University of Alabama
Department of Physics and Astronomy

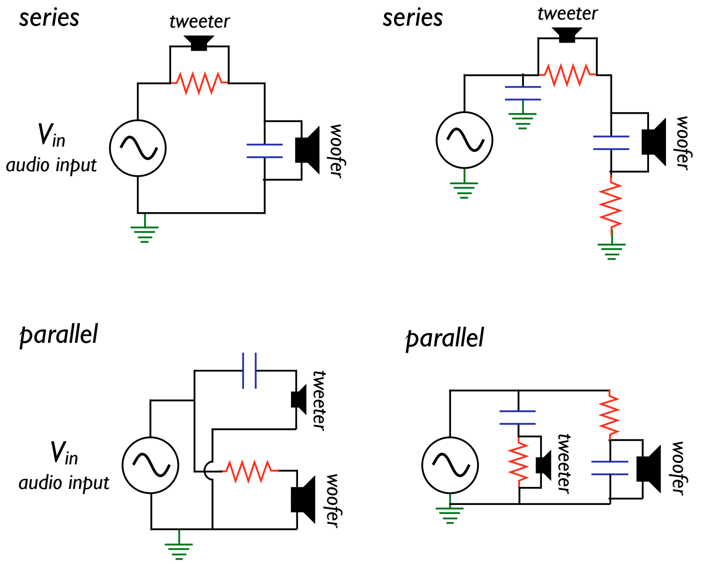
PH 102-2 / LeClair Spring 2008

Problem Set 8: SOLUTIONS

1. 15 points. Using capacitors, resistors, and inductors, sketch a circuit to split an audio signal composed of many frequencies in to a low frequency part and a high frequency part, for distribution to speakers. That is, filter the incoming signal into separate low frequencies and high frequencies to send to a woofer and tweeter, respectively. Such a circuit is known as an "audio crossover." You do not need to specify the values of your components.

Even with only passive components like capacitors, inductors, and resistors, there are many ways to go about this problem. For simplicity, we will use only resistors and capacitors. We know that capacitors allow higher frequency signals through easily, while blocking lower frequency signals. A resistor, on the other hand, lets all frequencies through equally. What we can do, then, is use a capacitor to direct high frequency signals away from the woofer and toward the tweeter.

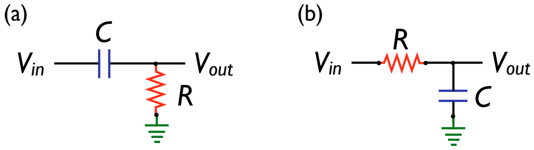
The circuit in the upper left portion of the circuit below shows the simplest possible crossover. If you look closely, it is identical to the low-pass filter in the next problem! All we have done is connect the output - which will be preferentially low-frequency signals - to the woofer. The tweeter takes the full range signal developed across the resistor, which will include the high frequencies. Another way of thinking of this circuit is that the capacitor "shorts out" the high frequency signals, and keeps them away from the woofer. One serious way that this circuit is lacking is that we don't send *only* high frequencies to the tweeter.



We could cure that problem by putting a low-pass and high-pass filter in series instead of just a resistor and capacitor, as shown in the upper right panel in the figure above. The low pass filter connects across the tweeter, and shorts the low frequency signals around it, such that it sees only high frequencies. The high pass filter does the reverse for the woofer, so it only sees low frequency signals. This is a perfectly workable crossover, and on paper it does just what we want. A major disadvantage (or, an advantage depending on your viewpoint) is that the two filters interact with each other - each filter is sending some signal to ground that really should be going to the other. Further, changes in any one component affect *both* high and low pass filter sections - to keep the same overall flat response, one must change all components, not just one, when one tries to tune the crossover frequency. Finally, this circuit is very sensitive to component variations and (in)accuracy. Still, the design is sufficiently simple to be appealing.

