## Exam II

I. Answer six of the eight questions below. All problems have equal weight.
2. Using the tick boxes, clearly mark your which problems you have chosen.
3. You are allowed 2 sides of a standard $8.5 \times 1$ I in piece of paper and a calculator.
-I. Find the current in the $6 \Omega$ resistor.


- 2. Four very long wires are directed perpendicular to the plane of the figure below. The points where these wires cross the plane of the drawing form a square of edge length 0.45 m , and each wire carries a current of magnitude 2.5 A . Find the magnetic field at the center of the square (i.e., at the origin). Be careful about your directions before deciding things cancel ...

- 3. An electron and a proton are both moving in a circle in the $x-y$ plane due to the force produced by a magnetic field perpendicular to the plane. (a) If the electron moves clockwise and the proton counterclockwise, is the magnetic field directed into the plane or out of the plane of the drawing below? (b)

If the circular paths followed by the electron and proton have the same radii, what is the ratio of their speeds?


- 4. Copper telephone wires were originally designed to carry speech only, using a band of frequencies from 300 Hz to 3400 Hz through a system called the PSTN (Public Switched Telephone Network). The ADSL (Asymmetric Digital Subscriber Line) uses frequencies very much higher than this speech band to carry fast data traffic using frequencies between 25 kHz and 1.1 MHz . To ensure that the higherfrequency transmissions do not interfere with normal phone electronics, filters like that shown below must be installed on all lines to separate data and voice signals.
(a) Is the filter below allowing speech (low frequency) or data (high frequency) to the output?
(b) Explain your reasoning for part (a).
(c) Inductors add in series to form an equivalent inductance $\mathrm{L}_{\text {eq }}=\mathrm{L}_{1}+\mathrm{L}_{2}+\ldots$. Estimate the cutoff frequency of this filter by considering only the upper set of three inductors and the capacitor. Based on the explanation above, you should find the cutoff to fall between the voice and data bands ...

- 5. An automobile travels at $88 \mathrm{~km} / \mathrm{h}$ along a level road. The vertical downward component of the Earth's magnetic field is $5.8 \times 10^{-5} \mathrm{~T}$. (a) What is the induced voltage between the right and left door handles separated by a distance of 1.8 m ? (b) Which side is positive, and which side is negative? - 6. Two inductors $L_{1}$ and $L_{2}$ are connected in parallel to a battery. These two inductors act as one equivalent inductance $L_{e q}$. To find $L_{e q}$ we first notice that because they are connected in parallel, the
voltage across the two must be the same, but the rate at which the current changes with time $\Delta \mathrm{I} / \Delta \mathrm{t}$ is different for the two inductors. Use these facts to write the total voltage across the two inductors in the form $V=-\mathrm{L}_{\mathrm{eq}} \frac{\Delta \mathrm{I}}{\Delta \mathrm{t}}$ to find $\mathrm{L}_{\mathrm{eq}}$ in terms of $\mathrm{L}_{1}$ and $\mathrm{L}_{2}$. (Do not fret about minus signs too much.)
- 7. The light from a red laser pointer has a wavelength of about 600 nm . (a) If this laser has a power of 0.10 mW , what is the momentum carried by a 3.0 s pulse of this radiation? Recall that for light, momentum is energy divided by the speed of light, and in general power is energy per unit time. (b) If the diameter of the (circular) beam is 0.5 mm , what is the amplitude of the electric field $\mathrm{E}_{\mathrm{mx}}$ for this radiation?
- 8. What is the apparent depth of a well $d_{\text {app }}$ in which there is water of depth $d_{\text {real }}=3 \mathrm{~m}$ when viewed from an angle of incidence of $30^{\circ}$ with respect to the normal? The refractive index of water is $n=1.33$, you may take the index of refraction of air to be 1.00 . The figure below may help; the actual path of the light ray is drawn as a solid line, the dotted line in the water is the apparent path according to the observer.


