UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 102-1 / LeClair Summer II 2008

Problem Set 3: Solutions

1. **10 points.** **(a)** What is the velocity of a beam of electrons which move undeflected in a region of space in which there exist both a uniform electric field $|\vec{\mathbf{E}}|=3.4\times10^5\,\mathrm{V/m}$ and a uniform magnetic field of $|\vec{\mathbf{B}}|=2\times10^{-3}\,\mathrm{T}$? **(b)** Show the orientation of the vectors $\vec{\mathbf{v}}$, $\vec{\mathbf{E}}$, and $\vec{\mathbf{B}}$ in a diagram. **(c)** What is the radius of the electron orbit when the electric field is removed, and only the magnetic field remains? You may ignore relativistic effects.

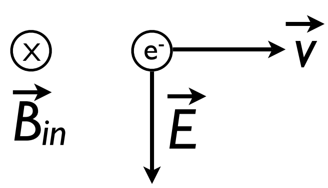
If the electron is to be undeflected, moving in a straight line path, then it must be traveling at constant velocity, and experience no net force. For this to be true, the magnetic and electric forces must cancel each other by acting in opposite directions with equal magnitude:

$$\begin{aligned}
&\sum \vec{\mathbf{F}} = 0 = \vec{\mathbf{F}}_e + \vec{\mathbf{F}}_B \\
\Longrightarrow \quad &\vec{\mathbf{F}}_e = -\vec{\mathbf{F}}_B \\
&-e\vec{\mathbf{E}} = -\left(-e\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right) \\
&\vec{\mathbf{E}} = \vec{\mathbf{v}}\times\vec{\mathbf{B}}
\end{aligned}$$

So the electric field must be pointing along the same direction as $\vec{\mathbf{v}}\times\vec{\mathbf{B}}$. One way for this to be true is if the electric field is along the same axis as the velocity, and the magnetic field is perpendicular to the velocity ($\theta_{vB}=90°$). In this case, we can equate the magnitude of the electric and magnetic fields:

$$\begin{aligned}
E &= vB\sin\theta_{vB} = vB \\
v &= \frac{E}{B} \approx 1.7\times10^8\,\mathrm{m/s}
\end{aligned}$$

One possible diagram of the field and velocity vectors is shown below. Remember: since an electron has a negative charge, the force is in the opposite direction it would be for a positive charge. You can verify that the magnetic force will be pointing down in this case, and the electric force will be pointing up.



If we take away the electric field, we simply have a single charge moving perpendicularly to a constant magnetic field. We have solved this problem already: the magnetic force will always be perpendicular to the velocity, which means circular motion will result. The magnetic force supplies the centripetal force in this case:

$$\sum |\vec{\mathbf{F}}| = \frac{mv^2}{r} = qvB$$
$$\implies \quad r = \frac{mv^2}{qvB} = \frac{mv}{qB} \approx 0.48\,\mathrm{m}$$

2. 15 points. Each of the twelve edges of the cube is a resistor of value R. What is the resistance between two *opposite corners*?

For now, see this document:

`http://www.radioelectronicschool.net/files/downloads/resistor_cube_problem.pdf`

I prefer the second solution (end of the document), using "logical reasoning," which physicists usually call "symmetry arguments." I hope to have my own version posted soon.

3. 10 points. Two resistors connected in series are measured to have an equivalent resistance of $1500\,\Omega$. The same two resistors in parallel are measured to have an equivalent resistance of $350\,\Omega$. What are the values of the resistors?