How can we do better? We can make a voltage divider to split the input signal into two, and send half to each filter and speaker. This is a *parallel* crossover. Rather than make a voltage divider out of resistors, we make one out of a pair of filters. In the lower left panel of the figure above, the audio input signal is split into two. On the branch going to the tweeter, we put a capacitor in series, which ensures that only the high-frequency part of the signal will take this path to the tweeter. The leftover low-frequency parts of the signal take the other branch to the woofer. This design is much more common, mainly because the two filter sections do not interact, which means they can be designed separately. Further, the sensitivity to component variation is far less. The lower right diagram is a re-rendering of the same circuit, adding a shorting capacitor and resistor for the woofer and tweeter. In this geometry, it is (perhaps) easier to see that both filters receive the same signal, and thus act independently.

2. 15 points. Two filters are constructed, as shown below, both using a resistor of $R = 1.0\,\mathrm{k}\Omega$ and a capacitor of $0.01\,\mu\mathrm{F}$. **(a)** Determine which filter is a low-pass, and which is a high-pass. Explain your reasoning. **(b)** Sketch the frequency response of each filter, and calculate the 3dB "cutoff" frequency of each. **(c)** Assume now that these filters are applied in series, one after the other. What would be the frequency response of the combined filter?



Circuit **(a)** is the high-pass filter, circuit **(b)** is the low-pass filter. It is crucial to remember that the reactance of a capacitor increases as frequency *decreases*. In circuit **(a)**, high frequency signals will pass through the capacitor easily, reaching the output, while low frequency signals will be rejected. In circuit **(b)**, both high and low frequencies will pass through the resistor, but the high frequency signals will see the capacitor as an easy path to ground, and thus never reach the output - only low frequencies reach the output.

Both of these filters are simple *RC* filters, and thus both have the *same* cutoff frequency. Don't forget to convert between angular frequency (ω) and frequency (f):

$$\begin{aligned}
2\pi f_{\text{cutoff}} &= \omega_{\text{cutoff}} = \frac{1}{\tau} = \frac{1}{RC} \\
&= \frac{1}{(10^3\,\Omega)\,(0.01\times10^{-6}\,\mathrm{F})} \\
&= 10^5\,\mathrm{Hz} \\
\Longrightarrow \quad f &= \frac{10^5}{2\pi}\,\mathrm{Hz} \approx 15.9\,\mathrm{kHz}
\end{aligned}$$

Below, we show the ratio of the input to output voltage for a single low-pass RC filter, a single high-pass RC filter, and both filters in series on linear plot (*left*), a linear-logarithmic plot (*middle*), and a dual logarithmic plot (*right*). The latter is the most common way filter response curves are plotted. For both filters, $R = 1.0\,\mathrm{k}\Omega$ and $C = 0.01\,\mu\mathrm{F}$. The curves sketched in your notes are on a linear scale to avoid confusion, but the dual logarithmic plot is closer to how we process, . *e.g.,* audio signals.

You might think that nothing gets through two filters in series, or only the cutoff frequency would get through. This would be true if the filters had a "sharp" cutoff frequency, above or below which the signal abruptly went to zero. For simple *RC* filters, this is not the case - the "cutoff" is really more of a "rolloff," the point at which the output signal voltage has been reduced by $1/\sqrt{2}$ relative to the input signal.[i] As a result, there is a region where the filter response curves overlap, and there is band of frequencies, roughly centered on the cutoff frequency, which will make it through *both* filters, albeit attenuated. The response is more easily visualized on a logarithmic or double logarithmic plot.

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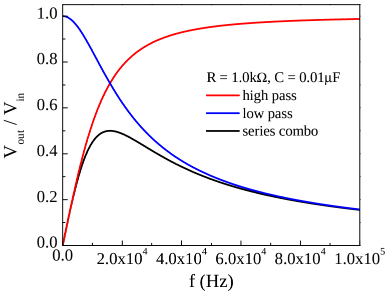
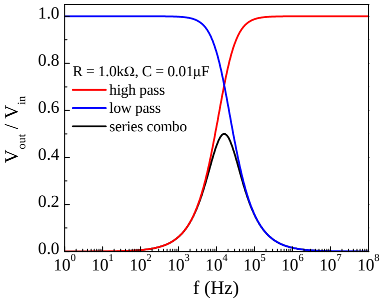
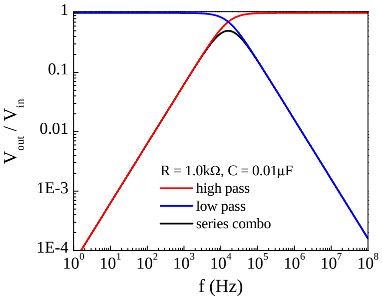
[i] Thus, at the cutoff frequency the output *power* is half the input.



