UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

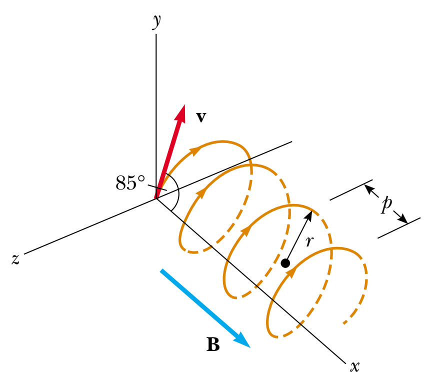
PH 102 / LeClair — Summer II 2012

Problem Set 4: Mostly Magnetic

Instructions:

1. Answer all questions below. Show your work for full credit.
2. All problems are due Tuesday 24 July 2012 by the end of the day (11:59pm if electronically submitted, by 5pm as a hard copy)
3. You may collaborate, but everyone must turn in their own work.

1. A uniform magnetic field of magnitude 0.150 T is directed along the positive x axis. A positron (a positively-charged electron) moving at 5.00×10^6 m/s enters the field along a direction that makes an angle of 85° with the x axis. The motion of the particle is expected to be a helix in this case. Calculate the pitch p and radius r of the trajectory.



The first thing to realize is that a helix is basically a curve described by circular motion in one plane, in this case the $y-z$ plane, and linear motion along the perpendicular direction, in this case the x axis. A helix of circular radius a and pitch p can be described parametrically by

$$x(t) = \frac{pt}{2\pi}$$
$$y(t) = a\cos t$$
$$z(t) = a\sin t$$

As we can see, the motion in the $y-z$ plane obeys $y^2+z^2=a^2$, describing a circle of radius a, and along the x axis we just have constant velocity motion. Since the x, y, and z motions are uncoupled (e.g., the equation for $x(t)$ has no y's or z's in it), things are in fact pretty simple.

The circular motion comes from the component of the velocity perpendicular to the magnetic field, the component of velocity lying in the $y-z$ plane, which we will call $v_{\perp}$. The pitch is just how far forward along the x axis the particle moves in one period of circular motion T. Thus, if the velocity along the x axis is v_x,

$$p = v_x T = (v \cos 85^\circ)\, T$$

We have already discovered that the period and radius of circular motion for a particle in a magnetic field does not depend on the particle's velocity, it only matters that there is always a velocity component perpendicular to the magnetic field

$$T = \frac{2\pi m}{qB} \qquad \text{and} \qquad r = \frac{mv_{\perp}}{qB}$$

Putting everything together,

$$p = \frac{2\pi mv}{Bq} \cos 85^\circ \approx 1.04 \times 10^{-4}\,\text{m}$$
$$r = \frac{mv}{qB} \sin 85^\circ \approx 1.89 \times 10^{-4}\,\text{m}$$

By the way, here is an interesting tidbit from MathWorld:[i]

> A helix, sometimes also called a coil, is a curve for which the tangent makes a constant angle with a fixed line. The shortest path between two points on a cylinder (one not directly above the other) is a fractional turn of a helix, as can be seen by cutting the cylinder along one of its sides, flattening it out, and noting that a straight line connecting the points becomes helical upon re-wrapping. It is for this reason that squirrels chasing one another up and around tree trunks follow helical paths.

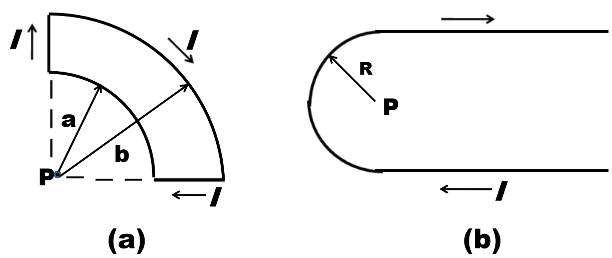
2. Find the magnetic field at point P for each of current configurations shown below. Hint for a: Magnetic due to the straight portions is zero at P. Hint for b: Two half-infinite wires make one infinite straight wire. Hint c: use superposition and symmetry!

(a) The easiest way to do solve this is by superposition – our odd current loop is just the same as two quarter circles plus two small straight segments. We know that the magnetic field at the center of a *full* circular loop of radius r carrying a current I is

$$B = \frac{\mu_o I}{2r} \qquad \text{(loop radius r)}$$

Since the magnetic field obeys superposition, we could just as well say that our full circle is built

[i] `http://mathworld.wolfram.com/Helix.html`



out of four equivalent quarter circles! The field from each quarter circle, by symmetry, must be one quarter of the total field, so the field at the center of a quarter circle must simply be

$$B = \frac{\mu_o I}{8r} \qquad \text{(quarter circle, radius r)}$$

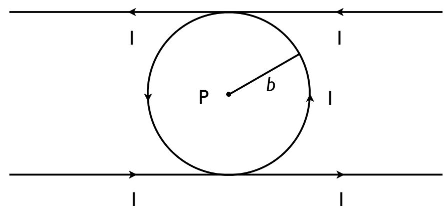
In other words: a quarter circle gives you a quarter of the field of a full circle. Here we have two quarter circle current segments contributing to the magnetic field at P: one of radius b, and one of radius a. The currents are in the opposite directions for the two loops, so their fields are in opposing directions. The outer loop of radius b has its field pointing into the page, and the inner loop of radius a has its field pointing out of the page, so if we (arbitrarily) call in plane direction positive, we can subtract the field of the smaller loop from that of the larger.

