

UNIVERSITY OF ALABAMA  
Department of Physics and Astronomy

PH 102-2 / LeClair

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## Problem Set 12 SOLUTIONS

**1. 10 points.** In the 1996 movie *Eraser*, a corrupt business Cyrez is manufacturing a handheld rail gun which fires aluminum bullets at nearly the speed of light. Let us be optimistic and assume the actual velocity is  $0.75c$ . We will also assume that the bullets are tiny, about the mass of a paper clip, or  $m = 5 \times 10^{-4}$  kg.

**(a)** What is the relativistic kinetic energy of such a bullet? **(b)** What *rest mass* would have to be completely converted to energy to supply this kinetic energy? Note that 1 kg of TNT has an equivalent energy content of about  $4 \times 10^9$  J.

Calculating the relativistic kinetic energy is straightforward. Let  $m_b$  be the mass of the bullet, and  $v$  its velocity.

$$\begin{aligned} KE &= (\gamma - 1) m_b c^2 \\ &= \left( \frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) m_b c^2 \\ &= \left( \frac{1}{\sqrt{1 - 0.75^2}} - 1 \right) (5 \times 10^{-4} \text{ kg}) (3 \times 10^8 \text{ m/s})^2 \\ &= (1.51 - 1) (4.5 \times 10^{13} \text{ kg} \cdot \text{m}^2/\text{s}^2) \\ &= 2.30 \times 10^{13} \text{ J} \approx 5700 \text{ kg of TNT} \end{aligned}$$

We would like to supply this much energy by converting some bit of rest mass  $m_r$  completely to energy. Thus, the kinetic energy above should be equated with the rest energy of a mass  $m_r$  used as propellant:

$$\begin{aligned} KE_{\text{bullet}} &= E_{\text{rest, propellant}} \\ (\gamma - 1) m_b c^2 &= m_r c^2 \\ m_r &= (\gamma - 1) m_b \\ m_r &\approx (1.51 - 1) (5 \times 10^{-4} \text{ kg}) \\ m_r &\approx 2.56 \times 10^{-4} \text{ kg} \end{aligned}$$

In other words, in order to supply enough energy for the bullet to reach  $0.75c$ , we would need the energy equivalent of a rest mass of  $2.56 \times 10^{-4}$  kg, about half the mass of the bullet itself.

**2. 10 points.** Two identical point charges  $+q$  are located on the  $y$  axis at  $y = +a$  and  $y = -a$ . **(a)** What is the electric potential for an arbitrary point  $(x, y)$ ?

We have two charges  $q$  separated by a distance  $2a$ . We will choose the origin  $\mathcal{O}$  to be precisely between the two charges, along the line connecting the charges, and we wish to calculate the electric potential at an arbitrary point  $\mathcal{P}(x, y)$ . We can readily write down the potential at the point  $\mathcal{P}$  – it is just a superposition of the potential due to each of the charges alone.

For a single charge (call it  $q_1$ ) at position  $(0, a)$ , the distance  $r_1$  to an arbitrary point  $(x, y)$  is readily found in two dimensions:

$$r_1 = \sqrt{x^2 + (y - a)^2}$$

Given the distance and the magnitude of the charge, finding the electric potential  $V_1$  is easy too:

$$V_1(x, y) = \frac{k_e q_1}{r_1} = \frac{k_e q_1}{\sqrt{x^2 + (y - a)^2}} \quad (1)$$

The potential due to the second charge has the same form, only the distance changes:

$$V_2(x, y) = \frac{k_e q_2}{r_2} = \frac{k_e q_2}{\sqrt{x^2 + (y + a)^2}} \quad (2)$$

The total potential at  $\mathcal{P}(x, y)$  is found by superposition - it is just the sum of the separate potentials due to each charge. Since electric potential is a scalar, we don't even need to worry about vectors:

$$V_{\text{tot}}(x, y) = V_1(x, y) + V_2(x, y) = k_e \left[ \frac{q_1}{\sqrt{x^2 + (y - a)^2}} + \frac{q_2}{\sqrt{x^2 + (y + a)^2}} \right]$$

**(extra)** How about the electric field? There is no trick to this one either, you just have to grind through it. First, consider one charge alone, and find the electric field due to it at point  $(x, y)$ . Then find the  $x$  and  $y$  components. Do the same for the other charge, and add the  $x$  and  $y$  components for each together to find the total field. It is messy.

