A few stray problems.

1. What is the wavelength of light emitted when a hydrogen atom makes a transition from $n_i = 5$ to $n_f = 2$?

The energy of the emitted photon is equal to the energy difference between the 5 and 2 levels. The photon energy is $hf = hc/\lambda$.

$$E = \frac{-13.6\,\text{eV}}{5^2} - \frac{-13.6\,\text{eV}}{2^2} \approx 2.96\,\text{eV} \tag{1}$$

$$\lambda = \frac{hc}{E} \approx 4.35 \times 10^{-7}\,\text{m} = 435\,\text{nm} \tag{2}$$

2. How many different photons can be emitted by hydrogen atoms that undergo transitions to the ground state from the $n = 5$ state? Enumerate their energies. (Hint: draw an energy level diagram, and remember that the level spacing is *not* equal. Answer: 10.)

All states from $n = 5$ to $n = 1$ have different energies, and their spacing is not equal. Thus, we have the following possible transitions to the ground state:

$$\begin{aligned}
&5 \to 1\\
&5 \to 4, 4 \to 1\\
&5 \to 4, 4 \to 3, 3 \to 1\\
&5 \to 4, 4 \to 3, 3 \to 2, 2 \to 1\\
&5 \to 4, 4 \to 2, 2 \to 1\\
&5 \to 3, 3 \to 1\\
&5 \to 3, 3 \to 2, 2 \to 1\\
&5 \to 2, 2 \to 1
\end{aligned}$$

The different photons that can be emitted correspond to the unique level transitions above:

$$\begin{aligned}
&5 \to 4, 5 \to 3, 5 \to 2, 5 \to 1\\
&4 \to 3, 4 \to 2, 4 \to 1\\
&3 \to 2, 3 \to 1\\
&2 \to 1
\end{aligned}$$

Thus, there are 10 possible transitions, and the energy differences are calculated according to the Bohr model

$$\Delta E_{nn'} = -13.6\,\text{eV}\left(\frac{1}{n^2} - \frac{1}{n'^2}\right) \tag{3}$$

where n and n' are the numbers of the initial and final states, respectively.

3. Electrons of energy 12.2 eV are fired at hydrogen atoms in a gas discharge tube. Determine the wavelengths of the lines that can be emitted by the hydrogen. *Hint: to what state can the hydrogen atom be excited, given an excess energy of* 12.2 *eV above its ground state?* (Answer: $656.3, 121.5, 102.6\,\text{nm}$)

If the hydrogen atom is in its ground state, it has an energy of $-13.6\,\text{eV}$. Giving it another 12.2 eV means it has an energy of $-1.4\,\text{eV}$, and the maximum state it can be excited to is thus

$$-1.4\,\text{eV} = \frac{-13.6\,\text{eV}}{n^2} \tag{4}$$

This gives $n = 3.1$, so the atom may be excited to the $n = 3$ state (the $n = 4$ state has an energy of $-0.85\,\text{eV}$, and cannot be reached). The hydrogen atom may then relax to the ground state from the third state by the following paths:

$$3 \rightarrow 1$$
$$3 \rightarrow 2, 2 \rightarrow 1$$

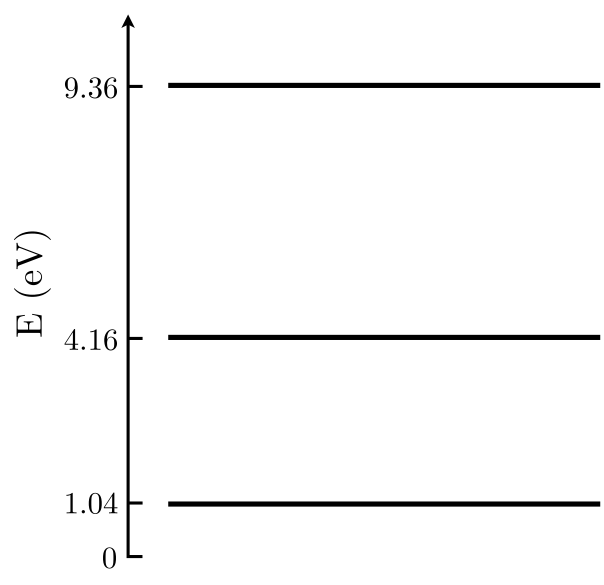
This gives us three possible photon energies:

$$E_3 - E_1 = -13.6\,\text{eV}\left(\frac{1}{3^2} - \frac{1}{1^2}\right) = 12.09\,\text{eV}$$
$$E_3 - E_2 = -13.6\,\text{eV}\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = 1.88\,\text{eV}$$
$$E_2 - E_1 = -13.6\,\text{eV}\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = 10.2\,\text{eV}$$

The photon wavelengths are then given by $\lambda = hc/\Delta E \approx 1240\,\text{eV}\cdot\text{nm}/\Delta E$ for

$$\lambda = \{659.6, 121.5, 102.6\}\ \text{nm} \tag{5}$$

▫ **4.** What are the possible **wavelengths** of photons that could be emitted from a system with an energy level diagram like that in the figure below?



The possible photon energies are the possible energy *differences*, $9.36 - 4.16 = 5.2\,\text{eV}$, $4.16 - 1.04 = 3.12\,\text{eV}$, $9.36 - 1.04 = 8.32\,\text{eV}$.