UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 102 / LeClair Summer II 2010

Problem Set 2: Solutions

1. The end of a charged rubber rod will attract small pellets of Styrofoam that, having made contact with the rod, will move violently away from it. Describe why that happens.

Solution: The electric field from the charged rubber rod will orient the charges in the overall electrically neutral Styrofoam pellets such that the opposite charges attracted closer to the rod, and the like charges farther away. This will cause the Styrofoam pellets to jump toward the rubber rod. Upon contact, charge from the rubber rod will neutralize opposing charges on the Styrofoam pellets, leaving the pellets with a net charge of the same sign as the rod. This will immediately cause a repulsive interaction, which causes the pellets to move violently away.

Say the rubber rod is charged negatively. This will cause the positive charges in the pellets to orient themselves closer to the rod, and the negative charges in the pellets to orient themselves farther away. This causes the pellets to be attracted to the rod. Once in contact, negative charge from the rubber rod will (through contact) move on to the pellets, neutralizing some of the positive charge. After neutralization, this leaves a net surplus of negative charge on the pellets, which will then be quickly repelled from the negatively charged rod.

2. A charge of q and a charge of $5q$ sit a distance d away. Where could a third charge of magnitude $2q$ sit between them and experience no net force?

Solution: Without calculating anything, we can first figure where the third charge should be relative to the first two. Let us put the $5q$ charge at the origin, with the q charge a distance d away along the $+x$ axis (i.e., the $5q$ on the left, the q on the right). To the left of the $5q$ charge, the electric forces on the $2q$ charge from both the $5q$ and q charges would be repulsive and toward $-x$ (left). There is no way that we can have two forces in the same direction cancel, so the $2q$ charge cannot be to the left of the origin.

To the right of the q charge (so, $x > d$), the force from both charges on the $2q$ charge would be repulsive to the right, and again there is no way we can have a net zero force. However, in between the two charges (so $0 < x < d$), the repulsive forces are in opposite directions from the $5q$ and q charges, and cancellation is possible. Further, since the electric force depends on the magnitude of the charges involved, we know we will have to place the $2q$ charge closer to the q charge than the $5q$.

Let the $2q$ charge sit at a distance x from the origin, such that $0 < x < d$. From the arguments above, we know that we must find $x > d/2$ so that the $2q$ charge is closest to the q charge. The distance from the $2q$

to the $5q$ charge is now just x, while the distance from the $2q$ to the q charge is $d-x$. The total force on the $2q$ charge is then just the sum of the individual forces from the $5q$ and q charges, taking into account that they point in opposite directions – the force from the $2q$ charge should be negative, since it points toward $-x$. The total force balance must be zero, hence

$$F_{tot} = \frac{k_e\,(5q)\,(2q)}{x^2} - \frac{k_e\,(2q)\,(q)}{(d-x)^2} = 0 \tag{1}$$

Rearranging and canceling, we solve for x, the distance between the $2q$ and $5q$ charges.

$$\frac{k_e\,(5q)\,(2q)}{x^2} = \frac{k_e\,(2q)\,(q)}{(d-x)^2} \tag{2}$$

$$\frac{5}{x^2} = \frac{1}{(d-x)^2} \tag{3}$$

$$\frac{\sqrt{5}}{x} = \frac{1}{d-x} \tag{4}$$

$$\sqrt{5}\,(d-x) = x \tag{5}$$

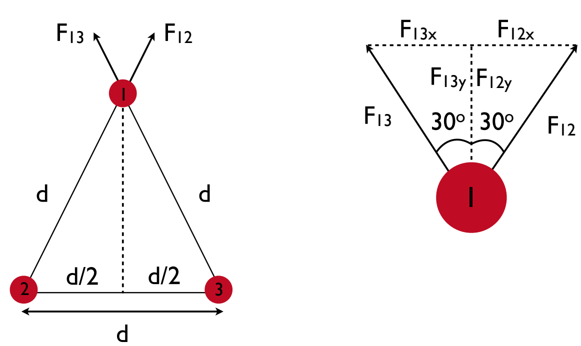
$$\left(1+\sqrt{5}\right)x = d \tag{6}$$

$$x = \frac{d}{1+\sqrt{5}} \tag{7}$$

Thus, the $2q$ must be a distance $d/\left(1+\sqrt{5}\right)$ from the $5q$ charge to feel no net force. You can verify that the forces are the same by plugging our result for x back into the expression for F_{tot} above.