Given the distance and the magnitude of the charge, finding  $\vec{\mathbf{E}}$  is easy too. For the first charge,:

$$|\vec{\mathbf{E}}_1| = \frac{k_e q_1}{r_1^2} = \frac{k_e q_1}{x^2 + (y - a)^2}$$

For the second charge, the total field at  $(x, y)$  is almost the same, only the distance changes:  $r_2 = \sqrt{x^2 + (y + a)^2}$ :

$$|\vec{\mathbf{E}}_2| = \frac{k_e q_2}{r_2^2} = \frac{k_e q_2}{x^2 + (y + a)^2}$$

The field due to both charges together can be found by superposition. Since electric field is a vector, we need to find the  $x$  and  $y$  components of  $\vec{\mathbf{E}}_1$  and  $\vec{\mathbf{E}}_2$  and add them together:  $E_{x, \text{total}} = E_{x,1} + E_{x,2}$ . In order to find the  $x$  components, we need to find the angles  $\theta_1$  and  $\theta_2$  that  $r_1$  and  $r_2$  make with the  $x$  axis, since  $E_x = E \cos \theta$ .

$$\cos \theta_1 = \frac{x}{r_1} = \frac{x}{\sqrt{x^2 + (y - a)^2}}$$

$$\cos \theta_2 = \frac{x}{r_2} = \frac{x}{\sqrt{x^2 + (y + a)^2}}$$

Thus, the  $x$  components of the field are:

$$E_{x,1} = E_1 \cos \theta_1 = \frac{k_e q_1}{x^2 + (y - a)^2} \cdot \frac{x}{r_1} = \frac{k_e q_1}{x^2 + (y - a)^2} \cdot \frac{x}{\sqrt{x^2 + (y - a)^2}} = \frac{k_e q_1 x}{[x^2 + (y - a)^2]^{\frac{3}{2}}}$$

$$E_{x,2} = E_2 \cos \theta_2 = \frac{k_e q_2}{x^2 + (y + a)^2} \cdot \frac{x}{r_2} = \frac{k_e q_2}{x^2 + (y + a)^2} \cdot \frac{x}{\sqrt{x^2 + (y + a)^2}} = \frac{k_e q_2 x}{[x^2 + (y + a)^2]^{\frac{3}{2}}}$$

$$E_{x, \text{total}} = E_{x,1} + E_{x,2}$$

For the  $y$  component, we need the sin of the relevant angles:

$$\sin \theta_1 = \frac{y}{r_1} = \frac{y}{\sqrt{x^2 + (y - a)^2}}$$

$$\sin \theta_2 = \frac{y}{r_2} = \frac{y}{\sqrt{x^2 + (y + a)^2}}$$

Thus, the  $y$  components of the field are:

$$E_{y,1} = E_1 \sin \theta_1 = \frac{k_e q_1}{x^2 + (y - a)^2} \cdot \frac{y}{r_1} = \frac{k_e q_1}{x^2 + (y - a)^2} \cdot \frac{y}{\sqrt{x^2 + (y - a)^2}} = \frac{k_e q_1 y}{[x^2 + (y - a)^2]^{\frac{3}{2}}}$$

$$E_{y,2} = E_2 \sin \theta_2 = \frac{k_e q_2}{x^2 + (y + a)^2} \cdot \frac{y}{r_2} = \frac{k_e q_2}{x^2 + (y + a)^2} \cdot \frac{y}{\sqrt{x^2 + (y + a)^2}} = \frac{k_e q_2 y}{[x^2 + (y + a)^2]^{\frac{3}{2}}}$$

$$E_{y,\text{total}} = E_{y,1} + E_{y,2}$$

We are told that both charges are the same,  $q_1 = q_2 \equiv q$ . Doing that, we can put the whole mess together:

$$\vec{E} = k_e q \left[ \frac{x}{[x^2 + (y + a)^2]^{\frac{3}{2}}} + \frac{x}{[x^2 + (y - a)^2]^{\frac{3}{2}}} \right] \hat{x} + k_e q \left[ \frac{y}{[x^2 + (y + a)^2]^{\frac{3}{2}}} + \frac{y}{[x^2 + (y - a)^2]^{\frac{3}{2}}} \right] \hat{y}$$