When we combine two resistors in series, they simply add to form a equivalent resistor. In parallel, they add inversely. This implies two equations:

$$R_1 + R_2 = 1500$$
$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{350}$$

It is more convenient if we rearrange the second one (find a common denominator for the left-hand side and invert) to look like this:

$$\frac{R_1 R_2}{R_1 + R_2} = 350$$

Now, plug the first one in to the second and massage it a bit:

$$\frac{R_1 R_2}{R_1 + R_2} = \frac{R_1 R_2}{1500} = 350$$
$$R_1 R_2 = 350 \cdot 1500$$

We can use our first equation a second time, noting that $R_2 = 1500 - R_1$:

$$R_1 R_2 = R_1 (1500 - R_1) = 1500 R_1 - R_1^2 = 350 \cdot 1500$$
$$\implies \quad R_1^2 - 1500 R_1 + 350 \cdot 1500 = 0$$

Now we have a quadratic that we can solve for R_1.

$$
\begin{aligned}
R_1 &= \frac{-(-1500) \pm \sqrt{(-1500)^2 - 4\cdot 1\cdot (350\cdot 1500)}}{2}\\
&= \frac{1500 \pm \sqrt{1500^2 - 1400\cdot 1500}}{2}\\
&= \frac{1500 \pm 1500\sqrt{1 - \frac{1400}{1500}}}{2}\\
&= 750\left[1 \pm \sqrt{1 - \frac{14}{15}}\right]\\
&= 750\left[1 \pm \sqrt{\frac{1}{15}}\right]\\
&\approx \{944, 556\}
\end{aligned}
$$

Now we have two solutions for R_1. What is that? No worries. Since we labeled R_1 and R_2 arbitrarily, and our equations are completely symmetric with regard to either, we have actually just found *both* R_1 and R_2. Try plugging them both in to the first equation, and you will see that we really only have one complete solution:

$$
\begin{aligned}
R_2 &= 1500 - R_1 = 1500 - 944 = 556 \qquad \text{1st solution}\\
R_2 &= 1500 - R_1 = 1500 - 556 = 944 \qquad \text{2nd solution}
\end{aligned}
$$

Thus, our two resistors have to be $944\,\Omega$ and $556\,\Omega$.

4. 10 points. A wire with a weight per unit length of $0.10\,\mathrm{N/m}$ is suspended directly above a second wire. The top wire carries a current of $30\,\mathrm{A}$ and the bottom wire carries a current of $60\,\mathrm{A}$. Find the distance of separation between the wires so that the top wire will be held in place by magnetic repulsion.

Since we have two parallel wires with currents flowing, we know we are going to have a magnetic force between them. Now the problem says that the magnetic force is repulsive (which implies that the currents are in opposite directions), which it must be in order for the the top wire to be held in place *against the force of gravity*. This means we want to balance the gravitational force on the top wire, acting downward, against the repulsive magnetic force between the two wires, acting upward on the top wire.

The top wire is quoted to have a *weight* per unit length of $0.10\,\mathrm{N/m}$, which we will call χ. A *weight* is already a force, mass times gravity, so the problem gives you the gravitational force per unit length $\chi = mg/l$ for some section of wire of length l and mass m. We can relate the more common *mass per unit length* $\lambda = m/l$ and the *weight per unit length* easily: $\lambda g = \chi$. Since the force between two parallel current carrying wires is also expressed in terms of force per unit length, we are nearly done.

Given the currents in the top and bottom wire (I_1 and I_2, respectively), the weight per unit length (χ), and the separation between the wires (d), we just have to set the weight per unit length equal to the magnetic force per unit length, and solve for d:

$$\begin{aligned}
\text{weight per length} &= \text{magnetic force per length}\\
\chi \equiv \frac{mg}{l} &= F_B\\
\chi &= \frac{\mu_0 I_1 I_2}{2\pi d}\\
\Longrightarrow d &= \frac{\mu_0 I_1 I_2}{2\pi\chi}
\end{aligned}$$

Plugging in the numbers we're given ...

$$\begin{aligned}
d &= \frac{\left[4\pi \times 10^{-7}\,\mathrm{N/A^2}\right][30\,\mathrm{A}]\,[60\,\mathrm{A}]}{2\pi \cdot 0.10\,\mathrm{N/m}}\\
&= \frac{2 \cdot 1800 \cdot 10^{-7}}{0.1}\,\mathrm{m}\\
&\approx 3.6 \times 10^{-3}\,\mathrm{m}
\end{aligned}$$

Of course, the *units* work out much easier if you know that μ_0 can be expressed in T·m/N or N/A^2, the two are equivalent: $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}$ and $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2}$. This equivalence makes some sense – the first set of units comes from thinking about the field created by a current-carrying wire, while the second comes from thinking about the force between *two* current-carrying wires.

