Name & ID ____

PH102 Final Exam

Instructions

1. **Answer six of the nine questions below.** All problems have equal weight.
2. Clearly mark your which problems you have chosen using the tick box.
3. You are allowed 2 sheets of standard 8.5x11 in paper and a calculator.

□ **1.** Three protons and three electrons are to be placed at the vertices of a regular octahedron of edge length a. We want to find the potential energy of the system, or the work required to assemble it starting with the particles infinitely far apart. There are essentially two different arrangements possible. What is the energy of each? Symbolic answer, please.

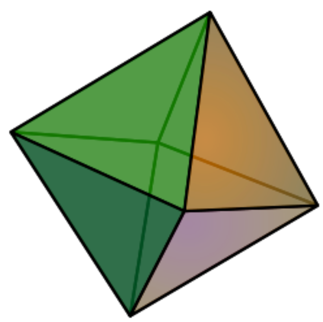

Figure 1: *An octahedron. It has eight faces and six vertices.*

□ **2.** An interstellar dust grain, roughly spherical with a radius of 3×10^{-7} m, has acquired a negative charge such that its electric potential is -0.15 Volts.

(a) How many extra electrons has it picked up?
(b) What is the strength of the electric field at its surface?

□ **3.** A 50 kV direct-current power line consists of two conductors 2 m apart. When this line is transmitting a power of 10 MW, how strong is the magnetic field halfway between the conductors?

□ **4.** A spaceship traveling at $0.70c$ away from the Earth launches a projectile of muzzle speed $0.90c$ (relative to the spaceship). What is the speed of the projectile relative to Earth if it is launched in the forward direction? In the backward direction?

□ **5.** Two positively charged particles separated by a distance d, each with charge q and mass m, are initially moving at the same speed v in opposite directions perpendicular to the line joining them. A magnetic field applied perpendicularly to the plane of the page will bend the paths of the particles into circles. What strength of magnetic field is necessary to make them collide head-on midway between the two starting points? (Ignore the electrical forces between the charges.)

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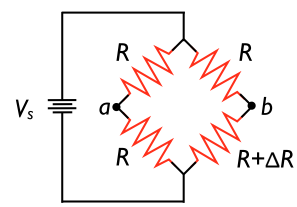
□ **6.** The bottom half of a beaker of depth 20 cm is filled with water ($n = 1.33$) and the top half is filled with oil ($n = 1.48$). If you look into this beaker from above, how far below the upper surface of the oil does the bottom of the beaker seem to be?

□ **7.** A particle of charge q and mass m, moving with a constant speed v perpendicular to a constant magnetic field B follows a circular path. If the angular momentum is quantized so that $|\vec{L}| = mvr = n\hbar$, determine the allowed radii for the particle in terms of the preceding quantities.

□ **8.** The circuit at right is known as a *Wheatstone Bridge*, and it is a useful circuit for measuring small changes in resistance. Perhaps you can figure out why. Three of the four branches on our bridge have identical resistance R, but the fourth has a slightly different resistance, ΔR more than the other branches, such that its total resistance is $R + \Delta R$.

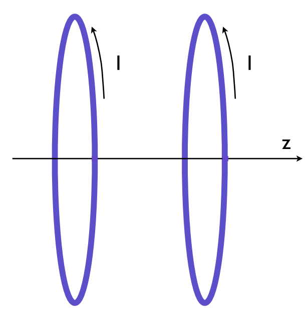
In terms of the source voltage V_s, base resistance R and change in resistance ΔR, what is the potential difference between points a and b? You may assume the voltage source and wires are perfect (no internal resistance and no voltage drop, respectively).