(b) A circular ring of charge of radius  $a$  has a total positive charge  $Q$  distributed uniformly around it. The ring is in the  $x=0$  plane with its center at the origin. What is the electric field (both magnitude and direction) along the  $x$  axis at an arbitrary point  $x=b$  due to the ring of charge? *Hint: Consider the total charge  $Q$  to be made up of many pairs of identical charges placed on opposite points on the ring.*

**3. 10 points.** The purely electrostatic crystallization energy for a mole of crystalline solid can be found by considering the electrostatic potential energy of a lattice of ionic charges. We found that it could be expressed as a factor  $M$  times the electrostatic energy of a single pair of ions:

$$E_{\text{cryst}} = -\frac{N_A z^2 e^2 M}{4\pi\epsilon_0 r}$$

where  $N_A$  is Avogadro's number,  $z$  is the charge per ion in the crystal,  $r$  is the distance between ions in the crystal, and  $M$  is the Madelung constant for the lattice as discussed in the notes. The values of  $M$ ,  $z$ , and  $r$  for several common solids crystallizing in the rocksalt (NaCl) structure are given below.

| Compound | chg/ion | $M$   | $r$ (pm) |
|----------|---------|-------|----------|
| LiF      | $e$     | 1.748 | 201.4    |
| LiCl     | $e$     | 1.748 | 257.0    |
| LiBr     | $e$     | 1.748 | 275.1    |
| NaCl     | $e$     | 1.748 | 282      |
| NaBr     | $e$     | 1.748 | 289.9    |

Note that  $1 \text{ pm} = 10^{-12} \text{ m}$ . As it turns out, the energy of crystallization determined from electrostatics is essentially the same energy required to separate a mole of solid into a gas of its ions - we need additionally only a small correction factor to account for the repulsion of the ions at small distances due to quantum effects.

(a) Calculate the crystallization energy, **in electron volts**, for the compounds in the table above. Which compound is the "most stable?"

We really just need to use the formula above and plug in the relevant numbers - being very careful with units. Note that the charge per ion,  $z$ , is given in *units* of the elementary charge  $e$  - thus,  $z=1$  for all of the compounds in the table, since they are all salts made of monovalent alkali metals and halogens. What we really mean by  $z$  is “how many units of elementary charge are there per ion,” and the answer is clearly 1 for these compounds. Probably, the table should have just listed 1 instead of  $e$  to make this less confusing. Anyway, start like this, for LiF, noting that  $1/4\pi\epsilon_0 = k_e$ :

$$\begin{aligned} E_{\text{LiF}} &= -\frac{N_A z^2 e^2 M}{4\pi\epsilon_0 r} = -\frac{k_e N_A z^2 e^2 M}{r} \\ &= -\frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) (6.02 \times 10^{23} \text{ mol}^{-1}) (1)^2 (1.602 \times 10^{-19} \text{ C})^2 (1.748)}{201.4 \times 10^{-12} \text{ m}} \\ &\approx -1.21 \times 10^6 \text{ J/mol} = -1210 \text{ kJ/mol} \end{aligned}$$

This is not far from the accepted value of  $-1036 \text{ kJ/mol}$ . Now we just do this for the other compounds in the table ... the results are summarized below, in  $\text{kJ/mol}$  and  $\text{eV/ion}$ .<sup>1</sup> The most stable compound is the one with the *lowest crystallization energy*, or the one that gains the most energy by crystallizing. In this case, it is LiF, which *gains*  $1210 \text{ kJ/mol}$  by crystallizing. NaBr gains “only”  $838 \text{ kJ/mol}$ , which makes it less stable. Remember: negative energies generally mean a bound stable state, and the more negative, the more stable.