3. Three charges q sit at the vertices of an equilateral triangle whose sides are length d. What is the net force on each charge? Roughly sketch the electric field lines around the set of charges.

Solution: Here is the situation:



Since the structure is symmetric, the force on each of the three charges will be the same – each of the three charges is in the same situation as the others. From the geometry, we can see that any given charge will have two repulsive forces on it from the other two charges, and these forces will be directed at a $60°$ angle with respect to each other.

Considering charge 1 above, the forces from charges 2 and 3 will be equal in magnitude. Further, from the symmetry of the problem, their x components (horizontal) will cancel, and the net force will be only in the vertical direction. Charges 2 and 3 are each a distance d from charge 1, so we can readily calculate the magnitude of either force if all three charges have magnitude q:

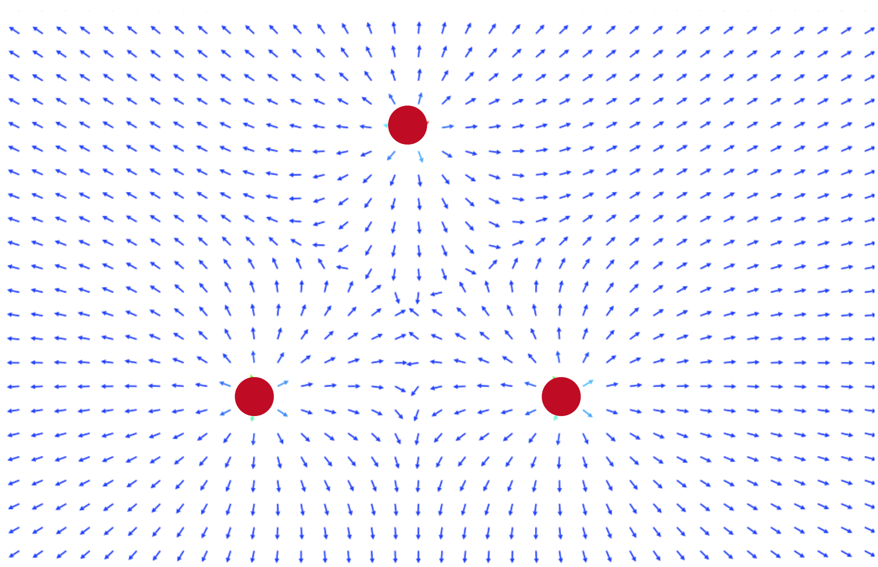
$$F_{12} = F_{13} = \frac{k_e q^2}{d^2} \tag{8}$$

The net force along the y direction is simply adding the y components of the two forces:

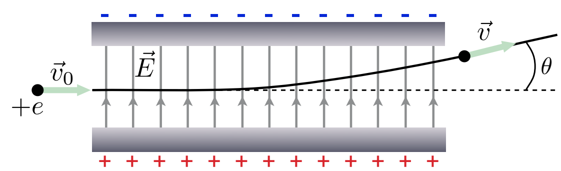
$$F_{y,net} = F_{12,y} + F_{13,y} = F_{12}\cos 30 + F_{13}\cos 30 = 2F_{12}\cos 30 = 2\frac{k_e q^2}{d^2}\frac{\sqrt{3}}{2} = \frac{\sqrt{3}k_e q^2}{d^2} \tag{9}$$

And, that is that.

Below is a quick calculation of the field, using tiny arrows to represent the electric force a positive charge would feel at various points in space on a square grid (i.e., an arrow for the electric field at every point in space). These are not the field lines exactly – that would be connecting the arrows with smooth curves flowing out from the charges, and encoding the field strength in the density of lines.