Constants:

$$
\begin{aligned}
\mathrm{k}_{e} & \equiv 1 / 4 \pi \epsilon_{\mathrm{o}}=8.98755 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} \cdot \mathrm{C}^{-2} \\
\epsilon_{\mathrm{o}} & =8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{N} \cdot \mathrm{~m}^{2} \\
\mu_{\mathrm{o}} & \equiv 4 \pi \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A} \\
\mathrm{c} & =\frac{1}{\sqrt{\mu_{0} \epsilon_{0}}}=2.99792 \times 10^{8} \mathrm{~m} / \mathrm{s} \\
e & =1.60218 \times 10^{-19} \mathrm{C} \\
\mathrm{~m}_{e} & =9.10938 \times 10^{-31} \mathrm{~kg} \\
\mathrm{~m}_{\mathrm{p}}+ & =1.67262 \times 10^{-27} \mathrm{~kg}
\end{aligned}
$$

Basic Equations:

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\begin{aligned}
\overrightarrow{\mathrm{F}}_{\text {net }} & =\frac{\Delta \overrightarrow{\mathrm{p}}}{\Delta \mathrm{t}}=\mathrm{m} \overrightarrow{\mathrm{a}} \quad \text { Newton's Second Law } \\
\overrightarrow{\mathrm{F}}_{\text {centr }} & =-\frac{m v^{2}}{\mathrm{r}} \hat{\mathrm{r}} \quad \text { Centripetal } \\
|\overrightarrow{\mathrm{F}}| & =\sqrt{\mathrm{F}_{x}^{2}+\mathrm{F}_{y}^{2}} \quad \text { magnitude } \quad \theta=\tan ^{-1}\left[\frac{\mathrm{~F}_{y}}{\mathrm{~F}_{x}}\right] \quad \text { direction } \\
0 & =\mathrm{a} x^{2}+\mathrm{b} x^{2}+\mathrm{c} \Longrightarrow x=\frac{-\mathrm{b} \pm \sqrt{\mathrm{b}^{2}-4 a c}}{2 \mathrm{a}}
\end{aligned}
$$

## Magnetism

$$
\begin{aligned}
\left|\overrightarrow{\mathrm{F}}_{\mathrm{B}}\right| & =\mathrm{q}|\vec{v}||\overrightarrow{\mathrm{B}}| \sin \theta_{v \mathrm{~B}} \\
\left|\overrightarrow{\mathrm{~F}}_{\mathrm{B}}\right| & =\mathrm{BIl} \sin \theta \text { wire } \\
|\overrightarrow{\mathrm{B}}| & =\frac{\mu_{0} \mathrm{I}}{2 \pi \mathrm{r}} \text { wire } \\
|\overrightarrow{\mathrm{B}}| & =\frac{\mu_{0} \mathrm{I}}{2 \mathrm{r}} \text { loop } \\
|\overrightarrow{\mathrm{B}}| & =\mu_{0} \frac{\mathrm{~N}}{\mathrm{~L}} \mathrm{I} \equiv \mu_{0} n \mathrm{I} \hat{z} \text { solenoid } \\
\frac{\left|\overrightarrow{\mathrm{F}}_{12}\right|}{l} & =\frac{\mu_{0} \mathrm{I}_{1} \mathrm{I}_{2}}{2 \pi \mathrm{~d}} 2 \text { wires, force per length }
\end{aligned}
$$

Current/resistors/circuits:

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\begin{aligned}
\mathrm{I} & =\frac{\Delta \mathrm{Q}}{\Delta \mathrm{t}}=\mathrm{nqA} v_{\mathrm{d}} \\
\mathrm{~J} & =\frac{\mathrm{I}}{\mathrm{~A}}=\mathrm{nq} v_{\mathrm{d}} \\
v_{\mathrm{d}} & =-\frac{e \tau}{\mathrm{~m}} \mathrm{E} \quad \tau=\text { scattering time } \\
\rho & =\frac{\mathrm{m}}{\mathrm{n} \mathrm{e}^{2} \tau} \\
\Delta \mathrm{~V} & =\frac{\rho l}{\mathrm{~A}} \mathrm{I}=\mathrm{RI} \\
\mathrm{R} & =\frac{\Delta \mathrm{V}}{\mathrm{I}}=\frac{\rho l}{\mathrm{~A}} \\
\mathscr{P} & =\mathrm{E} \cdot \Delta \mathrm{t}=\mathrm{I} \Delta \mathrm{~V}=\mathrm{I}^{2} \mathrm{R}=\frac{[\Delta \mathrm{V}]^{2}}{\mathrm{R}} \text { power } \\
\mathrm{R}_{\mathrm{eq}, \text { series }} & =\mathrm{R}+\mathrm{R}_{2} \\
\frac{1}{\mathrm{R}_{\mathrm{eq}, \text { par }}} & =\frac{1}{\mathrm{R}_{1}}+\frac{1}{\mathrm{R}_{2}} \\
\mathrm{X}_{\mathrm{C}} & =\frac{1}{2 \pi \mathrm{fC}} \quad \text { "resistance" of a capacitor for ac } \\
\mathrm{X}_{\mathrm{L}} & =2 \pi \mathrm{f} \quad \text { "resistance" of an inductor for ac }
\end{aligned}
$$

