

Final Exam answers relevant to Summer 2014 Exam 2**I. Relativity, Quantum, Atomic (choose 3 of 4)**

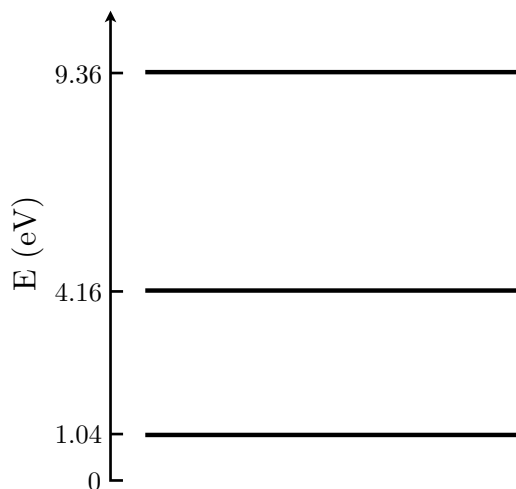
□ **1.** In a scattering experiment to reveal the atomic-scale structure of a material, electrons are accelerated through a potential difference of $\Delta V = 500$ kV. What is the smallest feature one could hope to see before quantum uncertainty spoils the resolution?

Solution: The potential difference gets you potential energy, potential energy change equals kinetic energy change. Kinetic energy gets you momentum, momentum gets you wavelength.

$$\Delta KE = \Delta PE = e\Delta V = \frac{p^2}{2m} \implies p = \sqrt{2me\Delta V} \quad (1)$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2me\Delta V}} \approx 1.7 \times 10^{-12} \text{ m} \quad (2)$$

□ **2.** What are the possible **wavelengths** of photons that could be emitted from a system with an energy level diagram like that in the figure below?



Solution: The possible photon energies are the possible energy *differences*, $9.36 - 4.16 = 5.2$ eV, $4.16 - 1.04 = 3.12$ eV, $9.36 - 1.04 = 8.32$ eV.

□ **3.** A muon is formed from a cosmic ray shower in the upper atmosphere at an altitude of 1 km. In its own reference frame, the muon has a lifetime of 2.2×10^{-6} s.

(a) If the muon moves at $0.98c$, what is its mean lifetime as measured by an observer on earth?

(b) Will it reach the surface of the earth?

(c) If you ignored relativity, would it reach the surface of the earth?

□ 4. The kinetic energy of an electron is increased by a factor of two. Neglecting relativistic effects, what factor does its wavelength change?

Solution: Use the kinetic energy-momentum relationship:

$$K = \frac{p^2}{2m} \implies p = \sqrt{2mK} \quad (3)$$

Now use the wavelength-momentum relationship:

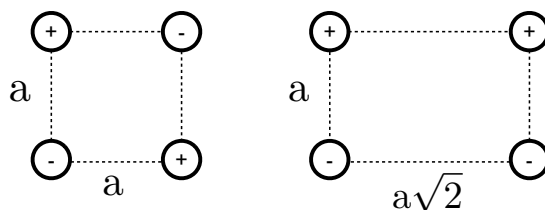
$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \quad (4)$$

If kinetic energy doubles, wavelength must decrease by a factor $\sqrt{2}$.

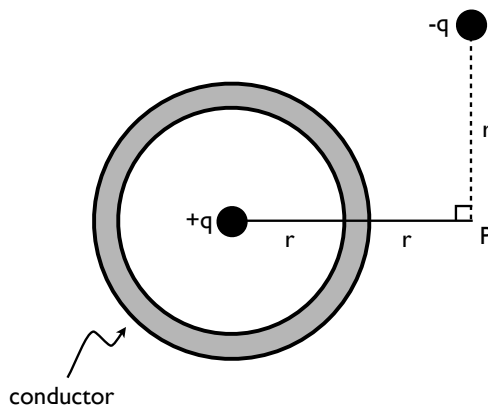
II. Electric forces, fields, and energy (choose 3 of 4)

□ 5. The two figures below show small sections of two different possible surfaces of a NaCl surface. In the left arrangement, the NaCl(100) surface, charges of $+e$ and $-e$ are arranged on a square lattice as shown. In the right arrangement, the NaCl(110) surface, the same charges are arranged in a rectangular lattice.