| Compound | $E_{\text{cryst}}$ (kJ/mol) | $E_{\text{cryst}}$ (eV/ion) |
|----------|-----------------------------|-----------------------------|
| LiF      | -1210                       | -12.5                       |
| LiCl     | -946                        | -9.81                       |
| LiBr     | -884                        | -9.17                       |
| NaCl     | -862                        | -8.94                       |
| NaBr     | -838                        | -8.69                       |

(b) Lime,  $\text{CaO}$ , is known to have the same structure as  $\text{NaCl}$  (and thus the same Madelung constant) and the edge length of the unit cell for  $\text{CaO}$  is  $481 \text{ pm}$ . Thus,  $\text{Ca-O}$  distance is  $241 \text{ pm}$ . Evaluate the energy of crystallization,  $E_{\text{cryst}}$  for  $\text{CaO}$ .

Now one thing you need to remember from chemistry is that both  $\text{Ca}$  and  $\text{O}$  are *doubly charged* ions -  $\text{Ca}$  is  $2+$  and  $\text{O}$  is  $2-$ . Thus, there are 2 unit charges per ion, and  $z=2$ . Other than that, we proceed as before. Since the crystal structure is the same, so is  $M$ , we need only change  $z$  and  $r$ :

$$\begin{aligned} E_{\text{CaO}} &= -\frac{N_A z^2 e^2 M}{4\pi\epsilon_0 r} = -\frac{k_e N_A z^2 e^2 M}{r} \\ &= -\frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) (6.02 \times 10^{23} \text{ mol}^{-1}) (2)^2 (1.602 \times 10^{-19} \text{ C})^2 (1.748)}{241 \times 10^{-12} \text{ m}} \\ &\approx -4034 \text{ kJ/mol} = -41.8 \text{ eV/ion} \end{aligned}$$

(c) Calculate the crystallization energy for  $\text{MgO}$ , for which  $z=2$  and  $r=286 \text{ pm}$ .

Proceed exactly as in (b):

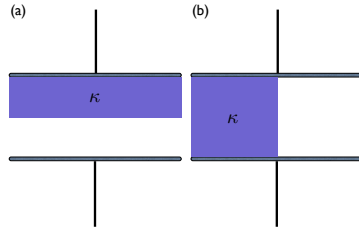
$$\begin{aligned} E_{\text{MgO}} &= -\frac{N_A z^2 e^2 M}{4\pi\epsilon_0 r} = -\frac{k_e N_A z^2 e^2 M}{r} \\ &= -\frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) (6.02 \times 10^{23} \text{ mol}^{-1}) (2)^2 (1.602 \times 10^{-19} \text{ C})^2 (1.748)}{286 \times 10^{-12} \text{ m}} \\ &\approx -3400 \text{ kJ/mol} = -35.2 \text{ eV/ion} \end{aligned}$$

We note that all of these numbers are within about 10% or so of what a modern density functional theory calculation would give - the 10% we miss has to do with the repulsive forces when we try to push ions too close together, which we covered briefly

<sup>1</sup>In order to get the latter, divide the  $\text{kJ/mol}$  result by  $N_A$  to get  $\text{J/ion}$ , and then divide by  $1.6 \times 10^{-19} \text{ J/eV}$  to get  $\text{eV/ion}$ .

in quantum physics. The bulk of the total energy of a crystal, however, is just tied up in electrostatic energy.