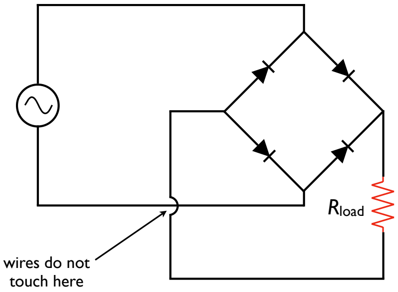


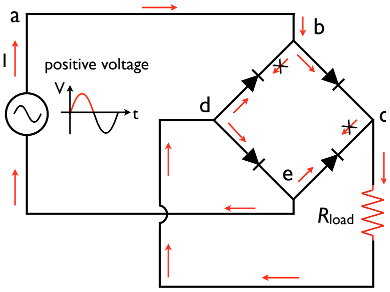


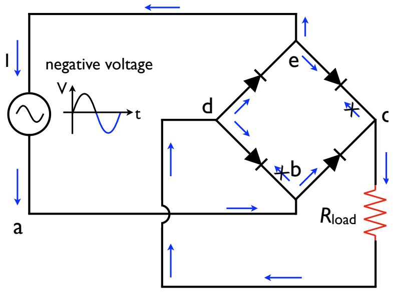
The latter "double log" plot makes everything into neat piece-wise linear graphs, which is convenient. It is also more representative to how we actually hear sound - our notion of pitch or a linear scale of notes is actually a *logarithmic* scale of frequencies, and our ears gauge sound intensity on a logarithmic scale as well, which is why sound intensity is usually reported in *decibels*, or dB.

But we digress.

3. 10 points. The circuit at right is known as a full-wave bridge rectifier - an ac source is connected to two opposite ends of a diode "bridge," and a load resistor is connected to the other two ends. If the ac voltage source is sinusoidal, sketch the voltage and current across the load resistor as a function of time. Refer to the previous problem to determine the current flow through the diodes.







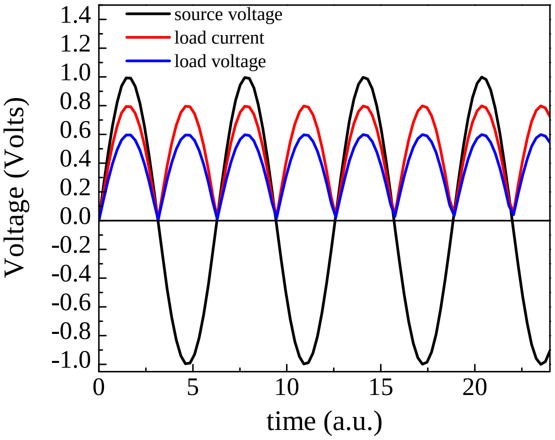
The easiest way to tackle this one is to simply trace out the path of the current from start to finish: once for positive voltages, and once for negative voltages. Using the middle panel above, start just above the source at point **(a)**.

Whenever the voltage is positive, let us say that current flows up out of the source, from a to b. At point b, the current encounters a junction. It cannot go left, because the diode does not conduct in that direction. Thus, it goes right, to point c. At point c, a second junction is reached. The current can only go down through the load in this case, since the diode to the left will not conduct in this direction. The current continues on to point d, at which point *both* diodes will actually conduct. What now?

The current will not take the upper path, since this will just lead it back to where it started. Further, since the resistive load introduced a voltage drop, point b is actually at a higher potential than point d, so current will not want to take this path. Thus, the current goes down through the lower diode to point e. At that point, the same logic applies - point c is at a higher potential than point e due to the voltage drop across the resistor, and thus the current will only go down and back to the source.