5. 5 **points.** A proton moves with velocity $\vec{\mathbf{v}} = (2\hat{\mathbf{x}} - 4\hat{\mathbf{y}} + 1\hat{\mathbf{z}})\,\mathrm{m/s}$ in a region where the magnetic field is $\vec{\mathbf{B}} = (1\hat{\mathbf{x}} + 2\hat{\mathbf{y}} - 3\hat{\mathbf{z}})\,\mathrm{T}$. What is the magnitude and direction of the magnetic force this charge experiences?

We know the velocity, and we know the magnetic field, so the magnetic force is easy enough to calculate:

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

For once, we want both the magnitude *and* direction of the force - that is, we want the whole force vector, not just the magnitude. How do we find the cross product of two vectors? There is a simple way. Say you have two vectors, $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$:

$$\begin{aligned}
\vec{\mathbf{a}} &= a_x\,\hat{\mathbf{x}} + a_y\,\hat{\mathbf{y}} + a_z\,\hat{\mathbf{z}}\\
\vec{\mathbf{b}} &= b_x\,\hat{\mathbf{x}} + b_y\,\hat{\mathbf{y}} + b_z\,\hat{\mathbf{z}}
\end{aligned}$$

An easy way to remember how to calculate the cross product of these two vectors, $\vec{\mathbf{c}} = \vec{\mathbf{a}} \times \vec{\mathbf{b}}$, is to take the determinant of the following matrix:

$$\begin{bmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}}\\ a_x & a_y & a_z\\ b_x & b_y & b_z \end{bmatrix}$$

Or, explicitly:

$$\vec{\mathbf{c}} = \det \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}}\\ a_x & a_y & a_z\\ b_x & b_y & b_z \end{vmatrix} = (a_y b_z - a_z b_y)\,\hat{\mathbf{x}} + (a_z b_x - a_x b_z)\,\hat{\mathbf{y}} + (a_x b_y - a_y b_x)\,\hat{\mathbf{z}}$$

Now, in the present case, we need to calculate $q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. Plugging in the vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$, and the proton's charge of $+e$:

$$\begin{aligned}
\vec{\mathbf{F}}_B = e\vec{\mathbf{v}} \times \vec{\mathbf{B}} &= e \det \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} \\
&= e\left[(v_yB_z - v_zB_y)\,\hat{\mathbf{x}} + (v_zB_x - v_xB_z)\,\hat{\mathbf{y}} + (v_xB_y - v_yB_x)\,\hat{\mathbf{z}}\right] \\
&= e\left(\Big[(-4\cdot -3) - (1\cdot 2)\Big]\,\hat{\mathbf{x}} + \Big[(1\cdot 1) - (2\cdot -3)\Big]\,\hat{\mathbf{y}} + \Big[(2\cdot 2) - (-4\cdot 1)\Big]\,\hat{\mathbf{z}}\right) \\
&= e\left(\Big[12 - 2\Big]\,\hat{\mathbf{x}} + \Big[1 + 6\Big]\,\hat{\mathbf{y}} + \Big[4 + 4\Big]\,\hat{\mathbf{z}}\right) \\
\implies \quad \vec{\mathbf{F}}_B &= e\,(10\,\hat{\mathbf{x}} + 7\,\hat{\mathbf{y}} + 8\,\hat{\mathbf{z}})
\end{aligned}$$

