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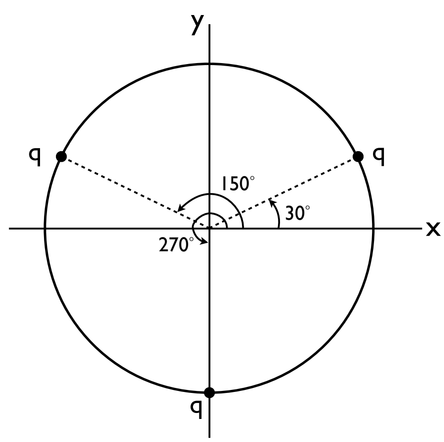
PH 102 Exam I Solutions

□ 1. Three identical charges of $q = -5.0\,\mu$C lie along a circle of radius 2.0 m at angles of 30°, 150°, and 270° as shown below. What is the resultant electric field at the center of the circle?

By symmetry, the field is zero at the center. If you draw the direction of the field from each charge at the center of the circle, there is a 120° between each of the three field vectors. If you break it down by components, you have one purely along y, and two that are 60° inclined from the x axis. Then you'll end up with something like this for the x component of the field:

$$E_x = -\frac{k_e q}{r^2} + \frac{k_e q}{r^2}\cos 60^\circ + \frac{k_e q}{r^2}\cos 60^\circ = 0 \tag{1}$$

where r is the radius of the circle. Similarly, $E_y = 0$.



□ 2. If the electric field strength in air exceeds 3.0×10^6 N/C, the air becomes a conductor and current may flow (i.e., a spark occurs). Using this fact, determine the maximum amount of charge that can be carried by a metal sphere 2.0 m in radius. Hint: recall that from Gauss' law that for points outside a spherically-symmetric charge distribution, the electric field is identical to a point charge at the center of the sphere.

Outside the sphere, Gauss' law says that the electric field is just that of a point charge, as we discussed in class. That means for a radius $r > 2$ m from the center, the field is

$$E = \frac{k_e q}{r^2} \tag{2}$$

where q is the charge on the sphere. Since we wish for E to exceed 3.0×10^6 N/C,

$$E = \frac{k_e q}{r^2} \geqslant 3.0 \times 10^6\,\text{N/C} \tag{3}$$

$$q \geqslant \frac{Er^2}{k_e} \approx 1.3 \times 10^{-3}\,\text{C} \tag{4}$$

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Now think back to the van de Graaff demo ...

□ 3. Four point charges each having charge Q are located at the corners of a square having sides of length a. What is the total potential energy of this system?

Using the principle of superposition, we know that the potential energy of a system of charges is just the sum of the potential energies for all the unique pairs of charges. The problem is then reduced to figuring out how many different possible pairings of charges there are, and what the energy of each pairing is. The potential energy for a single pair of charges, both of magnitude q, separated by a distance d is just:

$$PE_{\text{pair}} = \frac{k_e q^2}{a}$$

We need figure out how many pairs there are, and for each pair, how far apart the charges are. Once we've done that, we need to figure out the two different arrangements of charges and run the numbers.

In this case, there are not many possibilities. Label the upper left charge in each diagram "1" and number the rest clockwise. The possible pairings are then only

$$\begin{aligned} &q_1q_2, q_1q_3, q_1q_4 \\ &q_2q_3, q_2q_4 \\ &q_3q_4 \end{aligned}$$

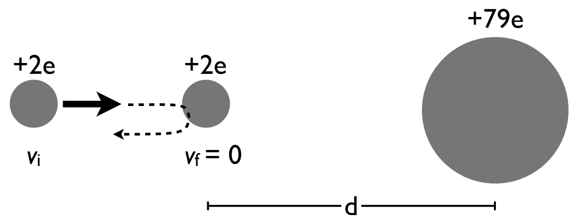
Since there are the same number of possibilities for either crystal, the total potential energy in either case is just adding all of these pairs' contributions together. Except for pairs q_2q_4 and q_1q_3, which are separated by a distance $a\sqrt{2}$, all others are separated by a distance a. Thus,

$$PE = \frac{k_e q_1 q_2}{a} + \frac{k_e q_1 q_3}{a\sqrt{2}} + \frac{k_e q_1 q_4}{a} + \frac{k_e q_2 q_3}{a} + \frac{k_e q_2 q_4}{a\sqrt{2}} + \frac{k_e q_3 q_4}{a} \tag{5}$$