Induction:

$$
\begin{aligned}
\Phi_{\mathrm{B}} & =\mathrm{B}_{\perp} \mathrm{A}=\mathrm{BA} \cos \theta_{\mathrm{BA}} \\
\Delta \mathrm{~V} & =-\mathrm{N} \frac{\Delta \Phi_{\mathrm{B}}}{\Delta \mathrm{t}} \\
\Delta \mathrm{~V} & =-\mathrm{L} \frac{\Delta \mathrm{I}}{\Delta \mathrm{t}} \\
\Delta \mathrm{~V} & =|\vec{v}||\overrightarrow{\mathrm{B}}| \mathrm{l}=|\overrightarrow{\mathrm{E}}| \mathrm{l} \text { motional voltage }
\end{aligned}
$$

EM Waves/reflection/refraction:

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\begin{aligned}
c & =\lambda f=\frac{|\vec{E}|}{|\vec{B}|} \\
I & =\text { intensity }=\frac{\text { energy }}{\text { time } \cdot \operatorname{area}}=\frac{E_{\max } B_{\max }}{2 \mu_{0}}=\frac{\text { power }(\mathscr{P})}{\text { area }}=\frac{E_{\max }^{2}}{2 \mu_{0} c} \\
n & =\frac{\text { speed of light in vacuum }}{\text { speed of light in a medium }}=\frac{c}{v} \\
\frac{\lambda_{1}}{\lambda_{2}} & =\frac{v_{1}}{v_{2}}=\frac{c / n_{1}}{c / n_{2}}=\frac{n_{2}}{n_{1}} \quad \text { refraction } \\
n_{1} \sin \theta_{1} & =n_{2} \sin \theta_{2} \quad \text { Snell's refraction } \\
\lambda f & =c \\
M & =\frac{h^{\prime}}{h}=-\frac{q}{p} \\
\frac{1}{f} & =\frac{1}{p}+\frac{1}{q}=\frac{2}{R} \quad \text { mirror \& lens }
\end{aligned}
$$

Electric Field, Force, Potential:

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\begin{aligned}
& \Delta \mathrm{PE}=\mathrm{q} \Delta \mathrm{~V}=-\mathrm{q}|\overrightarrow{\mathrm{E}}||\Delta \overrightarrow{\mathrm{x}}| \cos \theta=-\mathrm{q} \mathrm{E}_{x} \Delta \mathrm{x} \leftarrow \text { constant } \mathrm{E} \text { field } \\
& \vec{F}_{e, 12}=\quad \mathrm{q} \vec{E}_{12}=\frac{\mathrm{k}_{\mathrm{e}} \mathrm{q}_{1} \mathrm{q}_{2}}{\mathrm{r}_{12}^{2}} \hat{\mathbf{r}}_{12} \\
& \overrightarrow{\mathrm{E}} \quad=\quad k_{e} \frac{|q|}{r^{2}} \\
& V_{\text {point charge }}=k_{e} \frac{q}{r} \\
& P E_{\text {pair of point charges }}=k_{e} \frac{q_{1} q_{2}}{r_{12}} \\
& \text { Right-hand rule \# I } \\
& \text {. Point the fingers of your right hand along the direction of } \vec{v} \text {. } \\
& \text { 2. Point your thumb in the direction of } \vec{B} \text {. } \\
& \text { 3. The magnetic force on a }+ \text { charge points out from the back of your hand. }
\end{aligned}
$$

## Right-hand rule \# 2:

Point your thumb along the direction of the current (magnetic field). Your fingers naturally curl around the direction the magnetic field (current) circulates.

