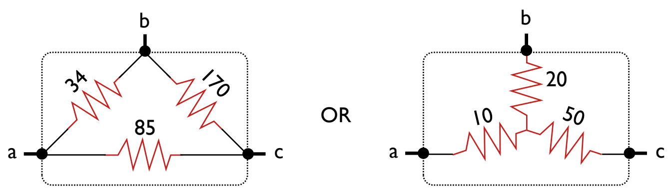
UNIVERSITY OF ALABAMA

Department of Physics and Astronomy

PH 102 / LeClair Summer II 2009

Problem Set 5: Solutions

1. A black box with three terminals, a, b, and c, contains nothing but three resistors and connecting wire. Measuring the resistance between pairs of terminals, you measure $R_{ab} = 30\,\Omega$, $R_{ac} = 60\,\Omega$, and $R_{bc} = 70\,\Omega$. Show that the box could be either of those below.



First, consider the box on the left side. Measuring between points a b (with point c unconnected), we would find a $34\,\Omega$ resistor in parallel with a series combination of $85\,\Omega$ and $170\,\Omega$. The series combination of $85\,\Omega$ and $170\,\Omega$ just gives $255\,\Omega$, and that in parallel with $34\,\Omega$ gives

$$R_{ab} = \frac{(34\,\Omega)\,(255\,\Omega)}{34\,\Omega + 255\,\Omega} = 30\,\Omega$$

Similarly, we can find $R_{bc} = 70\,\Omega$ and $R_{ac} = 60\,\Omega$.

For the box on the right, if we connect only points a and b then the $50\,\Omega$ resistor does nothing - it has one end disconnected. Thus, $R_{ab} = 30\,\Omega$, and similarly $R_{bc} = 70\,\Omega$, $R_{ac} = 60\,\Omega$. Since a measurement of the resistance between any two terminals yields the same result, the two boxes are indistinguishable.

Establishing the equivalence of these two configurations is more generally known as a "Y-Δ" transformation: `http://en.wikipedia.org/wiki/Y-%CE%94_transform`

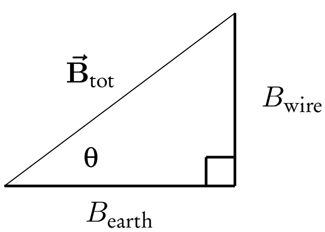
2. In a motorboat, the compass is mounted at a distance of $0.80\,\text{m}$ from a cable carrying a current of $20\,\text{A}$ from an electric generator to a battery. **(a)** What magnetic field does this current produce at the location of the compass? Assume the cable is a long, straight wire. **(b)** The horizontal (north) component of the Earth's magnetic field is $1.8 \times 10^{-5}\,\text{T}$. Since the compass points in the direction of the *net* horizontal magnetic field, the current will cause a deviation of the compass needle. Assume that the magnetic field of the current is horizontal and at a right angle to the horizontal component of the earth's magnetic field. Under these circumstances, by how many degrees will the compass deviate from true north?

At a distance of $d = 0.8\,\text{m}$ from the wire, the field must be:

$$
\begin{aligned}
B &= \frac{\mu_0 I}{2\pi d} \\
&= \frac{4\pi \times 10^{-7} \cdot 20}{2\pi \cdot 0.8} \\
&\approx 5\,\mu\mathrm{T}
\end{aligned}
$$

(b) The horizontal (north) component of the Earth's magnetic field is 1.8×10^{-5} T. Since the compass points in the direction of the *net* horizontal magnetic field, the current will cause a deviation of the compass needle. Assume that the magnetic field of the current is horizontal and at a right angle to the horizontal component of the earth's magnetic field. Under these circumstances, by how many degrees will the compass deviate from true north?

Without the field of the wire, the earth's magnetic field defines true north. Adding the wire's magnetic field to that, the compass will point along the resulting magnetic vector. Thus, we need to find the direction of the total magnetic field relative to the earth's field:



$$
\begin{aligned}
B_{\text{wire}} &= 5\,\mu\mathrm{T} \\
B_{\text{earth}} &= 18\,\mu\mathrm{T} \\
\tan\theta &= B_{\text{wire}}/B_{\text{earth}} \approx 0.278 \\
\theta &\approx 15.5^\circ
\end{aligned}
$$

3. Suppose you move along a current-carrying wire at the same speed $v_d \ll c$ as the drift speed of electrons in the wire. Do you now measure a magnetic field of zero?

No, because now the positive ions appear to move backwards, creating the same current.

4. A wire having a mass per unit length of 0.50 g/cm carries a 2.0 A current horizontally to the right. What are the direction and magnitude of the minimum magnetic field needed to lift this wire vertically upward?

Let the mass per unit be λ. This means that a segment of the wire of length l has mass $m = \lambda l$, resulting in a gravitational force on that segment of wire of $F_g = mg = \lambda l g$. We need the magnetic force on the wire F_b to balance this force precisely levitate the wire, at minimum.

If the current runs horizontally to the right, and the gravitational force acts in the downward direction, we need the magnetic force to act upwards. According to the right-hand rule, this requires a magnetic field pointing *into the page*. We can find the minimum magnitude of magnetic field required by equating the magnetic and gravitational forces, noting that the current is by construction perpendicular to the magnetic field:

$$
\begin{aligned}
F_b &= F_b \\
BIl\sin\theta = BIl = mg &= \lambda l g \\
\implies B &= \frac{\lambda g}{I}
\end{aligned}
$$

With the numbers given (after converting to proper units),

$$
B = \frac{\lambda g}{I} = \frac{[0.50\,\mathrm{g/cm}]\,[1\,\mathrm{kg}/1000\,\mathrm{g}]\,[100\,\mathrm{cm/m}]\,\left[9.81\,\mathrm{m/s^2}\right]}{2.0\,\mathrm{A}} \approx 0.245\,\mathrm{T}
$$

Here we had to use the handy fact that $1\,\mathrm{T} = 1\,\mathrm{kg/s^2A}$.