop falls into the region of magnetic field, more of its area is exposed to the field, which increases the total flux through the loop. This increase in magnetic flux will cause an induced potential difference around the loop, via Faraday's law, which will create a current that tries to counteract this change in magnetic flux. Since the flux is increasing, the induced current in the loop will try to act against the existing field to reduce the change in flux, which means the current will circulate counterclockwise to create a field out of the page.

Once there is a current flowing in the loop, each current-carrying segment will feel a magnetic force. The left and right segments of the loop will have equal and opposite forces, leading to no net effect, but the current flowing (to
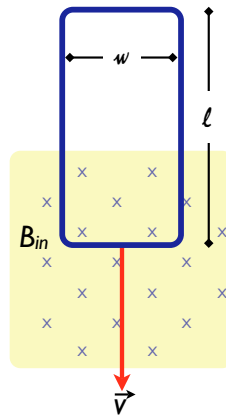
the right) in the bottom segment will lead to a force $F_B = BIw$ upward. Again, this is consistent with Faraday's (and Lenz's) law - any magnetic force on the loop must act in such a way to reduce the rate at which the flux changes, which in this case clearly means slowing down the loop. The upward force on the loop will serve to counteract the gravitational force, which is ultimately responsible for the flux change in this case anyway. The faster the loop falls, the larger the upward force it experiences, and at some point the magnetic force will balance the gravitational force perfectly, leading to no net acceleration, and hence constant velocity. This is the "terminal velocity." Of course, once the whole loop is inside the magnetic field, the flux is again constant, and the loop just starts to fall normally again.[i]

Quantitatively, we must first find the induced voltage around the loop, which will give us the current. The current will give us the force, which will finally give us the acceleration. As the loop falls into the magnetic field, at some instant t we will say that a length x of the loop has moved into the field, out of the total length l. At this time, the total flux through the loop is then:

$$\Phi_B = \vec{B} \cdot \vec{A} = BA = Bwx$$

From the flux, we can easily find the induced voltage from Faraday's law.

$$\Delta V = -\frac{\Delta \Phi_B}{\Delta t} = -Bw\frac{\Delta x}{\Delta t} = -Bwv$$

Here we made use of the fact that the rate at which the length of the loop exposed to the magnetic field changes is simply the instantaneous velocity, $\Delta x/\Delta t = v$. Once we have the induced voltage, given the resistance of the loop R, we know the current via Ohm's law:

$$I = \frac{\Delta V}{R} = -\frac{Bwv}{R}$$

From Lenz's law we know the current circulates counterclockwise. In the right-most segment of the loop, the current is flowing up, and the magnetic field into the page. The right-hand rule then dictates that the force on this current-carrying segment must be to the left. The left-most segment of the loop has a force equal in magnitude, since the current I, the length of wire, and the magnetic field are the same, but the force is in the opposite direction. Thus, taken together, the left and right segments of the loop contribute no net force. The

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[i]We would still have eddy currents, which would provide some retarding force, but for thin wires eddy current forces are probably going to be negligible. This is basically what we demonstrated with our conducting pendulums swinging through a magnetic field. The pendulums that had only thin segments of conductor (it looked like a fork) experienced very little damping compared to a plain flat plate.

bottom segment, however, experiences an upward force, since the current is to the right. For a constant magnetic field and constant current (true at least instantaneously), the force is easily found:

$$F_B = BIw$$

We can substitute our expression for I above:

$$F_B = BIw = -\frac{B^2w^2v}{R}$$

At the terminal velocity v_T, this upward force will exactly balance the downward gravitational force:

$$\sum F = mg - \frac{B^2w^2v_T}{R} = 0$$

$$\implies \quad v_T = \frac{mgR}{B^2w^2}$$

☐ 6. In a mass spectrometer, a beam of ions is first made to pass through a velocity selector with perpendicular $\vec{E}$ and $\vec{B}$ fields. Here, the electric field $\vec{E}$ is to the right, between parallel charged plates, and the magnetic field $\vec{B}$ in the same region is into the page. The selected ions are then made to enter a region of different magnetic field $\vec{B}'$, where they move in arcs of circles. The radii of these circles depend on the masses of the ions. Assume that each ion has a single charge e. Show that in terms of the given field values and the impact distance l the mass of the ion is

$$m = \frac{eBB'l}{2E}$$
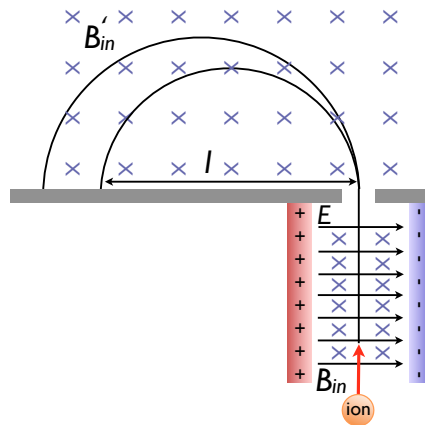
Standard mass spectrometer problem from the text, just be sure to use the new field B' in the region with no electric field, and note that $l=2r$.

☐ 7. An ocean current flows at a speed of $1\,\mathrm{m/s}$ in a region where the vertical component of the earth's magnetic field is $3.5 \times 10^{-5}\,\mathrm{T}$. The resistivity of seawater in that region is about $\rho = 0.25\,\Omega\,\mathrm{m}$. If there are no external electric fields present, what is the horizontal current density J in $\mathrm{A/m^2}$? NB – recall the general version of Ohm's law, viz. $E = \rho J$.

Ions in the seawater in motion experience a magnetic force qvB, which will separate positive and negative ions. This results in an electric force qE. In equilibrium, the two will balance, giving $E = vB$. Using Ohm's law, $J = E/\rho = vB/\rho \approx 1.4 \times 10^{-4}\,\text{A/m}^2$.

□ 8. A hair dryer intended for travelers operates at $115\,\text{V}$ and also at $230\,\text{V}$. A switch on the dryer adjusts the dryer for the voltage in use. At each voltage, the dryer delivers $1000\,\text{W}$ of heat.

(a) What must the resistance of the heating coils be for each voltage?
(b) For such a dryer, sketch a circuit consisting of two identical heating coils connected to a switch and the power outlet. Opening and closing the switch should give the proper resistance for each voltage.
(c) What is the current in the heating elements at each voltage?

$13.225\,\Omega$ for $115\,\text{V}$, $52.9\,\Omega$ for $230\,\text{V}$, giving $4.35\,\text{A}$ and $8.7\,\text{A}$ respectively.