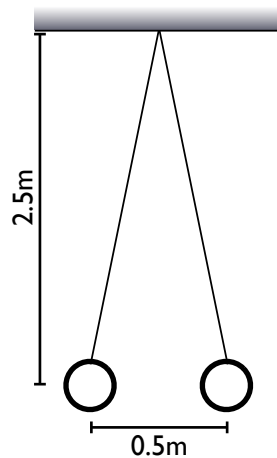


Problem Set 2: Solutions

1. Two volley balls, mass 0.3kg, tethered by nylon strings and charged with an electrostatic generator, hang as shown below. What is the charge on each in coulombs, assuming the charges are equal?



We will need a more detailed sketch including the forces present. Let the distance between the two charges at equilibrium be d . We need only consider one charge or the other – by symmetry the other charge will have all the same forces. Let us consider the left charge then. It will have three forces present: the repulsive force from the other charge (F_q), its weight (mg), and the tension in the string (T). Considering all that, here is our sketch and axis definitions:

The force due to the repulsion of the other charge a distance d away is given by Coulomb's law

$$F_q = \frac{k_e q^2}{d^2} \quad (1)$$

Positive, in this case, because the force is to the right and we have defined $+x$ to be to the right. Acting against this force in the $-x$ direction is the x component of the tension in the string, $-T \cos \theta$. The sum of the forces along the x direction is then

$$\sum F_x = F_q - T \cos \theta = \frac{k_e q^2}{d^2} - T \cos \theta = 0 \quad (2)$$

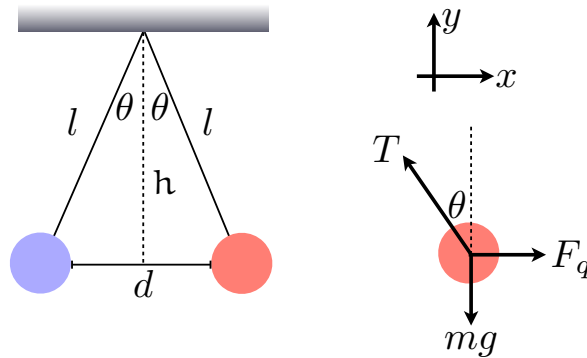


Figure 1: Forces on charge q due to the electric field (F_E), the repulsion from the other charge charge (F_q), its weight (mg), and the tension in the string (T).

The sum of the forces along x must be zero in equilibrium. We can find θ by trigonometry in terms of the quantities we were given, d and h :

$$\tan \theta = \frac{h}{d/2} = \frac{2h}{d} = \frac{1}{10} \quad (3)$$

As it turns out, we will only need the tangent of θ , so we'll leave it like this. How do we find the tension? From the force balance along y . In equilibrium, we know that the y component of the tension must be precisely balanced by the weight of the charge. The y component of tension is $T \cos \theta$ using the angles as defined in our sketch:

$$F_{y,\text{net}} = T \cos \theta - mg = 0 \quad \implies \quad T = \frac{mg}{\cos \theta} \quad (4)$$

Now we can plug this tension into our force balance along x , which will give us enough to solve for q in terms of known quantities:

$$F_{x,\text{net}} = F_q - T \sin \theta = 0 \quad (5)$$

$$0 = \frac{k_e q^2}{d^2} - \frac{mg}{\cos \theta} \sin \theta \quad (6)$$

$$\frac{k_e q^2}{d^2} = mg \tan \theta \quad (7)$$

$$\implies q = \sqrt{\frac{d^2 mg \tan \theta}{k_e}} = \sqrt{\frac{0.5^2 \cdot 0.3 \cdot 9.81 \cdot \frac{1}{10}}{9 \times 10^9}} \approx 2.86 \times 10^{-6} \text{ [C]} = 2.86 \mu\text{[C]} \quad (8)$$

When plugging in your numbers, be careful to convert everything to base SI units – m not cm, kg

not g.

2. On the unrealistic assumption that there are no other charged particles in the vicinity, at what distance below a proton would the upward force on an electron equal the electron's weight?

This just means that the attractive force between the proton (charge e) and electron (charge $-e$) at a distance d must balance the weight of the electron:

$$\frac{ke^2}{d^2} = mg \quad \implies \quad d = \sqrt{\frac{ke^2}{mg}} \approx 5.08 \text{ [m]} \quad (9)$$

3. (a) How much negative charge and how much positive charge are there on the electrons and the protons in a cup of water (0.25 kg)? Note Avogadro's number is $N_A = 6.022 \times 10^{23}$, and each oxygen atom has 8 electrons. **(b)** What is the magnitude of the attractive force exerted by the electrons in a cup of water on the protons in a second cup of water at a distance of 10 m?

