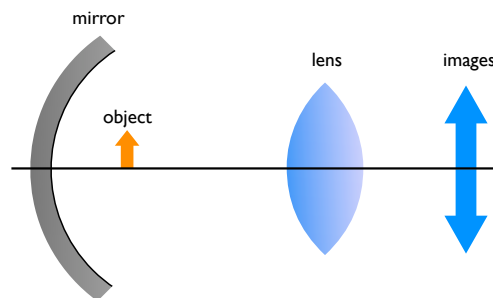


Problem Set 9: Optics

Instructions:

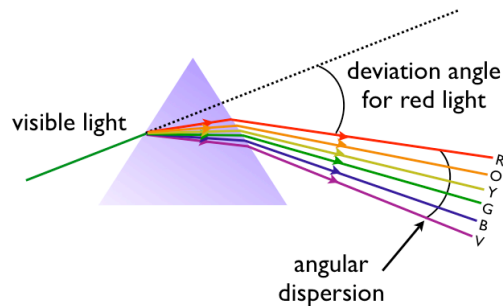
- Answer all questions below. Show your work for full credit.
- Due before 5pm, 31 Mar 2008.
- Problem sets may be turned in *via* email or hard copy
- Hard copies may be left in Dr. LeClair's mailbox (Gallalee 206) or office (Bevill 228)
- You may collaborate, but everyone must turn in their own work

1. **15 points.** An observer to the right of the mirror-lens combination shown in the figure sees two real images that are the same size and in the same location. One image is upright and the other is inverted. Both images are 1.70 times larger than the object. The lens has a focal length of 11.2 cm. The lens and mirror are separated by 40.0 cm. Determine the focal length of the mirror. (Don't assume that the figure is drawn to scale.)



2. **15 points.** The index of refraction for **violet** light in silica flint glass is $n_{\text{violet}} = 1.66$, and for **red** light it is $n_{\text{red}} = 1.62$. In air, $n = 1$ for both colors of light.

What is the **angular dispersion** of visible light (the angle between red and violet) passing through an equilateral triangle prism of silica flint glass, if the angle of incidence is 50° ? The angle of incidence is that between the ray and a line *perpendicular* to the surface of the prism. Recall that all angles in an equilateral triangle are 60° .



3. 15 points. As light from the Sun enters the atmosphere, it refracts due to the small difference between the speeds of light in air and in vacuum. The optical length of the day is defined as the time interval between the instant when the top of the Sun is just visibly observed above the horizon, to the instant at which the top of the Sun just disappears below the horizon. The geometric length of the day is defined as the time interval between the instant when a geometric straight line drawn from the observer to the top of the Sun just clears the horizon, to the instant at which this line just dips below the horizon. The day's optical length is slightly larger than its geometric length.

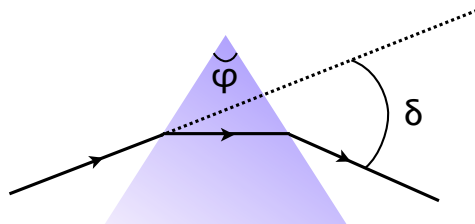
By how much does the duration of an optical day exceed that of a geometric day? Model the Earth's atmosphere as uniform, with index of refraction $n = 1.000293$, a sharply defined upper surface, and depth 8767 m. Assume that the observer is at the Earth's equator so that the apparent path of the rising and setting Sun is perpendicular to the horizon. You may take the radius of the earth to be 6.378×10^6 m. Express your answer to the nearest hundredth of a second.

4. 10 points. What is the apparent depth of a swimming pool in which there is water of depth 3 m, **(a)** When viewed from normal incidence? **(b)** When viewed at an angle of 60° with respect to the surface? The refractive index of water is 1.33.

5. 15 points. Light is deviated by a glass prism of index n as shown in the figure below. The ray in the prism is parallel to the base. Show that the refractive index is related to the deviation angle δ and the prism angle φ by the equation

$$n \sin \frac{\varphi}{2} = \sin \left(\frac{\varphi + \delta}{2} \right)$$

for this angle of incidence (*i.e.*, the angle of incidence such that the ray in the prism is parallel to the base). The deviation angle δ is a minimum for this angle of incidence, and is known as the angle of minimum deviation. *Hint: You can solve this and the first problem together, if you keep things as general as possible - this is just a special case of the first problem.*



6. 10 points. Prove that if two thin lenses are placed in contact, they are equivalent to a single lens of focal length

$$f_{\text{equiv}} = \frac{f_1 f_2}{f_1 + f_2}$$

where f_1 and f_2 are the focal lengths of the two thin lenses. In some sense, lenses in series add like capacitors do.

7. 10 points. A thin lens of focal length f is used to form an enlarged real image of an object on a screen. The separation of the screen and object is s . **(a)** Deduce an expression for the distance between the object and the lens. **(b)** What is the minimum ratio s/f for the formation of a real image?

8. 5 points. A thin convex lens of focal length f produces a real image of magnification m . Show that the object distance p is given by

$$p = \frac{m + 1}{m} f$$

9. 5 points. A transparent sphere of unknown composition is observed to form an image of the Sun on the surface of the sphere opposite the Sun. What is the refractive index of the sphere material? *Hint: assume the object distance is effectively infinite.* You may take the refractive index of air to be 1.0.