What is the electrical potential energy of each arrangement (symbolic answer only)? Which is more stable?



□ 6. A conducting spherical shell of radius r surrounds a point charge $+q$. Outside the shell, there is a point charge $-q$. At point P , a distance r below the $-q$ charge and $2r$ to the right of the $+q$ charge, what is the magnitude of the total electric field?



Solution: Gauss' law tells us that the point charge $+q$ and the spherical shell can be treated as single point charge $+q$. The conductor doesn't matter so long as you are outside of it. That means the field at P is just that of two point charges added together: one field E_+ due to a charge $+q$ at a distance $2r$, the other field E_- due to a charge $-q$ at a distance R .

$$E_+ = \frac{k_e q}{(2r)^2} = \frac{k_e q}{4r^2} \quad (5)$$

$$E_- = -\frac{k_e q}{r^2} \quad (6)$$

However, since they act at right angles to one another, we have to add them as vectors. This is simple enough due to the fact that they are perpendicular:

$$E_{tot} = \sqrt{E_+^2 + E_-^2} = \frac{k_e q}{r^2} \sqrt{\frac{1}{4^2} + 1} = \frac{\sqrt{17}}{4} \frac{k_e q}{r^2} \quad (7)$$

□ **7.** A household lamp has a cord with wire of diameter 1 mm and uses a total of 3 m of wire to connect to a source of voltage. If the lamp carries a steady current of 0.5 A, and the copper has a charge density of $n = 8.5 \times 10^{28}$ electrons/m³, how long would it take for one electron to travel the full length of the cord?

Solution: The length of time it will take is dictated by the drift velocity of the electrons v_d . We know this is related to the current:

$$I = nq v_d A \quad \text{or} \quad v_d = \frac{I}{nqA} \quad (8)$$

We are given n , and from the wire diameter d we can find the cross-sectional area A

$$A = \pi r^2 = \frac{1}{4} \pi d^2 \quad (9)$$

This is now enough to determine the velocity. At a velocity v_d the electrons will cover a distance $l = v_d t$ in a time t . If l is the length of the wire,

$$l = v_d t = \frac{I}{nqA} t \quad (10)$$

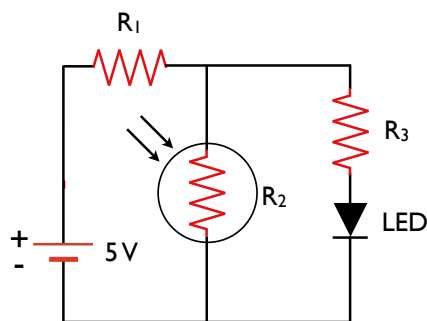
$$t = \frac{nqAl}{I} = \frac{nq\pi d^2 l}{4I} \approx 6.4 \times 10^4 \text{ sec} \approx 18 \text{ hr} \quad (11)$$

□ **8.** How many electrons should be removed from an initially uncharged spherical conductor of radius 0.300 m to produce a potential of 7.50 kV at the surface?

III. Circuits (choose 2 of 3)

□ **9.** The circuit below is a crude ‘electric eye’ I built a few nights ago. Its primary components are a photoresistor (R_2), and a light-emitting diode (LED). The photoresistor has the property that in the dark, its resistance is about $R_{2,\text{dark}} = 50\text{ k}\Omega$, while in bright light its resistance is dramatically lower, about $R_{2,\text{light}} = 0.3\text{ k}\Omega$. The (red) LED in the circuit will light up when its voltage drop exceeds 1.8 V . You may assume that the light from the LED does not reach the photoresistor to change its resistance.