All we need to do now is plug in q for all the charges, since they're all the same:

$$PE_a = \frac{k_e q^2}{a} + \frac{k_e\left(q^2\right)}{a\sqrt{2}} + \frac{k_e\left(q^2\right)}{a} + \frac{k_e\left(q^2\right)}{a} + \frac{k_e\left(q^2\right)}{a\sqrt{2}} + \frac{k_e q^2}{a} = \frac{k_e q^2}{a}\left(4+\sqrt{2}\right) \tag{6}$$

□ 4. In Rutherford's famous scattering experiments that led to the planetary model of the atom, alpha particles (having charge $+2e$ and masses of 6.64×10^{-27} kg) were fired toward a gold nucleus with charge $+79e$. An alpha particle, initially very far from the gold nucleus, is fired at a speed of $v_i = 2.00 \times 10^7$ m/s directly toward the nucleus, as shown below. How close does the alpha particle get to the gold nucleus before turning around? Assume the gold nucleus remains stationary, and that energy is conserved.



This one is just conservation of energy. The alpha particle (α) starts with some potential energy, and it can only work against its repulsion from the gold nucleus until it has converted all that kinetic energy into potential energy. At this point it comes to rest for an instant, and the repulsive force pushes it back to where it came from.

If the particle starts from an infinite distance away with purely kinetic energy, and stops with no kinetic energy at some distance r_f from the gold (Au) nucleus:

$$\Delta KE = -\Delta PE \tag{7}$$

$$\frac{1}{2}mv_i^2 = \frac{k_e q_\alpha q_{Au}}{r_f} \tag{8}$$

$$r_f = \frac{2k_e q_\alpha q_{Au}}{mv_i^2} \approx 2.74 \times 10^{-14}\,\mathrm{m} \tag{9}$$

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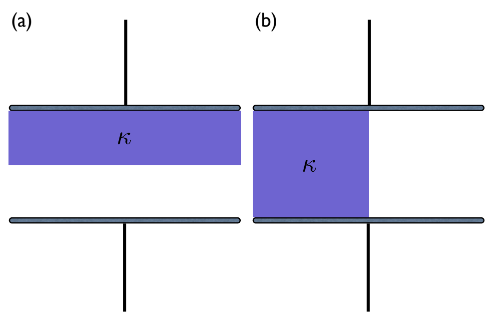
□ 5. A parallel plate capacitor has a capacitance C when there is vacuum between the plates. The gap between the plates is half filled with a dielectric with dielectric constant κ in two different ways, as shown below. Calculate the effective capacitance, in terms of C and κ, for both situations. Hint: try breaking each situation up into a combination of two separate capacitors.



(a) Dielectric parallel to the plates: $C_{\text{eff}} = \frac{2K}{1+K}C$.

It is easiest to think of this as two capacitors in series, both with half the plate spacing - one filled with dielectric, one with nothing. First, without any dielectric, we will say that the original capacitor has plate spacing d and plate area A. The capacitance is then:

$$C_0 = \frac{\epsilon_0 A}{d} \tag{10}$$

The upper half capacitor with dielectric then has a capacitance:

$$C_d = \frac{K\epsilon_0 A}{d/2} = \frac{2K\epsilon_0 A}{d} = 2KC_0 \tag{11}$$

The half capacitor without then has

$$C_{\text{none}} = \frac{\epsilon_0 A}{d/2} = \frac{2\epsilon_0 A}{d} = 2C_0 \tag{12}$$

Now we just add the two like capacitors in series:

$$\frac{1}{C_{\text{eff}}} = \frac{1}{2KC_0} + \frac{1}{2C0} \tag{13}$$

$$C_{\text{eff}} = \frac{4KC_0^2}{2KC_0 + 2C_0} \tag{14}$$

$$= \frac{2K}{1+K}C_0 \tag{15}$$

(b) Dielectric "perpendicular" to the plates: $C_{\text{eff}} = \frac{K+1}{2}C$.