Wheatstone Bridge

□ **9.** A Helmholtz coil consists of two identical circular coils separated by a distance equal to their radius R, as shown at right. Each carries current I in the same direction. Find the field at any point along the axis between the two coils (the z axis in the figure). *Hint: The field from a single loop of radius R a distance z along the axis is:*

$$B = \frac{\mu_o I}{2} \frac{R^2}{(z^2 + R^2)^{3/2}} \quad \text{(single loop)}$$



Helmholtz Coil

Constants:

$$\begin{aligned}
N_A &= 6.022\times 10^{23}\text{ things/mol}\\
k_e &= \frac{1}{4\pi\epsilon_o} = 8.98755\times 10^{9}\,\mathrm{N\cdot m^2\cdot C^{-2}}\\
\mu_o &\equiv 4\pi\times 10^{-7}\,\mathrm{T\cdot m/A}\\
\epsilon_o &= \frac{1}{4\pi k_e} = 8.85\times 10^{-12}\,\mathrm{C^2/N\cdot m^2}\\
e &= 1.60218\times 10^{-19}\,\mathrm{C}\\
h &= 6.6261\times 10^{-34}\,\mathrm{J\cdot s} = 4.1357\times 10^{-15}\,\mathrm{eV\cdot s}\\
\hbar &= \frac{h}{2\pi}\\
c &= \frac{1}{\sqrt{\mu_0\epsilon_0}} = 2.99792\times 10^{8}\,\mathrm{m/s}\\
m_{e^-} &= 9.10938\times 10^{-31}\,\mathrm{kg} = 0.510998\,\mathrm{MeV/c^2}\\
m_{p^+} &= 1.67262\times 10^{-27}\,\mathrm{kg} = 938.272\,\mathrm{MeV/c^2}\\
m_{n^0} &= 1.67493\times 10^{-27}\,\mathrm{kg} = 939.565\,\mathrm{MeV/c^2}\\
1\,\mathrm{u} &= 931.494\,\mathrm{MeV/c^2}\\
hc &= 1239.84\,\mathrm{eV\cdot nm}
\end{aligned}$$

Quadratic formula:

$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Basic Equations:

$$\begin{aligned}
\vec{F}_{net} &= m\vec{a}\ \text{ Newton's Second Law}\\
\vec{F}_{centr} &= -\frac{mv^2}{r}\,\hat{r}\ \text{ Centripetal}
\end{aligned}$$

Magnetism

$$\begin{aligned}
|\vec{F}_B| &= q|\vec{v}||\vec{B}|\sin\theta_{vB}\\
|\vec{F}_B| &= BIl\sin\theta\ \text{ wire}\\
\vec{B} &= \frac{\mu_0 I}{2\pi r}\,\hat{\theta}\ \text{ wire}\\
\vec{B} &= \frac{\mu_0 I}{2r}\,\hat{\theta}\ \text{ loop}\\
\vec{B} &= \mu_0\frac{N}{L}I\,\hat{z} \equiv \mu_0 nI\,\hat{z}\ \text{ solenoid}\\
\frac{|\vec{F}_{12}|}{l} &= \frac{\mu_0 I_1 I_2}{2\pi d}\ \text{ 2 wires, force per length}
\end{aligned}$$

Current:

$$\begin{aligned}
I &= \frac{\Delta Q}{\Delta t} = nqAv_d\\
J &= \frac{I}{A} = nqv_d\\
v_d &= \frac{-e\tau}{m}E \quad \tau = \text{scattering time}\\
\rho &= \frac{m}{ne^2\tau}\\
\Delta V &= \frac{\rho l}{A}I = RI\\
R &= \frac{\Delta V}{I} = \frac{\rho l}{A}\\
\mathscr{P} &= E\cdot\Delta t = I\Delta V = I^2R = \frac{[\Delta V]^2}{R}\quad \text{power}
\end{aligned}$$

Ohm:

$$\begin{aligned}
\Delta V &= IR\\
\mathscr{P} &= E\cdot\Delta t = I\Delta V = I^2R = \frac{[\Delta V]^2}{R}\quad \text{power}
\end{aligned}$$