This is the full force vector, which already contains magnitude and direction. Basically, we are done, and if you stopped here, that is fine. If we do want the overall magnitude, we can find that easily:

$$|\vec{\mathbf{F}}_B| = \sqrt{F_x^2 + F_y^2 + F_z^2} = e\sqrt{10^2 + 7^2 + 8^2} = 14.6e \approx 2.34 \times 10^{-19}\,\mathrm{N}$$

If we want, we can also find the angle the vector makes with each axis. If θ is the angle $\vec{\mathbf{F}}_B$ makes with respect to the x axis, ϕ the angle with respect to the y axis, and ψ the angle with respect to the z axis, then:

$$\cos\theta = \frac{F_x}{|\vec{\mathbf{F}}|} \qquad \cos\phi = \frac{F_y}{|\vec{\mathbf{F}}|} \qquad \cos\psi = \frac{F_z}{|\vec{\mathbf{F}}|}$$

Plugging in our numbers ...

$$\theta = 46.8^\circ \qquad \phi = 61.4^\circ \qquad \psi = 56.8^\circ$$

6. 10 points. A lightbulb marked "75 W [at] 120 V" is screwed into a socket at one end of a long extension cord, in which each of the two conductors has a resistance of $0.800\,\Omega$. The other end of the extension cord is plugged into a 120 V outlet. Draw a circuit diagram and find the actual power delivered to the bulb in this circuit.

7. 15 points. Consider two parallel wires of infinite length, separated by a distance r, each with a uniform positive charge density λ. Both wires are moving in the same direction with velocity v, also parallel to the wires. If the electrical repulsion is exactly balanced by the magnetic attraction caused by the current, what is v? Remember that in chapter 3 of the notes, we derived the electric field from a long, charged wire. *Hint: find electric and magnetic forces per unit length of the wire.*

For now, see this page:

`http://www.builtonfacts.com/2008/07/15/the-light-fantastic/`

Don't believe everything you find on the internet, but this person knows their stuff. It is a rather terse solution, but if you were following along when we set this one up in class, it should be enough. I hope to have my own solution posted soon.

8. 10 points. Consider two solenoids, one of which is a tenth-scale model of the other. The larger solenoid is 2 m long, and 1 m in diameter, and is wound with 1 cm-diameter copper wire. When the coil is connected to a 120 V dc generator, the magnetic field at the center is exactly 0.1 T. The scaled-down version is exactly one-tenth the size in every linear dimension, including the diameter of the wire. The number of turns is the same in both coils, and both are designed to provide the same central field.

(a) Show that the voltage required is the same, namely, 120 V
(b) Compare the coils with respect to the power dissipated, and the difficulty of removing this heat by some cooling means.

This is basically a scaling problem: when everything is shrunk by 10 times, what happens to the required voltage for a given field? First, let's consider the large solenoid. Let's say it has length $L = 2\,\mathrm{m}$, radius $r = 0.5\,\mathrm{m}$, contains N turns of wire, and it provides a field $B = 0.1\,\mathrm{T}$ with a current I. We know we can relate the field and the current:

$$B = \mu_0 \frac{N}{L} I$$

The solenoid is just a long single strand of wire wrapped around a cylinder. If we say that the total length of wire used to wrap the solenoid is l, and the wire's diameter is d, then we can calculate the resistance of the solenoid:

$$R = \frac{\varrho l}{A} = \frac{\varrho l}{\pi d^2/4}$$

Here we have used the wire's resistivity ϱ, and its cross-sectional area $A = \pi r^2 = \pi d^2/4$. Given the resistance and voltage of $\Delta V = 120\,\mathrm{V}$, we can calculate the current:

$$I = \frac{\Delta V}{R} = \frac{\Delta V \pi d^2/4}{\varrho l}$$

Now if we plug that into our first solenoid equation above, we can relate voltage and magnetic field:

$$B = \mu_0 \frac{N}{L} I = \mu_0 \frac{N}{L} \frac{\Delta V \pi d^2/4}{\varrho l} = \frac{\mu_0 \pi}{4\varrho} \frac{N \Delta V d^2}{Ll}$$

Now, what about the small solenoid? Every dimension is a factor of 10 smaller. If *all* the dimensions are 10 times smaller, the number of turns that fit within 1/10 the length is the *same* as the big solenoid if the wire diameter is also 1/10 as large! In other words, both coils will have the same number of turns - the space for the wire is 10 times smaller, but so is the wire.