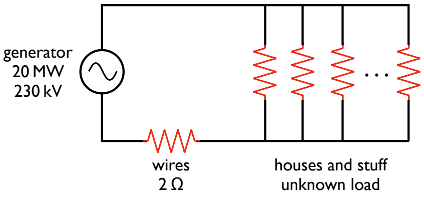
If we want to consider what happens for *negative* voltages, we just trace the diagram backwards - for negative voltages, the current out of the source flow the opposite direction. Using the same logic as above at every junction, we find that *the current goes through the load in the same direction* as it did for positive voltages. Thus, no matter what, the resistive load sees the same polarity of current. What this does is change an alternating current (which changes sign) into a direct current (which does not change sign). The direct current is still very "bumpy," but by using a filter at the position of the load (say, shorting the load with a capacitor, forming an *RC* filter like those above!) we can smooth out the "bumps" as much as we like, until the current is essentially constant. Finally: remember that the resistor still follows Ohm's law. The current and voltage will still be proportional.[ii]

A nice discussion of power supplies, including an animated GIF of the schematic above, can be found at: `http://www.kpsec.freeuk.com/powersup.htm`.



4. 10 points. An electricity-generating station needs to deliver energy at a rate of 20 MW to a city 1.0 km away. **(a)** If the resistance of the wires is 2.0 Ω and the energy costs about 10¢/kWh, estimate what it costs the utility company for the energy converted to internal energy in the wires during one day. A common voltage for commercial power generators is 22 kV, but a step-up transformer is used to boost the voltage to 230 kV before transmission. **(b)** Repeat the calculation for the situation in which the power plant delivers the energy at its original voltage of 22 kV.

We can think of the generator as nothing more than an ac voltage source with a particular constant power output. Our generator is connected to many, many loads - houses, factories, whatever - and has a series resistance to represent the wires:



[ii] We have neglected one thing: diodes actually have a built-in voltage drop of about 0.6 V, and they will not actually conduct until their forward voltage exceeds this value. First, this means that the bridge will not work at all unless the signals are sufficiently larger than 0.6 V! This further means that the output will actually have small "gaps" between the peaks, since wherever the source voltage becomes lower than 0.6 V, *none* of the diodes will conduct, no matter what the direction. The effect is basically the same as if we shifted the voltage zero upward by 0.6 V, and clipped off the downward-pointy parts of the load current and voltage, a problem known as "diode clipping." Again, we digress.

In order to find the power lost in the wires, we need to find the current going through the resistor. Now, we don't know what the other resistances are in the circuit, or even how many of them there are. What we *do* know, is that whatever the current flowing out of the source is, it will be the same in the wires, since they are just in series. The current in the generator can be found from the power:

$$\mathscr{P}_{\text{gen}} = I\Delta V$$

$$\implies \quad I = \frac{\mathscr{P}}{\Delta V} = \frac{20 \times 10^6\,\text{W}}{230 \times 10^3\,\text{V}} \approx 87.0\,\text{A}$$

Now that we have the current in the generator, and thus the wires, we can easily find the power dissipated in the wires:

$$\mathscr{P}_{\text{wires}} = I^2R = (87.0\,\text{A})^2\,(2.00\,\Omega) \approx 15100\,\text{W} = 15.1\,\text{kW}$$

This power is energy per unit time. The actual energy lost over some time period is just power times time. Over the course of one day, 24 hours, the energy expressed in kW·hr is just:

$$\text{energy in one day} = (15.1\,\text{kW})\,(24\,\text{hr}) = 362\,\text{kW}\cdot\text{hr}$$

The cost is just determined by the price per kW·hr:

$$\text{money lost} = [362\,\text{kW}\cdot\text{hr}]\,[\$0.01/\text{kW}\cdot\text{hr}] \approx \$36.20$$

All we have to do for the second part is repeat the calculation for 22 kV. Since the voltage is roughly 10 times less, the current will be about 10 times more. The power - going as the square of the current - will be about 100 times more, however, and so will be the energy and money lost. You should find about \$3970. Now you can see why it is best to transmit power at the highest feasible voltage, and use a transformer to step it down near its intended distribution point.

5. 10 points. Any two adjacent conductors can be considered as a capacitor, although the capacitance will be small unless the conductors are close together or long. This (unwanted) effect is termed "stray" or "parasitic" capacitance. Stray capacitance can allow signals to leak between otherwise isolated circuits (an effect called crosstalk), and it can be a limiting factor for proper functioning of circuits at high frequency. A stray capacitance can result when you touch or come close the wires in a circuit - your body provides a capacitive path between the circuit of interest and an adjacent noise source.