What about the straight bits of wire? For those segments, the direction field is zero. Since the magnetic field "circulates" around the wire, along the wire axis it must be zero. Even if it were not, by symmetry the two straight bits would have to give equal and opposite contributions and cancel each other anyway. There is no field contribution at P from the straight segments! Thus, the total field is just that due to the quarter circle bits,

$$\vec{B} = \frac{\mu_o I}{8b} - \frac{\mu_o I}{8a} = \frac{\mu_o I}{8}\left(\frac{1}{b} - \frac{1}{a}\right)$$

(b) There is a sneaky way to solve this problem using symmetry. Qualitatively, we can immediately observe that the field must point out of the plane of the page. Think of the hairpin as being broken up into three sections: an upper semi-infinte wire, a half circle, and a lower semi-infinite wire. All three segments give the same direction of field at P. Further, if we were to rotate the entire hairpin in the plane of the page, this will not change. Do that in your head once . . . rotate the entire setup, say, 90° clockwise, and you will find that the magnetic field at P will not change. If this is the case, let us consider the particular case where we have the same arrangement of wires rotated a full 180°, and add this to the existing setup, as shown below:

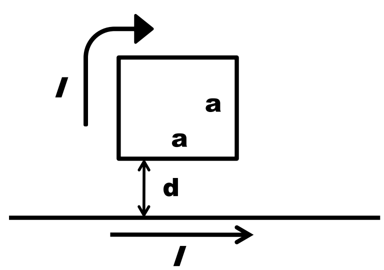
Since both the "normal" and "rotated" hairpins give the same field, this arrangement just gives



us twice the field of the original arrangement. With this arrangement, we can break the problem down into two infinite wires and a single circular loop of radius r. We have already calculated the field due to straight wires and loops; at point P, these three fields superimpose, and the calculation is trivial:

$$2B = \frac{\mu_o I}{2\pi b} + \frac{\mu_o I}{2\pi b} + \frac{\mu_o I}{2b} \qquad \Longrightarrow \qquad B = \frac{\mu_o I}{4\pi b}\left(1 + \frac{2}{\pi}\right)$$

3. Find the force on a square loop (side a) placed as shown below, near an infinite straight wire. Both loop and wire carry a steady current I.



We know how to find the force between parallel segments of wire. If the parallel segments are of length l, we know

$$F = \frac{\mu_o I_1 I_2 l}{2\pi d} \tag{1}$$

Here we have a top parallel segment of length a a distance $d+a$ away, and a bottom parallel segment a distance d away. The bottom segment has its current *antiparallel* to the main wire, making this a repulsive force, while the top segment has its current parallel to the main wire, giving an attractive force. In finding the net force, this means we should subtract the force on the top segment from that on the bottom, since they act in opposite directions.

What about the sides? We have to think carefully about the directions now. Charges in the left segment move at a right angle to the field from the main wire (which pokes out of the page at the position of the left segment), and will feel a force to the right. Charges in the right segment, by the same logic, will feel a force to the left. These forces will be equal and opposite, and the net effect on the loop, if it is rigid, will cancel. (Were the loop flexible, it would be squished along the horizontal direction.

Overall, the net force is just the difference between that on the top segment and that on the bottom:

$$F_{\text{net}} = F_{\text{bott}} - F_{\text{top}} = \frac{\mu_o I^2 a}{2\pi d} - \frac{\mu_o I^2 a}{2\pi\,(d+a)} = \frac{\mu_o I^2 a}{2\pi}\left(\frac{1}{d} - \frac{1}{d+a}\right) = \frac{\mu_o I^2}{2\pi}\frac{a^2}{d\,(d+a)} \tag{2}$$

4. What is the induced EMF between the ends of the wingtips of a Boeing 737 when it is flying over the magnetic north pole? The internet has most of the numbers you require.

The induced voltage[ii] can be found by considering the motion of the conducting metal plane in a perpendicular magnetic field, and making a few seemingly outlandish (but justifiable) assumptions.

First, at the south magnetic pole, the magnetic field will be essentially straight down. If the 737 is flying level over the ground, this means that its metal (conducting) skin is in motion relative to a magnetic field. This in turn means that there will be a motionally-induced voltage. If the field is straight down, and the 737 travels straight forward, then positive charges will experience a force in the port (left) direction, and negative charges toward the starboard (right). This means that the wingtips will have a potential difference between them due to the magnetic force on the charges in the conducting skin. If the wingspan is l meters, the airplane's velocity v and the vertical magnetic field B, then we know the potential difference due to motion in a magnetic field is $\Delta V = Blv$.

The wingspan of a 737 is roughly 30 m, and its cruising speed is about 200 m/s.[iii] Currently, the earth's magnetic field[iv] at the south magnetic pole[v] is about 60 μT. Putting this together,

$$\Delta V = Blv = (60\,\mu\text{T})\,(30\,\text{m})\,(200\,\text{m/s}) \approx 0.36\,\text{V}$$

[ii] I try to avoid using the term"EMF" and usually just use "voltage" instead. EMF is a bit antiquated and tends to confuse students in my opinion. If you see "EMF" just read it as "potential difference" or "voltage."

[iii] http://en.wikipedia.org/wiki/Boeing_737

[iv] http://www.ngdc.noaa.gov/geomag/magfield.shtml

[v] http://en.wikipedia.org/wiki/South_Magnetic_Pole

5. Show that, if the condition $R_1 R_2 = L/C$ is satisfied by the components of the circuit below, the difference in voltage between points A and B will be zero at any frequency.