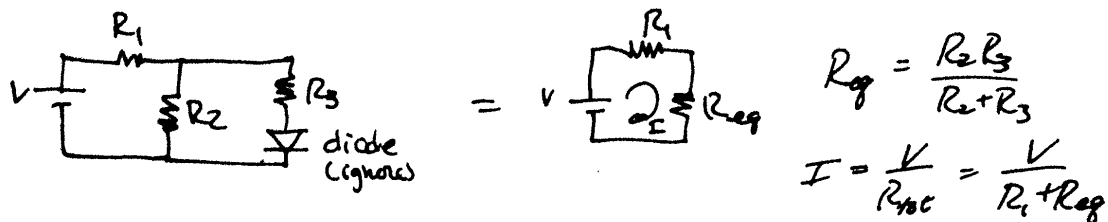
Take $R_1 = 3\text{ k}\Omega$ and $R_3 = 370\ \Omega$. The LED has a resistance of $175\ \Omega$ when lit, and $1\text{ M}\Omega$ when not. When the photoresistor is exposed to bright light (i.e., in its low resistance state), should the LED be lit or not? Explain your reasoning. *Hint: the voltage across the LED is mostly determined by the potential drop across R_1 , which is determined by the net current flowing out of the battery.*



Solution: Not really relevant to the summer 2014 exam 2, but here’s a rough qualitative explanation. When the photoresistor is lit, its resistance is very low, and it takes a lot of current from the battery (compared to the much higher R_3 and diode resistances in series). If the photoresistor resistance becomes too low, the battery can’t supply enough power to light the LED. When the photoresistor is dark, it has a high resistance, and takes little current from the battery, leaving it plenty of capacity to light the LED.

□ **10.** Refer to the previous question and its figure. Ignore the LED (pretend it is just a wire), and let $R_1 = 300\ \Omega$ and $R_3 = 200\ \Omega$. The photoresistor behaves the same as in the previous question. **(a)** What is the voltage on R_3 when the photoresistor is dark? **(b)** What is the voltage on R_3 when the photoresistor is illuminated? *Hint: note that R_2 and R_3 are in parallel.*

□ **11.** The circuit on the next page is part of an instrument in my laboratory, a Keithley Instruments model 428 current amplifier. The section of the instrument in the diagram below is a filter with a selectable frequency cutoff. The frequency range is selected by opening or closing various switches shown in the diagram. The upper three switches (U_1 , U_2 , and U_3) selectively bypass resistors R_{134} , R_{150} , and R_{138} to vary the series resistance. Switches U_{106} , U_{115} allow one to



$V_{in} R_2 = V_{in} R_3 = V_{in} R_{eq}$ ignoring diode

$V_{R3} = V_{Req} = I R_{eq} = \frac{V}{R_1 + R_{eq}} \cdot R_{eq} = V \left(\frac{1}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} \right) \frac{R_2 R_3}{R_2 + R_3}$



$V_{Req} = \frac{R_2 R_3}{R_1 (R_2 + R_3) + R_2 R_3} = \frac{R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$

$R_1 = 300\Omega \quad R_3 = 200\Omega$

$R_2 = \begin{cases} 50k\Omega & \text{dark} \\ 0.3k\Omega & \text{lit} \end{cases}$

$\Rightarrow V_{dark} = 0.40A$

$V_{lit} = 0.29A$

connect various capacitances to ground before the signal output. (The two switches labeled U106 open and close together.)

- (a) What sort of filter is this? Justify your answer in a sentence or two.
- (b) What is the cutoff frequency when switches U115 and U1 are **closed**, but all other switches are open?

N.B. - the inverted triangles in this diagram are ground connections. Recall $p = 10^{-12}$, $\mu = 10^{-6}$, and $k = 10^3$.