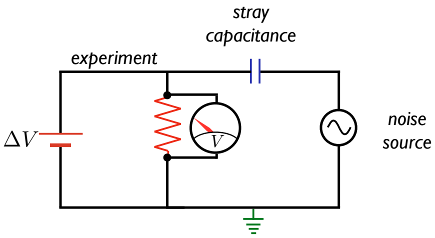
(a) Explain, referencing the figure below, why the stray capacitance allows unwanted ac signals to couple into the circuit, but does not allow dc signals. **(b)** Suggest a method for minimizing this effect (*hint: last week's homework*).

In the circuit shown, the noise source and experiment share a common ground point, which means the circuits are connected at one point. Connecting the circuits at a second point will allow signals to pass from one circuit to the other. If this connection is made by a stray capacitance, constant dc currents can pass through the ground connection, but not through the capacitance. Thus, dc signals still have only one path into the experimental circuit, there is no closed loop for them to couple into the experiment. Ac signals, on the other hand, *can* travel through the capacitor, and now have a closed loop to travel between the experiment and noise source. The higher the frequency, the easier the signals can couple into the experiment.

The solution to this problem is simply to use coaxial shielded cable. The stray capacitance arises when wires from the experiment and noise source come too close together, and the intervening region makes a capacitor out of the adjacent wires. Time-varying electric fields from one circuit can be coupled across the intervening region into the other circuit, just like ac currents can pass through a capacitor. If the experimental circuit's wires are encased in a conducting shell, the electric fields from nearby circuits is shielded out, since the field inside a conductor must be zero. Of course, another solution is just to put the experiment far, far away from any potential noise sources, but this is not always practical.

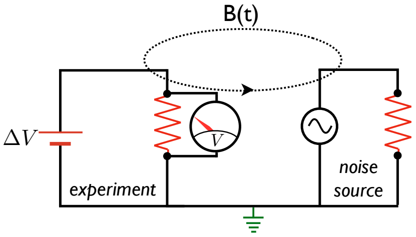
See the solutions to last week's homework, in which we discuss shielded cable in more detail.

6. 10 points. The previous problem dealt with unintentional capacitive coupling. A similar phenomena is *inductive* coupling, and the basis for this effect is illustrated in the figure at left. Time-varying magnetic fields (for instance, due to



the 60 Hz current present all around you!) can couple the circuit of interest to a nearby circuit, inducing an unwanted signal.

(a) Explain, referencing the figure below, why inductive coupling also allows unwanted ac signals to couple into the circuit, but does not allow dc signals. **(b)** Suggest a method for minimizing this effect (*hint: last week's homework*).



Remember that any current-carrying wire has a magnetic field circulating around it. A time-varying current in a noise source will thus create a time-varying magnetic field around its wires. If this time-varying magnetic field penetrates the loop of the experimental circuit, it will result in a time-varying flux through the experimental circuit's loop. Faraday's law tells us that a time-varying flux leads to an induced voltage, which means that the time-varying current in the noise source will inductively couple a noisy signal into the experimental circuit.

This doesn't work for dc currents. A static current in the noise source creates a static current in the wires of that circuit, which gives a constant magnetic field. This constant magnetic field, even if it intercepts the loop of the experimental circuit, will only lead to a constant magnetic flux. Since the flux does not vary in time, there is no induced voltage, and no additional signal is coupled into the circuit. Only ac or time-varying signals will be inductively coupled into the experiment. Of course, noise is by definition time-varying, so this problem is much more general than it sounds.
The solution to this problem is to use twisted-pair wiring, also covered in detail on last week's homework. Twisting the wires creates many small, alternating loops, which have far smaller and opposing induced voltages that cancel each other out overall.

One can solve *both* capacitive and inductive coupling problems by having twisted pair wiring surrounded by a conducting sheath - usually referred to as "shielded twisted pair" wiring. One can also just take two normal coaxial cables and twist them together - for most purposes, this is sufficient.

7. 10 points. An audio amplifier delivers to a speaker alternating voltage at audio frequencies. If the source voltage has an amplitude of 15.0 V and an internal resistance of 8.20 Ω, and the speaker can be considered equivalent to a 10.4 Ω resistor, what is the time-averaged power transferred to it? The source, its internal resistance, and the speaker are in series.