In this case, we think of the half-filled capacitor as two capacitors in parallel, one filled with dielectric, one with nothing. Now each half capacitor has half the plate area, but the same spacing. The upper half capacitor with dielectric then has a capacitance:

$$C_d = \frac{K\epsilon_0 \frac{1}{2}A}{d} = \frac{K\epsilon_0 A}{2d} = \frac{1}{2}KC_0 \tag{16}$$

The half capacitor without then has

$$C_{\text{none}} = \frac{\epsilon_0 \frac{1}{2}A}{d} = \frac{\epsilon_0 A}{2d} = \frac{1}{2}C_0 \tag{17}$$

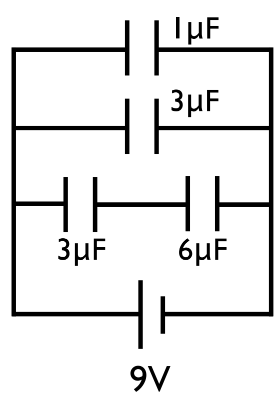
Now we just add our parallel capacitors:

$$C_{\text{eff}} = \frac{1}{2}KC_0 + \frac{1}{2}C_0 \tag{18}$$

$$= \frac{1}{2}(K+1)C_0 \tag{19}$$

$$= \frac{K+1}{2}C_0 \tag{20}$$

□ 6. Find (a) the equivalent capacitance of this circuit, and (b) the total charge stored in this circuit. Note $\mu = 10^6$.



Combine the upper two capacitors that are purely in parallel to make a single one of 4 μF. Combine the lower two purely in series to make a single one of 2 μF. These two equivalent capacitors are then in parallel, and add to 6 μF.

If the total voltage is 9 V on the equivalent capacitance of 6 μF, the total charge is $Q = C\Delta V = 54\,\mu C$.

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□ 7. In a time interval of 7.00 s, the amount of charge that passes through a light bulb is 2.51 C. (a) What is the current in the bulb? (b) How many electrons pass through the bulb in 5.00 sec?

Current is charge per unit time, so $I = 2.51\,\mathrm{C}/7\,\mathrm{s} \approx 0.359\,\mathrm{A}$.

In 5 s, the number of charges passing through is $\Delta Q = I\Delta t \approx 1.79\,\mathrm{C}$. One electron is $1.6 \times 10^{-19}\,\mathrm{C}$, so the number of electrons is $N = 1.79/1.6 \times 10^{-19} = 1.1 \times 10^{19}$.

□ 8. A 0.05 kg sample of a conducting material is all that is available. The resistivity of the material is measured to be $1.1 \times 10^{-7}\,\Omega\,\mathrm{m}$, and its density is $7860\,\mathrm{kg/m^3}$. The material is to be shaped into a solid cylindrical wire that has a total resistance of $1.5\,\Omega$. What length and diameter of wire are required?

Not yet available . . . but a problem like this will not be on the first exam in Summer 2011.

Constants:

$$k_e \equiv 1/4\pi\epsilon_o = 8.98755 \times 10^9\,\mathrm{N \cdot m^2 \cdot C^{-2}}$$
$$\epsilon_o = 8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}$$
$$e = 1.60218 \times 10^{-19}\,\mathrm{C}$$
$$m_e = 9.10938 \times 10^{-31}\,\mathrm{kg}$$