Solution: This is a low-pass filter. The capacitors present a low reactance to high frequencies, and take them to ground. Low frequencies see the capacitors as a high reactance, and try to avoid

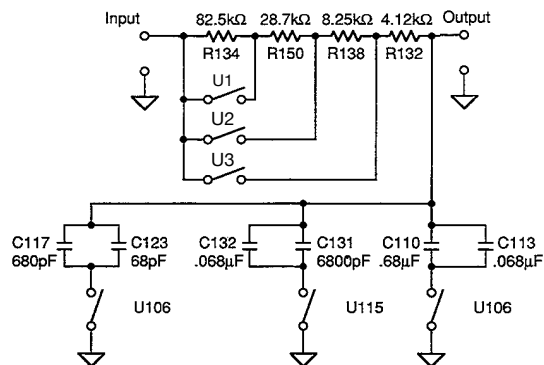


Figure 1: A portion of the input filter in a Keithley 428 current preamplifier. From the K428 instrument manual.

them, so they make it to the output. If $U115$ is closed, that connects capacitors $C131$ and $C132$ to ground, but no others. In parallel, they have an equivalent capacitance of 74.8 nF (watch the metric prefixes ...). If $U1$ is closed, that shorts out resistor $R134$, leaving resistors $R150$, $R138$, and $R132$ in series, which have an equivalent resistance of $41.07\text{ k}\Omega$. Thus, with this combination of switches closed, the low-pass filter is one with $R = 41.07\text{ k}\Omega$ and $C = 74.8\text{ nF}$. The cutoff frequency is when the two have equal reactances:

$$X_C = X_R \quad (12)$$

$$\frac{1}{2\pi fC} = R \quad (13)$$

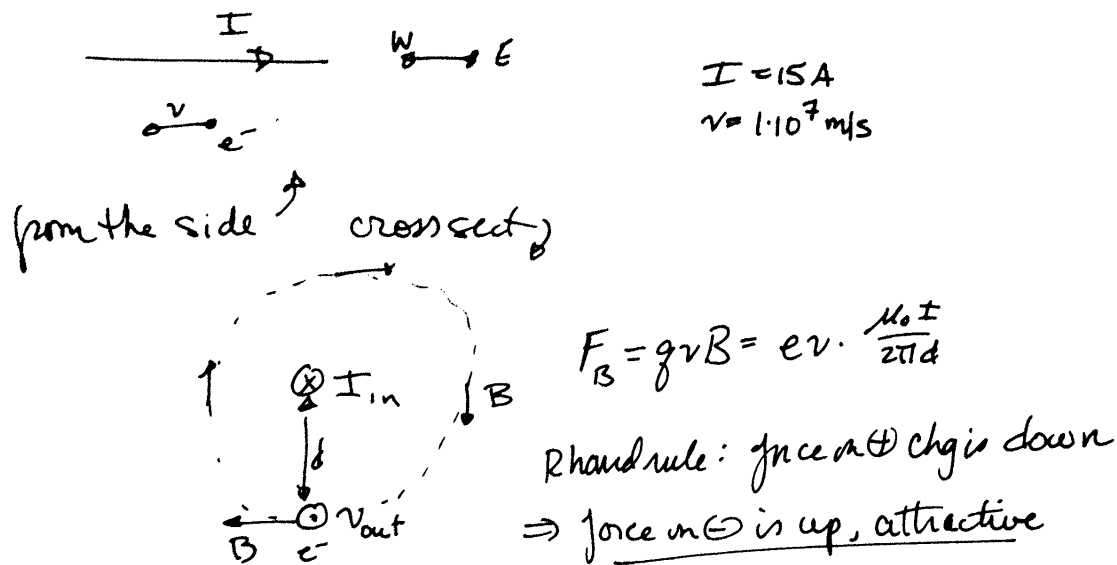
$$f = \frac{1}{2\pi RC} \approx 52\text{ Hz} \quad (14)$$

$$(15)$$

IV. Magnetism, induction (choose 2 of 3)

□ **12.** A wire carries a current $I = 15\text{ A}$ to the east. A distance d directly below it, an electron travels west at $1.0 \times 10^7\text{ m/s}$. **(a)** What is the force on the electron in magnitude, and is it attracted or repelled from the wire? **(b)** If the electron is to travel in a straight line parallel to the wire, what is d ?