EM Waves:

$$\begin{aligned}
c &= \lambda f = \frac{|\vec{E}|}{|\vec{B}|}\\
\mathcal{I} &= \left[\frac{\text{photons}}{\text{time}}\right]\left[\frac{\text{energy}}{\text{photon}}\right]\left[\frac{1}{\text{Area}}\right]\\
\mathcal{I} &= \frac{\text{energy}}{\text{time}\cdot\text{area}} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{\text{power }(\mathscr{P})}{\text{area}} = \frac{E^2_{max}}{2\mu_0 c}
\end{aligned}$$

Electric Potential:

$$\begin{aligned}
\Delta V &= V_B - V_A = \frac{\Delta PE}{q}\\
\Delta PE &= q\Delta V = -q|\vec{E}||\Delta\vec{x}|\cos\theta = -qE_x\Delta x\\
&\quad \uparrow\ \text{constant E field}\\
V_{\text{point charge}} &= k_e\frac{q}{r}\\
PE_{\text{pair of point charges}} &= k_e\frac{q_1q_2}{r_{12}}\\
PE_{\text{system}} &= \text{sum over unique pairs of charges} = \sum_{\text{pairs } ij}\frac{k_e q_i q_j}{r_{ij}}\\
-W &= \Delta PE = q(V_B - V_A)
\end{aligned}$$

Optics:

$$\begin{aligned}
\mathscr{E} &= hf = \frac{hc}{\lambda}\\
n &= \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v}\\
\frac{\lambda_1}{\lambda_2} &= \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}\quad \text{refraction}\\
n_1\sin\theta_1 &= n_2\sin\theta_2\quad \text{Snell's refraction}\\
\lambda f &= c\\
M &= \frac{h'}{h} = -\frac{q}{p}\\
\frac{1}{f} &= \frac{1}{p}+\frac{1}{q} = \frac{2}{R}\quad \text{mirror \& lens}\\
\frac{n_1}{p}+\frac{n_2}{q} &= \frac{n_2-n_1}{R}\quad \text{spherical refracting}\\
q &= -\frac{n_2}{n_1}p\quad \text{flat refracting}\\
\frac{1}{f} &= \left(\frac{n_2-n_1}{n_1}\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]\quad \text{lensmaker's}
\end{aligned}$$

Electric Force & Field

$$\begin{aligned}
\vec{F}_{e,12} &= q\vec{E}_{12} = \frac{k_e q_1 q_2}{r^2_{12}}\,\hat{r}_{12}\\
\vec{E} &= k_e\frac{|q|}{r^2}\\
\Phi_E &= |\vec{E}|A\cos\theta_{EA} = \frac{Q_{\text{inside}}}{\epsilon_0}\\
\Delta PE &= -W = -q|\vec{E}||\Delta\vec{x}|\cos\theta = -qE_x\Delta x\\
&\quad \uparrow\ \text{constant E field}
\end{aligned}$$

Capacitors:

$$\begin{aligned}
Q_{\text{capacitor}} &= C\Delta V\\
C_{\text{parallel plate}} &= \frac{\epsilon_0 A}{d}\\
E_{\text{capacitor}} &= \frac{1}{2}Q\Delta V = \frac{Q^2}{2C}\\
C_{\text{eq, par}} &= C_1 + C_2\\
C_{\text{eq, series}} &= \frac{C_1C_2}{C_1+C_2}\\
C_{\text{with dielectric}} &= \kappa C_{\text{without}}
\end{aligned}$$

Resistors:

$$
\begin{aligned}
I_{\text{V source}} &= \frac{\Delta V_{\text{rated}}}{R+r} \\
\Delta V_{\text{V source}} &= \Delta V_{\text{rated}} \frac{R}{r+R} \\
I_{\text{I source}} &= I_{\text{rated}} \frac{r}{r+R} \\
R_{\text{eq, series}} &= R_1 + R_2 \\
\frac{1}{R_{\text{eq, par}}} &= \frac{1}{R_1} + \frac{1}{R_2}
\end{aligned}
$$

