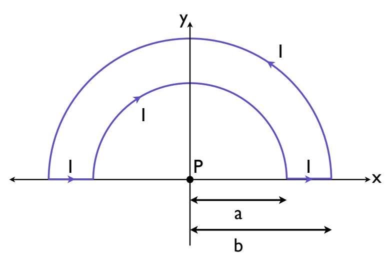
UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 102 / LeClair Summer II 2009

Problem Set 6: Solutions

1. Find the magnetic field at point P due to the current distribution shown below. *Hint: Break the loop into segments, and use superposition.*



Problem 3: A current loop

The easiest way to do solve this is by superposition – our odd current loop is just the same as two semicircles plus two small straight segments. We know that the magnetic field at the center of a *full* circular loop of radius r carrying a current I is

$$B = \frac{\mu_o I}{2r} \qquad \text{(loop radius r)}$$

Since the magnetic field obeys superposition, we could just as well say that our full circle is built out of two equivalent half circles! The field from each half circle, by symmetry, must be half of the total field, so the field at the center of a semicircle must simply be

$$B = \frac{\mu_o I}{4r} \qquad \text{(semicircle, radius r)}$$

In other words: a half circle gives you half the field of a full circle. Here we have two semicircular current segments contributing to the magnetic field at P: one of radius b, and one of radius a. The currents are in the opposite directions for the two loops, so their fields are in opposing directions. Based on the axes given, it is the outer loop of radius b that has its field pointing out of the page in the $\hat{\mathbf{z}}$ direction, and the inner loop of radius a in the $-\hat{\mathbf{z}}$ direction.

What about the straight bits of wire? For those segments, the direction field is zero. Since the magnetic field "circulates" around the wire, along the wire axis it must be zero. Even if it were not, by symmetry the two straight bits would have to give equal and opposite contributions and cancel each other anyway. There is no field contribution at P from the straight segments! Thus, the total field is just that due to the semicircular bits,

$$\vec{\mathbf{B}} = \frac{\mu_o I}{4b}\,\hat{\mathbf{z}} - \frac{\mu_o I}{4a}\,\hat{\mathbf{z}} = \frac{\mu_o I}{4}\left(\frac{1}{b} - \frac{1}{a}\right)\hat{\mathbf{z}}$$

2. You want to confine an electron of kinetic energy 3.0×10^4 eV by making it circle inside a solenoid of radius 0.1 m under the influence of the force exerted by the magnetic field. The solenoid has 12000 turns of wire per meter. What minimum current must you put through the wire if the electron is not to hit the wall of the solenoid?

If we have a charged particle (charge e) moving with velocity v perpendicular to a magnetic of magnitude B, we know the particle will undergo circular motion with a radius

$$r = \frac{mv}{qB}$$

We want this radius to be equal to or smaller than the radius of the solenoid $R = 0.1$ m, such that the circular orbit fits inside the solenoid. The kinetic energy given tells us the velocity of the electron:

$$K = \frac{1}{2}mv^2$$
$$v = \sqrt{\frac{2K}{m}}$$

Thus,

$$r = \frac{m}{eB}\sqrt{\frac{2K}{m}} \leq R$$

The larger the B field, the smaller the radius. For a solenoid, we know that the B field produced is proportional to the current I in the solenoid, $B = \mu_o n I$, where n is the number of turns per unit length (given). Substituting in the equation above and solving for I,

$$R \leq \frac{m}{eB}\sqrt{\frac{2K}{m}} = r$$
$$R \leq \frac{m}{e\mu_o n I}\sqrt{\frac{2K}{m}}$$
$$I \geq \frac{\sqrt{2Km}}{e\mu_o n R} \approx 0.39\,\text{A}$$

To get the numbers to come out, we have to remember to convert our energy units ($1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$) …

3. Consider an electron orbiting a proton and maintained in a fixed circular path of radius $R = 5.29 \times 10^{-11}$ m by the Coulomb force. Treating the orbiting charge as a current loop, calculate the resulting torque when the system is in a magnetic field of 0.400 T directed perpendicular to the magnetic moment of the electron.

First, need to know the current that corresponds to one orbiting electron. From the current I, magnetic field B, and the orbital radius R we can find the torque. An electron in a circular orbit of radius R has a period of $T = 2\pi R/v$, where v it he electron's velocity. If a single electron charge $-e$ orbits once every T seconds, then the current is by definition

$$I = \frac{\Delta q}{\Delta t} = \frac{-e}{T} = \frac{-ev}{2\pi R}$$

We can find the velocity from the condition for circular motion. The only force present (that we know of) is the electric force, which must then provide the centripetal force on the electron. The electric force is just that of two point charges e and $-e$ separated by a distance R.

$$\begin{aligned} F_{\text{centr}} &= F_E \\ \frac{-m_e v^2}{r} &= \frac{-k_e e^2}{R^2} \\ \implies \qquad v &= \sqrt{\frac{k_e e^2}{m_e R}} \end{aligned}$$

We can now substitute this in our expression for current above:

$$I = \frac{-ev}{2\pi R} = \frac{-e}{2\pi R}\sqrt{\frac{k_e e^2}{m_e R}} = \frac{-e^2}{2\pi}\sqrt{\frac{k_e}{m_e R^3}}$$

Finally, since the magnetic field is perpendicular to the electron's magnetic moment, the magnitude of the torque is given by $\tau = IAB$ where A is the area of the "current loop" formed by the orbiting electron, $A = \pi R^2$. Thus,

$$\tau = IAB = \frac{-e^2}{2\pi}\sqrt{\frac{k_e}{m_e R^3}}\pi R^2 B = -\frac{1}{2}e^2 B\sqrt{\frac{k_e R}{m_e}} \approx -3.7 \times 10^{-24}\,\text{N}\cdot\text{m}$$

The negative sign reminds us that current is the direction that *positive* charge flows, and thus the direction of the torque is given by the right hand rule consistent with the current, which is *opposite* the direction that the electron orbits.

4. The electric field of a long, straight line of charge with λ coulombs per meter is

$$E = \frac{2k_e\lambda}{r}$$

where r is the distance from the wire. Suppose we move this line of charge parallel to itself at speed v. **(a)** The moving line of charge constitutes an electric current. What is the magnitude of this current? **(b)** What is the magnitude of the magnetic field produced by this current? **(c)** Show that the magnitude of the magnetic field is proportional to the magnitude of the electric field, and find the constant of proportionality.

The current can be found by thinking about how much charge passes through a given region of space per unit time. If we were standing next to the wire, in a time Δt, the length of wire that passes by us would be $v\Delta t$. The corresponding charge is then $\Delta q = \lambda v \Delta t$, and thus the current is

$$I = \frac{\Delta q}{\Delta t} = \frac{\lambda v \Delta t}{\Delta t} = \lambda v$$

From the current, we can easily find the magnetic field a distance r from the wire.

$$B = \frac{\mu_o I}{2\pi r} = \frac{\mu_o \lambda v}{2\pi r}$$

If the wire were sitting still (or we were traveling parallel to it at the same velocity v), it would produce the electric field given above. Rearranging the given expression, we can relate λ and E, $\lambda = Er/2k_e$. Substituting this in our expression for the magnetic field,

$$B = \frac{\mu_o \lambda v}{2\pi r} = \frac{\mu_o E r v}{4\pi k_e r} = \mu_o \epsilon_o v E = \frac{v}{c^2} E$$

For the last step, we noted that $\epsilon_o = 1/4\pi k_e$ and $c^2 = 1/\epsilon_o \mu_o$.