□ **13.** A large solenoid of length 1.0 m , a radius of 0.01 m , and $N_1 = 1000$ turns carries a current of $I = 40\text{ A}$ and fully encloses a smaller solenoid of length 0.5 m , radius of 0.005 m , and $N_2 = 5000$ turns (see the crude figure on the following page). The larger solenoid creates a uniform magnetic field over the area of the smaller solenoid. During a power outage, the current in the outer coil suddenly decreases to zero over a time $\Delta t = 0.1\text{ s}$. What voltage is induced between the end points of the inner solenoid?



for straight line motion: $a=0$ or $\sum F_y = 0$

$$\sum F_y = F_B - mg = 0 \quad \text{magn force balances weight}$$

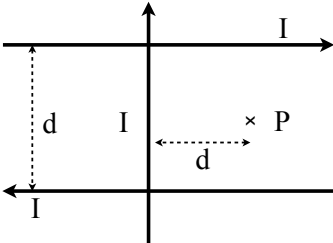
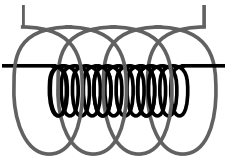
$$\Rightarrow F_B = \frac{ev\mu_0 I}{2\pi d} = mg$$

$$\Rightarrow d = \frac{\mu_0 ev I}{2\pi mg}$$

□ 14. Three wires carry a current I , as shown in the figure below (the wires cross but do not touch). The vertical wire crosses the parallel horizontal wires at a 90° angle, and the two horizontal wires are a distance d apart. What is the magnetic field at point P , a distance d from the vertical wire and halfway between the horizontal wires?

Solution: From the second right hand rule, we can figure out that the field from all three wires is into the page at point P . Since they all have the same direction, we just have to add the three fields together. We have two wires at a distance $d/2$ and one at a distance d , all carrying a current I , so the total field is

$$B_{\text{tot}} = B_1 + B_2 + B_3 = \frac{\mu_0 I}{2\pi(d/2)} + \frac{\mu_0 I}{2\pi(d/2)} + \frac{\mu_0 I}{2\pi d} = \frac{\mu_0 I}{2\pi d} (2 + 2 + 1) = \frac{5\mu_0 I}{2\pi d} \quad (16)$$



V. Optics, EM waves (choose 2 of 3)

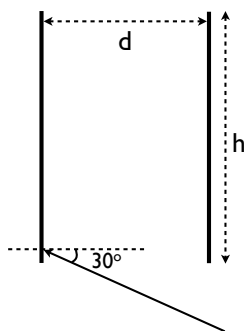
□ **15.** A small object is placed at a distance of 10 cm from a converging lens, and an image is found to form on the opposite side of the lens a distance of 8 cm from the lens. **(a)** What is the focal length of the lens? **(b)** What is the magnification factor?

Solution: We know the image and object distances, focal length is no problem:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{10} + \frac{1}{8} \implies f = 4.44 \text{ cm} \quad (17)$$

Magnification is just $M = -q/p = -8/10 = -0.8$.

□ **16.** Two parallel mirrors of height $h = 1$ m are a distance $d = 0.1$ m apart. A light ray enters through the bottom at an angle of 30° . How many times will the incident beam be reflected by each of the parallel mirrors?



Solution: Just geometry. Each time the light bounces once, it has moved upward by an amount $\delta = d \tan 30$. If the light bounces off n times, it has moved upward by $n\delta$, which needs to be greater to or equal than h . Thus,

$$n\delta = h \quad (18)$$

$$n = \frac{h}{\delta} = \frac{h}{d \tan 30} = 17.3 \quad (19)$$

17 bounces won't quite do it, 18 are required.