In order to find the relationship for the small solenoid, we will use the same symbols, but everything for the small solenoid will have a prime $'$. The number of turns in the small solenoid is N', and in for the large solenoid it is just N. The voltage on the little solenoid is $\Delta V'$, and on the large one we have just ΔV. Using the results from above, magnetic field for the small solenoid is then easily found by substitution:

$$B' = \frac{\mu_0 \pi}{4\varrho} \frac{N' \Delta V (d')^2}{L' l'} = B$$

We don't have to bother with a prime on the resistivity, both coils have the same sort of wire. Remember, our desired condition is that $B' = B$. We know that $N' = N$, and all the dimensions are 10 times smaller - the length of the solenoid, the wire diameter, and therefore also the length of wire required. We have the same number of *turns* in each coil, but in the smaller coil the circumference of each turn is 10 times smaller, which means overall, the total length of wire required l is 10 times smaller. Thus:

$$\begin{aligned}
B' &= \frac{\mu_0 \pi}{4\varrho} \frac{N' \Delta V' (d')^2}{L' l'} \\
&= \frac{\mu_0 \pi}{4\varrho} \frac{N \Delta V' (d')^2}{L' l'} && \text{note that } N' = N \\
&= \frac{\mu_0 \pi}{4\varrho} \frac{N \Delta V' (\frac{d}{10})^2}{\frac{L}{10} \frac{l}{10}} && \text{scale all dimensions by } \frac{1}{10} \\
&= \frac{\mu_0 \pi}{4\varrho} \frac{N \Delta V' d^2}{L l}
\end{aligned}$$

Now, we want to enforce the condition that the field is the same in both solenoids:

$$\begin{aligned}
B' &= B \\
\Longrightarrow \quad \frac{\mu_0 \pi}{4\varrho} \frac{N \Delta V' d^2}{L l} &= \frac{\mu_0 \pi}{4\varrho} \frac{N \Delta V d^2}{L l} \\
\Longrightarrow \quad \Delta V' &= \Delta V
\end{aligned}$$

Thus, a solenoid shrunk by 10 times in every dimension will require the same applied voltage for the same magnetic field. What about the power consumption? The current in the large solenoid was

$$I = \frac{\Delta V}{R} = \frac{\Delta V \pi d^2/4}{\varrho l}$$

In the small solenoid, we now know that the voltage is the same, but the resistance is not, so we should have:

$$I' = \frac{\Delta V}{R'} = \frac{\Delta V \pi (d')^2/4}{\varrho l'} = \frac{\Delta V \pi (\frac{d}{10})^2/4}{\varrho \frac{l}{10}} = \frac{1}{10} \frac{\Delta V \pi d^2/4}{\varrho l} = \frac{1}{10} I$$

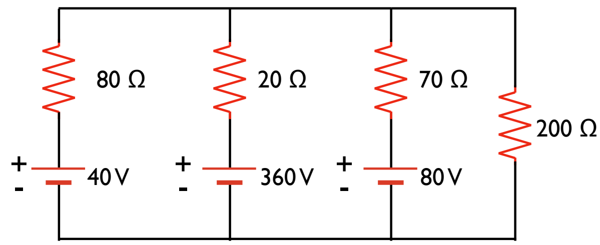
The current in the little solenoid is 10 times less - sensible, since the total length of wire is 10 times smaller, but the area of the wire is 100 times smaller. The power required for each is the product of current and voltage:

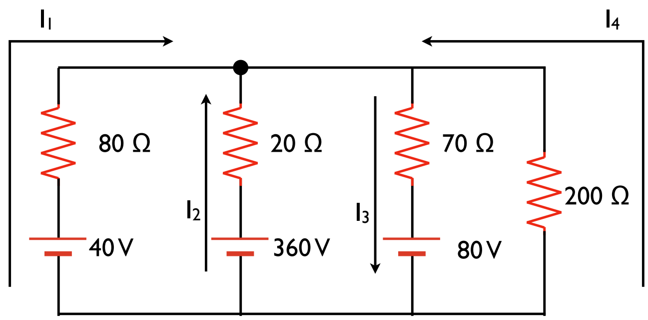
$$\begin{aligned}
\mathscr{P}_{\text{big}} &= I \Delta V \\
\mathscr{P}_{\text{small}} &= I' \Delta V = \frac{1}{10} I \Delta V = \frac{1}{10} \mathscr{P}_{\text{big}}
\end{aligned}$$

Not only is the larger solenoid ten times larger, it requires ten times more power, and therefore dissipates ten times more heat. The cooling requirements will be far more formidable for the larger solenoid. For instance, if we decide to use water cooling, the flow rate will need to be at least 10 times larger for the large solenoid to extract a heat load ten times larger. Not to mention the fact that we have to acquire a much larger power supply in the first place - practically speaking, the difference between a 5 A current source and a 50 A current source is significant. Keep in mind that your normal household outlets deliver 120 V at a maximum of $\sim$15 A.

9. 15 points. In the circuit below, determine the current in each resistor and the voltage across the $200\,\Omega$ resistor.

As usual, we need to define the currents and their directions in every branch of the circuit:





We can pick the current directions arbitrarily - if we get the direction wrong, the current will just come out negative. First, we can apply the "junction rule" at the point indicated by the black dot above: currents I_1, I_2, and I_4 enter that junction, and current I_3 leaves it. Thus:

$$I_1 + I_2 + I_4 = I_3$$

Next, we can apply the "loop rule" to each of the inner loops. First, we'll take the left-most of the three small loops (containing currents I_1 and I_2), and walk around it clockwise, starting from the 40 V battery. Adding up the voltage sources and sinks:

$$40\,\mathrm{V} - 80\,\Omega\, I_1 + 20\,\Omega\, I_2 - 360 = 0$$

Next, the middle loop, containing currents I_2 and I_3, again going clockwise, starting from the 360 V battery:

$$360\,\mathrm{V} - 20\,\Omega\, I_2 - 70\,\Omega\, I_3 - 80\,\mathrm{V} = 0$$

Finally, we have the right-most loop, containing currents I_3 and I_4. Again, go around clockwise, this time starting from the 80 V battery:

$$80\,\mathrm{V} + 70\,\Omega\, I_3 + 200\,\Omega\, I_4 = 0$$

Now we have four equations, and four unknowns:

$$\begin{aligned} I_1 + I_2 - I_3 + I_4 &= 0 \\ -80I_1 + 20I_2 &= 320 \\ 20I_2 + 70I_3 &= 280 \\ 70I_3 + 200I_4 &= -80 \end{aligned}$$

Now, solve the first equation for I_1 and plug it into the second:

$$-80I_1 + 20I_2 = -80\left(I_3 - I_2 - I_4\right) + 20I_2 = 100I_2 - 80I_3 + 80I_4 = 320$$

We have now eliminated I_1, which leaves us with only three equations with three unknowns:

$$\begin{aligned} 100I_2 - 80I_3 + 80I_4 &= 320 \\ 20I_2 + 70I_3 &= 280 \\ 70I_3 + 200I_4 &= -80 \end{aligned}$$