RC **circuits**

$$
\begin{aligned}
Q_C(t) &= Q_0\left[1-e^{-t/\tau}\right] && \text{charging} \\
Q_C(t) &= Q_0 e^{-t/\tau} && \text{discharging} \\
Q(t) &= C\Delta V(t) \\
\tau &= RC
\end{aligned}
$$

Vectors:

$$
\begin{aligned}
|\vec{F}| &= \sqrt{F_x^2 + F_y^2} \quad \text{magnitude} \\
\theta &= \tan^{-1}\left[\frac{F_y}{F_x}\right] \quad \text{direction}
\end{aligned}
$$

Induction:

$$
\begin{aligned}
\Phi_B &= B_\perp A = BA\cos\theta_{BA} \\
\Delta V &= -N\frac{\Delta\Phi_B}{\Delta t} \\
L &= N\frac{\Delta\Phi_B}{\Delta I} = \frac{N\Phi_B}{I} \\
\Delta V &= |\vec{v}||\vec{B}|l = |\vec{E}|l \quad \text{motional voltage}
\end{aligned}
$$

ac Circuits

$$
\begin{aligned}
\tau &= L/R && \text{RL circuit} \\
\tau &= RC && \text{RC circuit} \\
X_C &= \frac{1}{2\pi f C} && \text{"resistance" of a capacitor for ac} \\
X_L &= 2\pi f L && \text{"resistance" of an inductor for ac} \\
\omega_{\text{cutoff}} &= \frac{1}{\tau} = 2\pi f
\end{aligned}
$$

Relativity

$$
\begin{aligned}
\gamma &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \\
\Delta t'_{\text{moving}} &= \gamma \Delta t_{\text{stationary}} = \gamma \Delta t_p \\
L'_{\text{moving}} &= \frac{L_{\text{stationary}}}{\gamma} = \gamma(x - vt) \\
\Delta t' &= t'_1 - t'_2 = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right) \\
v_{\text{obj}} &= \frac{v + v'_{\text{obj}}}{1 + \frac{v v'_{\text{obj}}}{c^2}} \qquad v'_{\text{obj}} = \frac{v_{\text{obj}} - v}{1 - \frac{v v_{\text{obj}}}{c^2}} \\
KE &= (\gamma - 1)mc^2 \\
E_{\text{rest}} &= mc^2 \qquad p = \gamma m v \\
E^2 &= p^2c^2 + m^2c^4
\end{aligned}
$$

Right-hand rule #1

1. Point the fingers of your right hand along the direction of $\vec{v}$.
2. Point your thumb in the direction of $\vec{B}$.
3. The magnetic force on a + charge points out from the back of your hand.

Right-hand rule #2:
Point your thumb along the direction of the current (magnetic field). Your fingers naturally curl around the direction the magnetic field (current) circulates.

Unit	Symbol	equivalent to
newton	N	$kg{\cdot}m/s^2$
joule	J	$kg{\cdot}m^2/s^2 = N{\cdot}m$
watt	W	$J/s = m^2{\cdot}kg/s^3$
coulomb	C	$A{\cdot}s$
amp	A	C/s
volt	V	$W/A = m^2{\cdot}kg/{\cdot}s^3{\cdot}A$
farad	F	$C/V = A^2{\cdot}s^4/m^2{\cdot}kg$
ohm	Ω	$V/A = m^2{\cdot}kg/s^3{\cdot}A^2$
tesla	T	$Wb/m^2 = kg/s^2{\cdot}A$
electron volt	eV	1.6×10^{-19} J
-	$1\,T \cdot m/A$	$1\,N/A^2$
-	$1\,T \cdot m^2$	$1\,V \cdot s$
-	$1\,N/C$	$1\,V/m$

Power	Prefix	Abbreviation
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T