□ **17.** The intensity of moonlight when it reaches the Earth's surface is approximately 0.02 W/m^2 (for a full moon). What are the amplitudes (maximum intensity) of the corresponding electric and magnetic fields?

Solution: We can write intensity in terms of either the E or B field maxima:

$$I = \frac{E_m}{2\mu_0 c} = \frac{cB_m^2}{2\mu_0} \quad (20)$$

With the known intensity, first find one and then the other (or, find one and note $E = cB$ for an EM wave). $E_m \approx 3.88 \text{ V/m}$, $B_m = E_m/c \approx 13 \text{ nT}$.

Constants:

$$\begin{aligned}
g &\approx 9.81 \text{ m/s} \\
N_A &= 6.022 \times 10^{23} \text{ things/mol} \\
k_e &= \frac{1}{4\pi\epsilon_0} = 8.98755 \times 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2} \\
\mu_0 &\equiv 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \\
\epsilon_0 &= \frac{1}{4\pi k_e} = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2 \\
e &= 1.60218 \times 10^{-19} \text{ C} \\
h &= 6.6261 \times 10^{-34} \text{ J} \cdot \text{s} = 4.1357 \times 10^{-15} \text{ eV} \cdot \text{s} \\
\hbar &= \frac{h}{2\pi} \\
c &= \frac{1}{\sqrt{\mu_0\epsilon_0}} = 2.99792 \times 10^8 \text{ m/s} \\
m_{e^-} &= 9.10938 \times 10^{-31} \text{ kg} = 0.510998 \text{ MeV}/c^2 \\
m_{p^+} &= 1.67262 \times 10^{-27} \text{ kg} = 938.272 \text{ MeV}/c^2 \\
m_{n^0} &= 1.67493 \times 10^{-27} \text{ kg} = 939.565 \text{ MeV}/c^2 \\
1 \text{ u} &= 931.494 \text{ MeV}/c^2 \\
hc &= 1239.84 \text{ eV} \cdot \text{nm}
\end{aligned}$$

Quadratic formula:

$$0 = ax^2 + bx + c \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Basic Equations:

$$\begin{aligned}
\vec{F}_{\text{net}} &= m\vec{a} \text{ Newton's Second Law} \\
\vec{F}_{\text{centr}} &= -\frac{mv^2}{r} \hat{r} \text{ Centripetal}
\end{aligned}$$

Magnetism

$$\begin{aligned}
|\vec{F}_B| &= q|\vec{v}||\vec{B}|\sin\theta_{vB} \\
|\vec{F}_B| &= BIl\sin\theta \text{ wire} \\
B &= \frac{\mu_0 I}{2\pi r} \text{ wire} \\
B &= \frac{\mu_0 I}{2r} \text{ loop} \\
B &= \mu_0 \frac{N}{L} I \hat{z} \equiv \mu_0 n I \hat{z} \text{ solenoid} \\
\frac{|\vec{F}_{12}|}{l} &= \frac{\mu_0 I_1 I_2}{2\pi d} \text{ 2 wires, force per length}
\end{aligned}$$

Current & Resistance:

$$\begin{aligned}
I &= \frac{\Delta Q}{\Delta t} = nqAv_d \\
v_d &= -\frac{e\tau}{m} E \quad \tau = \text{scattering time} \\
e &= \frac{m}{ne^2\tau} \\
\Delta V &= \frac{eI}{A} = RI \\
R &= \frac{\Delta V}{I} = \frac{eI}{A} \\
\mathcal{P} &= \frac{\Delta E}{\Delta t} = I\Delta V = I^2 R = \frac{[\Delta V]^2}{R} \text{ power}
\end{aligned}$$