4. A parallel plate capacitor has a capacitance  $C$  when there is vacuum between the plates. The gap between the plates is half filled with a dielectric with dielectric constant  $\kappa$  in two different ways, as shown below. Calculate the effective capacitance, in terms of  $C$  and  $\kappa$ , for both situations. *Hint: try breaking each situation up into two equivalent capacitors.*



*Dielectric parallel to the plates.* It is easiest to think of this as two capacitors in series, both with half the plate spacing - one filled with dielectric, one with nothing. First, without *any* dielectric, we will say that the original capacitor has plate spacing  $d$  and plate area  $A$ . The capacitance is then:

$$C_0 = \frac{\epsilon_0 A}{d} \quad (3)$$

The upper half capacitor with dielectric then has a capacitance:

$$C_d = \frac{K\epsilon_0 A}{d/2} = \frac{2K\epsilon_0 A}{d} = 2KC_0 \quad (4)$$

The half capacitor without then has

$$C_{\text{none}} = \frac{\epsilon_0 A}{d/2} = \frac{2\epsilon_0 A}{d} = 2C_0 \quad (5)$$

Now we just add the two like capacitors in series:

$$\frac{1}{C_{\text{eff}}} = \frac{1}{2KC_0} + \frac{1}{2C_0} \quad (6)$$

$$C_{\text{eff}} = \frac{4KC_0^2}{2KC_0 + 2C_0} \quad (7)$$

$$= \frac{2K}{1+K} C_0 \quad (8)$$

*Dielectric "perpendicular" to the plates.* In this case, we think of the half-filled capacitor as two capacitors in *parallel*, one filled with dielectric, one with nothing. Now each half capacitor has half the plate *area*, but the same spacing. The upper half capacitor with dielectric then has a capacitance:

$$C_d = \frac{K\epsilon_0 \frac{1}{2} A}{d} = \frac{K\epsilon_0 A}{2d} = \frac{1}{2} KC_0 \quad (9)$$

The half capacitor without then has

$$C_{\text{none}} = \frac{\epsilon_0 \frac{1}{2} A}{d} = \frac{\epsilon_0 A}{2d} = \frac{1}{2} C_0 \quad (10)$$

Now we just add our parallel capacitors:

$$C_{\text{eff}} = \frac{1}{2}KC_0 + \frac{1}{2}C_0 \quad (11)$$

$$= \frac{1}{2}(K+1)C_0 \quad (12)$$

$$= \frac{K+1}{2}C_0 \quad (13)$$

**5. 10 points.** The battery in your car stores about  $10^3 \text{ W} \cdot \text{h}$  worth of energy at a potential difference of 12 V. One day while doing yard work, you decide to run your 200 W car stereo without the engine running. In order to start your car, you need to deliver 500 A at 7.2 V for at least 1 sec. How long can you run your stereo before you can no longer start the car?

This is basically a conservation of energy problem: your battery has only so much energy available, and you want to use it all up between the stereo and starting the car. The total energy of the battery should first be converted into  $\text{W} \cdot \text{s}$ , or Joules:

$$E_{\text{batt}} = (10^3 \text{ W} \cdot \text{h}) (3600 \text{ s/h}) = 3.6 \times 10^6 \text{ J}$$

The power required to start the car can be found from the given current and voltage requirements. Then, noting that energy is power times time, we can find how many Joules should be required to start the car:

$$\mathcal{P}_{\text{start}} = (500 \text{ A}) (7.2 \text{ V}) = 3600 \text{ W}$$

$$E_{\text{start}} = \mathcal{P}_{\text{start}} \Delta t_{\text{start}} = 3600 \text{ J}$$

The power used by the stereo is just its power times the time we are able to run it:

$$E_{\text{stereo}} = \mathcal{P}_{\text{stereo}} \Delta t_{\text{stereo}} = (200 \text{ W}) \Delta t_{\text{stereo}}$$

Now we just set up the total energy balance, and solve for our one unknown - how long we can afford to run the stereo:

$$E_{\text{batt}} = E_{\text{start}} + E_{\text{stereo}}$$

$$3.6 \times 10^6 \text{ J} = 3600 \text{ J} + (200 \text{ W}) \Delta t_{\text{stereo}}$$

$$\Delta t_{\text{stereo}} \approx 18000 \text{ s} \approx 5 \text{ h}$$

Of course, the reality of things is always different - can a nearly dead battery really still generate 500 A? When entering the real world, one should always give these estimates a hefty safety margin. Better yet, buy an iPod. It is far more stylish and efficient, if not cheaper.