What we want is the current through the $200\,\Omega$ resistor, viz., I_4. The fastest way to get it is to use Cramer's rule[i] (though if you are unfamiliar with this, you can just keep plugging one equation into the other and solve for I_4 that way). First, write the equations in matrix form, using the order I_2, I_3, I_4:

$$\begin{bmatrix} 100 & -80 & 80 \\ 20 & 70 & 0 \\ 0 & 70 & 200 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 320 \\ 280 \\ -80 \end{bmatrix}$$
$$\mathbf{aI} = \mathbf{V}$$

The matrix $\mathbf{a}$ times the column vector $\mathbf{I}$ gives the column vector $\mathbf{V}$, and we can use the determinant of the matrix $\mathbf{a}$ with Cramer's rule to find the currents. For each current, we construct a new matrix, which is the same as the matrix $\mathbf{a}$ except that the the corresponding column is replaced the column vector $\mathbf{V}$. Thus, for I_4, we replace column 3 in $\mathbf{a}$ with $\mathbf{V}$. We find the current then by taking the new matrix, calculating its determinant, and dividing that by the determinant of $\mathbf{a}$. Below, we have highlighted the columns in $\mathbf{a}$ which have been replaced to make this more clear:

$$I_4 = \frac{\begin{vmatrix} 100 & -80 & 320 \\ 20 & 70 & 280 \\ 0 & 70 & -80 \end{vmatrix}}{\det \mathbf{a}}$$

Now we need to calculate the determinant of each new matrix, and divide that by the determinant of $\mathbf{a}$.[ii] First, the determinant of $\mathbf{a}$.

$$\det \mathbf{a} = (100)(70)(200) - (100)(0)(70) + (-80)(0)(0) - (-80)(20)(200) \\ + (80)(20)(70) - (80)(70)(0) = 1.832 \times 10^6$$

We can now find the currents readily from the determinants of the modified matrices above and that of $\mathbf{a}$ we

[i] We didn't really need to eliminate one equation first, we could have just used Cramer's rule to start with. However, finding the determinant of a 3x3 matrix is much easier than finding the determinant of a 4x4 - it is easier in the end to eliminate one equation when you can.

[ii] Again, the Wikipedia entry for 'determinant' is quite instructive.

just found. We really only want I_3, so we can find that directly:

$$
\begin{aligned}
I_4 &= \frac{\begin{vmatrix} 100 & -80 & 320 \\ 20 & 70 & 280 \\ 0 & 70 & -80 \end{vmatrix}}{\det \mathbf{a}} \\
&= \frac{100(70)(-80) - 100(280)(-70) + (-80)(280)(0) - (-80)(20)(-80) + 320(20)(-70) - 320(70)(0)}{1.72 \times 10^6} \\
&= \frac{-2.2 \times 10^6}{1.832 \times 10^6} \\
&= -1.20\,\mathrm{A}
\end{aligned}
$$

We can repeat this for I_2 and I_3 as well:

$$
I_3 = \frac{\begin{vmatrix} 100 & 320 & 80 \\ 20 & 280 & 0 \\ 0 & -80 & 200 \end{vmatrix}}{\det \mathbf{a}} = 2.29\,\mathrm{A}
$$

$$
I_2 = \frac{\begin{vmatrix} 320 & -80 & 80 \\ 280 & 70 & 0 \\ -80 & 70 & 200 \end{vmatrix}}{\det \mathbf{a}} = 5.99\,\mathrm{A}
$$

Finally, I_1 we can get from our original "junction rule" equation:

$$
I_1 = I_3 - I_2 - I_4 = -2.50\,\mathrm{A}
$$

The negative sign just means we picked the directions of I_1 and I_4 incorrectly above - e.g., I_4 should be going down through the $200\,\Omega$ resistor, not up.

Lastly, knowing the current through the $200\,\Omega$ resistor, we can calculate the voltage drop across it readily:

$$
\Delta V_{200} = I_4 R = (1.2\,\mathrm{A})(200\,\Omega) = 240\,\mathrm{V}
$$