EM Waves:

$$\begin{aligned}
c &= \lambda f = \frac{|\vec{E}|}{|\vec{B}|} \\
I &= \left[\frac{\text{photons}}{\text{time}} \right] \left[\frac{\text{energy}}{\text{photon}} \right] \left[\frac{1}{\text{Area}} \right] \\
I &= \frac{\text{energy}}{\text{time} \cdot \text{area}} = \frac{E_{\text{max}} B_{\text{max}}}{2\mu_0} = \frac{\text{power } (\mathcal{P})}{\text{area}} = \frac{E_{\text{max}}^2}{2\mu_0 c}
\end{aligned}$$

Electric Potential:

$$\begin{aligned}
\Delta V &= V_B - V_A = \frac{\Delta PE}{q} \\
\Delta PE &= q\Delta V = -q|\vec{E}||\Delta\vec{x}|\cos\theta = -qE_x\Delta x \\
&\quad \uparrow \text{constant E field} \\
V_{\text{point charge}} &= k_e \frac{q}{r} \\
PE_{\text{pair of point charges}} &= k_e \frac{q_1 q_2}{r_{12}} \\
PE_{\text{system}} &= \text{sum over unique pairs of charges} = \sum_{\text{pairs } ij} \frac{k_e q_i q_j}{r_{ij}} \\
-W &= \Delta PE = q(V_B - V_A)
\end{aligned}$$

Optics/Photons:

$$\begin{aligned}
E &= hf = \frac{hc}{\lambda} \\
n &= \frac{\text{speed of light in vacuum}}{\text{speed of light in a medium}} = \frac{c}{v} \\
\frac{\lambda_1}{\lambda_2} &= \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1} \text{ refraction} \\
n_1 \sin\theta_1 &= n_2 \sin\theta_2 \text{ Snell's refraction} \\
\lambda f &= c \\
M &= \frac{h'}{h} = -\frac{q}{p} \text{ spherical mirror \& lens} \\
\frac{1}{f} &= \frac{1}{p} + \frac{1}{q} = \frac{2}{R} \text{ spherical mirror \& lens} \\
\frac{n_1}{p} + \frac{n_2}{q} &= \frac{n_2 - n_1}{R} \text{ spherical refracting} \\
q &= -\frac{n_2}{n_1} p \text{ flat refracting} \\
\frac{1}{f} &= \left(\frac{n_2 - n_1}{n_1} \right) \left[\frac{1}{R_1} - \frac{1}{R_2} \right] \text{ lensmaker's}
\end{aligned}$$

Electric Force & Field

$$\begin{aligned}
\vec{F}_{e,12} &= q\vec{E}_{12} = \frac{k_e q_1 q_2}{r_{12}^2} \hat{r}_{12} \\
\vec{E} &= k_e \frac{|q|}{r^2} \\
\Phi_E &= |\vec{E}|A \cos\theta_{EA} = \frac{Q_{\text{inside}}}{\epsilon_0} \\
\Delta PE &= -W = -q|\vec{E}||\Delta\vec{x}|\cos\theta = -qE_x\Delta x \\
&\quad \uparrow \text{constant E field}
\end{aligned}$$

Capacitors:

$$\begin{aligned}
Q_{\text{capacitor}} &= C\Delta V \\
C_{\text{parallel plate}} &= \frac{\epsilon_0 A}{d} \\
E_{\text{capacitor}} &= \frac{1}{2} Q\Delta V = \frac{Q^2}{2C} \\
C_{\text{eq, par}} &= C_1 + C_2 \\
\frac{1}{C_{\text{eq, series}}} &= \frac{1}{C_1} + \frac{1}{C_2} \\
C_{\text{with dielectric}} &= \kappa C_{\text{without}}
\end{aligned}$$