**6. 10 points.** How many different resistance values can be constructed from a  $2 \Omega$ ,  $4 \Omega$ , and a  $6 \Omega$  resistor? (You only have one of each.) Show how you would get each resistance value either individually or by combining them.

There is no trick: just find them by brute force. There are 16 different values of resistance.

| combination                                | number of ways | resistances ( $\Omega$ ) |
|--------------------------------------------|----------------|--------------------------|
| single resistor                            | 3              | 2, 4, 6                  |
| 2 in series                                | 3              | 6, 8, 10                 |
| 2 in parallel                              | 3              | 1.3, 1.5, 2.4            |
| 3 in series                                | 1              | 12                       |
| 3 in parallel                              | 1              | 1.1                      |
| 1 in parallel with series comb. of other 2 | 3              | 1.7, 2.7, 3.0            |
| 1 in series with parallel comb. of other 2 | 3              | 4.4, 5.5, 7.3            |

There are 17 combinations overall, but a resistance of  $6\ \Omega$  shows up twice. Thus, there are 16 *different* resistance values. If we insist that all three resistors are used (which we did not), then of course there are only 8.

**7. 10 points.** An electron is moving at a speed of  $0.01c$  on a circular orbit of radius  $10^{-10}$  m around a proton. (a) What is the strength of the resulting magnetic field at the center of the orbit? (The numbers given are typical, in order of magnitude, for an electron in an atom.)

The easiest way to proceed is to consider the electron in its circular orbit to be a very tiny current loop. But what is the current? Think back to the definition of current:  $I = \Delta Q / \Delta t$ . The electron has a constant speed  $v$  in its orbit, which means that the “current” corresponding to this single charge is just the amount of charge  $-e$  divided by how long it takes to orbit the nucleus - the circumference divided by the electron’s velocity.

$$\begin{aligned} I &= \frac{\Delta Q}{\Delta t} \\ &= \frac{-e}{2\pi r/v} = \frac{-ev}{2\pi r} \\ &= \frac{-(1.6 \times 10^{-19} \text{ C})(0.1 \cdot 3 \times 10^8 \text{ m/s})}{2\pi (10^{-10} \text{ m})} \\ &\approx -7.6 \times 10^{-4} \text{ A} = -760 \mu\text{A} \end{aligned}$$

Given this current around a circular loop, we know how calculate the field at the center:

$$B_{\text{center}} = \frac{\mu_0 I}{2r} = \frac{-\mu ev}{4\pi r^2} \approx -4.8 \text{ T}$$

(b) If the nucleus of the atom (at the center of the orbit) consists of a single proton, what would its precession frequency be? *Hint: from the nucleus’ point of view, it is orbiting the electron in a circular path.* Recall  $\omega = qB/m$  and  $\omega = 2\pi f$ .

From the electron’s reference frame, it is the proton that is revolving around *it*, experiencing the magnetic field due to the electron. This leads to a precessional motion, for which we can calculate the frequency given the charge ( $+e$ ) and mass ( $m_p$ ) of a proton:

$$f = \frac{\omega}{2\pi} = \frac{e|B|}{2\pi m} = \frac{\mu e^2 v}{8\pi^2 m r^2} \approx 73 \text{ MHz}$$

In fact, this is a crude estimate of the hydrogen proton resonance frequency. We are off by a factor of about 2.79, having neglected all of quantum physics. The missing factor is basically the proton’s “ $g$ -factor”, which is a bit too much to go in to right now ...

**8. 10 points.** A hydrogen atom initially in its ground state ( $n=1$ ) absorbs a photon and ends up in the state for which  $n=3$ . If the atom eventually returns to the ground state, what photon energies could the atom emit?