The molecular mass of water is 18 g/mol, so 250 g of water amounts to 250/18 mol. For each mole of water, there are 6.02×10^{23} molecules. Each molecule has two hydrogen atoms (with one electron) and one oxygen atom (with eight electrons), with a total of ten electrons per molecule. Thus, we can calculate the total amount of negative charge in the cup of water readily:

$$\begin{aligned} [\text{neg. chg.}] &= 0.25 \text{ kg} \left[\frac{1000 \text{ g}}{1 \text{ kg}} \right] \left[\frac{1 \text{ mol}}{18 \text{ g}} \right] \left[\frac{6.02 \times 10^{23} \text{ molecules}}{\text{mol}} \right] \left[\frac{10 \text{ electrons}}{\text{molecule}} \right] \left[\frac{-1.6 \times 10^{-19} \text{ C}}{\text{electron}} \right] \\ &= 1.3 \times 10^7 \text{ C} \end{aligned}$$

Clearly, since the cup of water is overall electrically neutral, the positive charge on the protons is just the opposite of this.

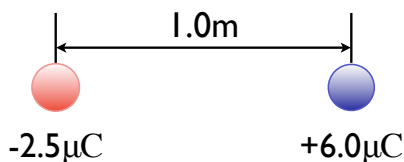
If we treat the total charge in each glass of water as point charges, then in the first cup of water we have one point charge of $-1.3 \times 10^{-7} \text{ C}$ and another of $1.3 \times 10^{-7} \text{ C}$, separated by 10 m. The force is then:

$$\begin{aligned} |\vec{F}| &= \frac{k_e q_1 q_2}{r_{12}^2} \\ &= \frac{[9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2] [-1.3 \times 10^{-7} \text{ C}] [+1.3 \times 10^{-7} \text{ C}]}{[10 \text{ m}]^2} \\ &= -1.5 \times 10^{22} \text{ N} \end{aligned}$$

This is an *enormous* force, equivalent to a weight of a billion billion tons! Thankfully, this attractive

force on the protons is precisely canceled by an equally large repulsive force exerted by the protons in one cup on the protons in the other.

4. Two charges, $-2.5 \mu\text{C}$ and $6.0 \mu\text{C}$, are separated by 1 m as shown below. Determine all points (other than those an infinite distance away) at which the net electric field is zero.



By symmetry, we can figure out on which side the field should be zero. In between the two charges, the field from the positive and negative charges *add together*. The force on a fictitious positive test charge placed in between the two would experience a force to the left due to the positive charge, and another force to the left due to the negative charge. There is no way the fields can cancel here.

If we place a positive charge to the *right of the positive charge*, it will feel a force to the right from the positive charge, and a force to the left from the negative charge. The directions are opposite, but the fields still cannot cancel because the test charge is closest to the larger charge.

This leaves us with points to the left of the negative charge. The forces on a positive test charge will be in opposite directions here, and we are closer to the smaller charge. What position gives zero field? First, we will call the position of the negative charge $x = 0$, which means the positive charge is at $x = 1$ m. We will call the position where electric field is zero x . The distance from this point to the negative charge is just x , and the distance to the positive charge is $1 + x$. Now write down the electric field due to each charge:

$$E_{\text{neg}} = \frac{k_e(-2.5 \mu\text{C})}{x^2} \quad (10)$$

$$E_{\text{pos}} = \frac{k_e(6 \mu\text{C})}{(1 + x)^2} \quad (11)$$

The field will be zero when $E_{\text{neg}} + E_{\text{pos}} = 0$:

$$E_{\text{neg}} + E_{\text{pos}} = 0 \quad (12)$$

$$\frac{k_e(-2.5 \mu\text{C})}{x^2} + \frac{k_e(6 \mu\text{C})}{(1+x)^2} = 0 \quad (13)$$

$$\frac{\cancel{k_e}(-2.5 \mu\text{C})}{x^2} + \frac{\cancel{k_e}(6 \mu\text{C})}{(1+x)^2} = 0 \quad (14)$$

$$\frac{-2.5}{x^2} + \frac{6}{(1+x)^2} = 0 \quad (15)$$

$$\implies \frac{2.5}{x^2} = \frac{6}{(1+x)^2} \quad (16)$$

$$(17)$$

Cross multiply, apply the quadratic formula:

$$2.5(1+x)^2 = 6x^2 \quad (18)$$

$$2.5 + 5x + 2.5x^2 = 6x^2 \quad (19)$$

$$3.5x^2 - 5x - 2.5 = 0 \quad (20)$$

$$\implies x = \frac{-(-5) \pm \sqrt{5^2 - 4(-2.5)(3.5)}}{2(3.5)} \quad (21)$$

$$x = \frac{5 \pm \sqrt{25 + 35}}{7} \quad (22)$$

$$x = \frac{5 \pm 7.75}{7} = 1.82, -0.39 \quad (23)$$

Which root do we want? We wrote down the distance x the distance to the *left* of the negative charge. A negative value of x is then in the wrong direction, in between the two charges, which we already ruled out. The positive root, $x = 1.82$, means a distance 1.82 m to the *left* of the negative charge. This is what we want.