Resistors:

$$\begin{aligned}
I_{V \text{ source}} &= \frac{\Delta V_{\text{rated}}}{R+r} \\
\Delta V_{V \text{ source}} &= \Delta V_{\text{rated}} \frac{R}{r+R} \\
I_{I \text{ source}} &= I_{\text{rated}} \frac{r}{r+R} \\
R_{\text{eq, series}} &= R_1 + R_2 \\
\frac{1}{R_{\text{eq, par}}} &= \frac{1}{R_1} + \frac{1}{R_2}
\end{aligned}$$

Vectors:

$$\begin{aligned}
|\vec{F}| &= \sqrt{F_x^2 + F_y^2} \text{ magnitude} \\
\theta &= \tan^{-1} \left[\frac{F_y}{F_x} \right] \text{ direction}
\end{aligned}$$

Induction:

$$\begin{aligned}
\Phi_B &= B_{\perp} A = BA \cos \theta_{BA} \\
\Delta V &= -N \frac{\Delta \Phi_B}{\Delta t} \\
L &= N \frac{\Delta \Phi_B}{\Delta I} = \frac{N \Phi_B}{I} \\
\Delta V &= |\vec{v}| |\vec{B}| l = |\vec{E}| l \text{ motional voltage}
\end{aligned}$$

ac Circuits

$$\begin{aligned}
\tau &= L/R \quad \text{RL circuit} \\
\tau &= RC \quad \text{RC circuit} \\
X_C &= \frac{1}{2\pi fC} \quad \text{"resistance" of a capacitor for ac} \\
X_L &= 2\pi fL \quad \text{"resistance" of an inductor for ac} \\
\omega_{\text{cutoff}} &= \frac{1}{\tau} = 2\pi f_{\text{cutoff}}
\end{aligned}$$

Relativity

$$\begin{aligned}
\gamma &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \\
\Delta t'_{\text{moving}} &= \gamma \Delta t_{\text{stationary}} = \gamma \Delta t_p \\
L'_{\text{moving}} &= \frac{L_{\text{stationary}}}{\gamma} \\
\Delta t' &= t'_1 - t'_2 = \gamma \left(\Delta t - \frac{v \Delta x}{c^2} \right) \\
v_{\text{obj}} &= \frac{v + v'_{\text{obj}}}{1 + \frac{v v'_{\text{obj}}}{c^2}} \quad v'_{\text{obj}} = \frac{v_{\text{obj}} - v}{1 - \frac{v v_{\text{obj}}}{c^2}} \\
\text{KE} &= (\gamma - 1) m c^2 \\
E_{\text{rest}} &= m c^2 \quad p = \gamma m v \\
E^2 &= p^2 c^2 + m^2 c^4
\end{aligned}$$

Right-hand rule #1

1. Point the fingers of your right hand along the direction of \vec{v} .
2. Point your thumb in the direction of \vec{B} .
3. The magnetic force on a + charge points out from the back of your hand.

Right-hand rule #2:

Point your thumb along the direction of the current (magnetic field). Your fingers naturally curl around the direction the magnetic field (current) circulates.

Unit	Symbol	equivalent to
newton	N	kg·m/s ²
joule	J	kg·m ² /s ² = N·m
watt	W	J/s = m ² ·kg/s ³
coulomb	C	A·s
amp	A	C/s
volt	V	W/A = m ² ·kg/s ³ ·A
farad	F	C/V = A ² ·s ⁴ /m ² ·kg
ohm	Ω	V/A = m ² ·kg/s ³ ·A ²
tesla	T	Wb/m ² = kg/s ² ·A
electron volt	eV	1.6 × 10 ⁻¹⁹ J
-	1 T · m/A	1 N/A ²
-	1 T · m ²	1 V · s
-	1 N/C	1 V/m

Power	Prefix	Abbreviation
10 ⁻¹²	pico	p
10 ⁻⁹	nano	n
10 ⁻⁶	micro	μ
10 ⁻³	milli	m
10 ⁻²	centi	c
10 ³	kilo	k
10 ⁶	mega	M
10 ⁹	giga	G
10 ¹²	tera	T