There are two different ways the electron can go from the third to the first level: it can jump from 3 to 1 directly, or it can first go from 3 to 2, and then from 2 to 1. In the former case, the electron must lose an energy  $\Delta E_1 = E_3 - E_1$  by emitting a photon. In the latter case, the electron must first emit a photon of energy  $\Delta E_2 = E_3 - E_2$ , followed by one of energy  $\Delta E_3 = E_2 - E_1$ . This makes a total of three different photons that can be emitted:

$$\begin{aligned} \Delta E_1 &= E_3 - E_1 = \frac{-13.6 \text{ eV}}{3^2} - \frac{-13.6 \text{ eV}}{1^2} \approx 12.1 \text{ eV} \\ \Delta E_2 &= E_3 - E_2 = \frac{-13.6 \text{ eV}}{3^2} - \frac{-13.6 \text{ eV}}{2^2} \approx 1.89 \text{ eV} \\ \Delta E_3 &= E_2 - E_1 = \frac{-13.6 \text{ eV}}{2^2} - \frac{-13.6 \text{ eV}}{1^2} \approx 10.2 \text{ eV} \end{aligned}$$

**9. 10 points.** Initially, the radioactive decay rate of a particular group of nuclei is 300 counts per second. After 5 minutes, the decay rate drops to about 38 counts per second.

(a) What approximately is the half life of this nucleus?

(b) What will be the approximate decay rate (in counts per second) after 2.5 *additional* minutes?

The radioactive decay rate (or the number of atoms in the sample, for that matter) follows an exponential law:

$$R(t) = R_0 \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

where  $R_0$  is the initial rate,  $R(t)$  the rate after a time  $t$ , and  $T_{1/2}$  is the half life. If the rate is reduced from 300 to 38 counts per second after 5 minutes, or 300 seconds:

$$\begin{aligned} 38 &= 300 \left(\frac{1}{2}\right)^{300/T_{1/2}} \\ \frac{38}{300} &= \left(\frac{1}{2}\right)^{300/T_{1/2}} \\ \ln \frac{38}{300} &= \frac{300}{T_{1/2}} \ln \frac{1}{2} \\ T_{1/2} &= \frac{300 \ln \frac{1}{2}}{\ln \frac{38}{300}} \approx 100 \text{ s} \end{aligned}$$

After an additional 2.5 minutes, a total of 450 seconds have elapsed. Using the now known half-life with the initial decay rate, we can find the rate after 450 seconds:

$$\begin{aligned} R(t) &= R_0 \left(\frac{1}{2}\right)^{t/T_{1/2}} \\ R(450 \text{ s}) &= 300 \left(\frac{1}{2}\right)^{450/100} \approx 13.5 \text{ counts/sec} \end{aligned}$$

**10. 10 points.** Calculate the binding energy *in MeV* of a deuteron (the atom  ${}^2_1\text{H}$ ), given that its atomic mass is 2.014102 u. Note that  $m_{p^+} = 1.007825 \text{ u}$ , and  $m_{n^0} = 1.008665 \text{ u}$ .

The binding energy is the difference in the mass of the bare protons and neutrons and the nucleus itself, all times  $c^2$  to get an energy. For a deuteron, we have one neutron and one proton, so we add those two masses together, and subtract off the mass of the deuteron itself:

$$\begin{aligned} \text{Binding Energy} &= \left[ \sum_{\substack{\text{all } p^+ \\ \text{and } n^0}} mc^2 \right] - m_{\text{atom}}c^2 \\ &= \left[ (1 p^+) \cdot \left(1.007825 \frac{\text{u}}{p^+}\right) + (1 n^0) \cdot \left(1.008665 \frac{\text{u}}{n^0}\right) \right] c^2 - 2.014102 \text{ u} \cdot c^2 \\ &= 0.002388 \text{ u} \cdot c^2 \\ &= 0.002388 \text{ u} \cdot c^2 \left( \frac{931 \text{ MeV}}{\text{u} \cdot c^2} \right) \\ &= 2.224 \text{ MeV} \end{aligned}$$

Note that in that last step, we used the (given) conversion of 931 MeV/u·c<sup>2</sup> to get from a mass difference to an energy ...