Quadratic formula:

$$0 = ax^2 + bx^2 + c \Longrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Basic Equations:

$$\begin{aligned} \vec{\mathbf{F}}_{\text{net}} &= \frac{d\vec{p}}{dt} = m\vec{a} && \text{Newton's Second Law} \\ \vec{\mathbf{F}}_{\text{centr}} &= -\frac{mv^2}{r}\hat{\mathbf{r}} && \text{Centripetal} \end{aligned}$$

Vectors:

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} \quad \text{magnitude} \qquad \theta = \tan^{-1}\left[\frac{F_y}{F_x}\right] \quad \text{direction}$$

Current:

$$\begin{aligned} I &= \frac{\Delta Q}{\Delta t} = nqAv_d \\ v_d &= \frac{e\tau}{m}E \quad \tau = \text{scattering time} \\ \rho &= \frac{m}{ne^2\tau} \\ \Delta V &= \frac{\rho l}{A}I = RI \\ R &= \frac{\Delta V}{I} = \frac{\rho l}{A} \\ \mathscr{P} &= E \cdot \Delta t = I\Delta V = I^2 R = \frac{[\Delta V]^2}{R} \quad \text{power} \end{aligned}$$

Ohm:

$$\begin{aligned} \Delta V &= IR \\ \mathscr{P} &= E \cdot \Delta t = I\Delta V = I^2 R = \frac{[\Delta V]^2}{R} \quad \text{power} \end{aligned}$$

Electric Potential:

$$\begin{aligned} \Delta V &= V_B - V_A = \frac{\Delta PE}{q} \\ \Delta PE &= q\Delta V = -q|\vec{E}||\Delta\vec{x}|\cos\theta = -qE_x\Delta x \\ & \quad \uparrow \text{ constant E field} \\ V_{\text{point charge}} &= k_e\frac{q}{r} \\ PE_{\text{pair of point charges}} &= k_e\frac{q_1 q_2}{r_{12}} \\ PE_{\text{system}} &= \text{sum unique pairs} = \sum_{\text{pairs } ij} \frac{k_e q_i q_j}{r_{ij}} \\ -W &= \Delta PE = q(V_B - V_A) \end{aligned}$$

Electric Force & Field

$$\begin{aligned} \vec{F}_{e,12} &= q\vec{E}_{12} = \frac{k_e q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} \\ \vec{E} &= k_e\frac{|q|}{r^2} \\ \Phi_E &= |\vec{E}|A\cos\theta_{EA} = \frac{Q_{\text{inside}}}{\epsilon_0} \quad \text{Gauss} \\ \Delta PE &= -W = -q|\vec{E}||\Delta\vec{x}|\cos\theta = -qE_x\Delta x \\ & \quad \uparrow \text{ constant E field} \end{aligned}$$

Capacitors:

$$\begin{aligned} Q_{\text{capacitor}} &= C\Delta V \\ C_{\text{parallel plate}} &= \frac{\epsilon_0 A}{d} \\ E_{\text{capacitor}} &= \frac{1}{2}Q\Delta V = \frac{Q^2}{2C} \\ C_{\text{eq, par}} &= C_1 + C_2 \\ C_{\text{eq, series}} &= \frac{C_1 C_2}{C_1 + C_2} \\ C_{\text{with dielectric}} &= \kappa C_{\text{without}} \end{aligned}$$

Unit	Symbol	equivalent to
newton	N	$\mathrm{kg{\cdot}m/s^2}$
joule	J	$\mathrm{kg{\cdot}m^2/s^2 = N{\cdot}m}$
watt	W	$\mathrm{J/s = m^2{\cdot}kg/s^3}$
coulomb	C	$\mathrm{A{\cdot}s}$
amp	A	$\mathrm{C/s}$
volt	V	$\mathrm{W/A = m^2{\cdot}kg/{\cdot}s^3{\cdot}A}$
farad	F	$\mathrm{C/V = A^2{\cdot}s^4/m^2{\cdot}kg}$
ohm	Ω	$\mathrm{V/A = m^2{\cdot}kg/s^3{\cdot}A^2}$
-	1 N/C	1 V/m

Power	Prefix	Abbreviation
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T