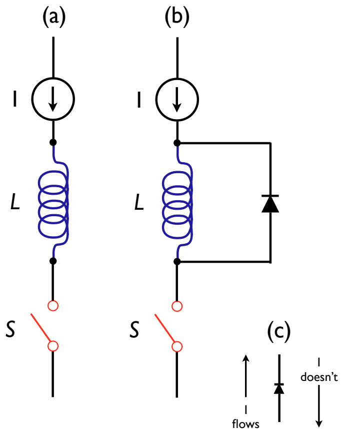
8. 5 points. Why does a capacitor act as a short circuit at high frequencies? Why does it act as an open circuit at low frequencies?

The reactance of a capacitor is $X_C = 1/\omega C$. At high frequencies, the reactance is low, and signals can pass through the capacitor easily. At high enough frequencies, the reactance becomes negligibly small, just like a short circuit. At low frequencies, the reactance is high, and signals do not pass through the capacitor easily. At sufficiently low frequencies, the reactance becomes so high that only a negligible portion of any signal will pass through it, just as if it were replaced by a broken wire. For dc signals under steady-state conditions, capacitors effectively do nothing.

9. 5 points. Why does an inductor act as a short circuit at low frequencies? Why does it act as an open circuit at high frequencies?

The reactance of a capacitor is $X_L = \omega L$. At low frequencies, the reactance is low, and signals can pass through the inductor easily. At low enough frequencies, the reactance becomes negligibly small, just like a short circuit. For dc signals under steady-state conditions, and ideal inductor is equivalent to a wire - a short circuit. At high frequencies, the reactance is high, and signals do not pass through the inductor easily. At sufficiently high frequencies, the reactance becomes so high that only a negligible portion of any signal will pass through it, just as if it were replaced by a broken wire.

10. 10 points. A current source I is used to drive a large inductor (say, a wound wire electromagnet) as shown at right. Driving inductive loads can be problematic - what happens when you open the switch providing current to an inductor in circuit **(a)** Why does adding a diode across the inductor, circuit **(b)**, add protection? Recall diodes only allow current through in one direction, as shown in **(c)**.



Because inductors have the property $V = -\Delta I/\Delta t$, it is not possible to turn off the current suddenly - if $\Delta t = 0$, that would imply an infinite voltage across the inductor, a violation of numerous physical laws (and good common sense). What does happen is that the voltage across the inductor rises rapidly after the switch is opened, and keeps rising until it *forces* a current to flow - for example, by making a spark jump across the poles of the switch. This is BAD, and may let the magic smoke out. Electronic devices controlling inductive loads can be damaged in this way, since essentially some component has to "break down" to satisfy the inductor's desire for constant current.

In the second circuit shown, a diode is used to protect the magic smoke in our devices, and stop the "inductive kick" that may damage other components. When the switch is initially closed in this circuit, current flows through the inductor. Current does not take the path through the diode, since it is not conducting in the direction of current flow. Now, what happens when the switch is suddenly opened? The inductor tries to keep current flowing toward the switch, as it had been moments before, and develops a negative voltage. This means that the bottom of the inductor becomes positive relative to the top - opposite the case when steady current is flowing - and a large current tries to flow *up* through the diode to maintain continuity.

Without the diode, the inductor would try to pull this large current from the switch or a nearby component, leading to a nasty spark somewhere in the circuit.[iii] When we have the diode protection, however, after the switch is thrown the back-current from the inductor can flow up through the diode - it is conducting in this direction. The back-current from the inductor will flow through the diode, creating a voltage drop from bottom to top. This voltage drop makes the top switch terminal at a slightly higher voltage than the supply or the top of the inductor, and no current will be "pulled"

[iii] In older cars, this is more or less how an ignition coil works.

from the switch. Basically, the back-current will be short-circuited by the diode, but the forward current during normal operation will not be.

The back-voltage on an inductive load can easily be 1000 V, enough to kill nearly any solid state electronics. Problem is, inductive loads are rather common, in the form of *relays*, which are basically current-controled switches. It is virtually certain that you have used a relay at some point today, and it is equally certain that said relay had a protection diode or its RC equivalent.

`http://en.wikipedia.org/wiki/Relay`