4. An ion milling machine uses a beam of gallium ions ($m = 70\,\mathrm{u}$) to carve microstructures from a target. A region of uniform electric field between parallel sheets of charge is used for precise control of the beam direction. Single ionized gallium atoms with initially horizontal velocity of $1.8\times10^{4}\,\mathrm{m/s}$ enter a $2.0\,\mathrm{cm}$-long region of uniform electric field which points vertically upward, as shown below. The ions are redirected by the field, and exit the region at the angle θ shown. If the field is set to a value of $E=90\,\mathrm{N/C}$, what is the exit angle θ?



Solution: A singly-ionized gallium atom has a charge of $q = +e$, and the mass of $m = 70\,\mathrm{u}$ means 70 *atomic mass units*, where one atomic mass unit is $1\,\mathrm{u}=1.66\times10^{-27}\,\mathrm{kg}$.

What we really have here is a particle under the influence of a constant force, just as if we were to throw a ball horizontally and watch its trajectory under the influence of gravity (the only difference is that since we have negative charges, things can "fall up"). To start with, we will place the origin at the ion's initial position, let the positive x axi run to the right, and let the positive y axis run straight up. Thus, the particle starts with a velocity purely in the x direction: $\vec{\mathbf{v}}_0=v_x\,\hat{\mathbf{x}}$.

While the particle is in the electric-field-containing region, it will experience a force pointing along the $+y$ direction, with a constant magnitude of qE. Since the force acts only in the y direction, there will be a net acceleration only in the y direction, and *the velocity in the x direction will remain constant.* Once outside the region, the particle will experience no net force, and it will therefore continue along in a straight line. It will have acquired a y component to its velocity due to the electric force, but the x component will still be v_x. Thus, the particle exits the region with velocity $\vec{\mathbf{v}}=v_x\,\hat{\mathbf{x}}+v_y\,\hat{\mathbf{y}}$. The angle at which the particle exits the plates, measured with respect to the x axis, must be

$$\tan\theta = \frac{v_y}{v_x}$$

Thus, just like in any mechanics problem, finding the angle is reduced to a problem of finding the final velocity components, of which we already know one. So, how do we find the final velocity in the y direction? Initially, there is no velocity in the y direction, and while the particle is traveling between the plates, there is a net force of qE in the y direction. Thus, the particle experiences an acceleration

$$a_y = \frac{F_y}{m} = \frac{qE_y}{m}$$

The electric field is purely in the y direction in this case, so $E_y = 90\,\mathrm{N/C}$. Now we know the acceleration in the y direction, so if we can find out the time the particle takes to transit the plates, we are done, since the the transit time Δt and acceleration a_y determine v_y:

$$v_y = a_y \Delta t$$

Since the x component of the velocity is not changing, we can find the transit time by noting that the distance covered in the x direction must be the x component of the velocity times the transit time. The distance covered in the x direction is just the width of the plates, so:

$$d_x = v_x \Delta t = 2.0\,\mathrm{cm} \quad \Longrightarrow \quad \Delta t = \frac{d_x}{v_x}$$

Putting the previous equations together, we can express v_y in terms of known quantities:

$$v_y = a_y \Delta t = a_y \frac{d_x}{v_x} = \frac{qE_y}{m}\frac{d_x}{v_x} = \frac{qE_y d_x}{m v_x}$$

Finally, we can now find the angle θ as well:

$$\tan\theta = \frac{v_y}{v_x} = \frac{\frac{qE_y d_x}{m v_x}}{v_x} = \frac{qE_y d_x}{m v_x^2}$$

And that's that. Now we plug in the numbers we have, watching the units carefully:

$$\begin{aligned}
\theta = \tan^{-1}\left[\frac{qE_y d_x}{m v_x^2}\right] &= \tan^{-1}\left[\frac{\left(1.6\times10^{-19}\,\mathrm{C}\right)(90\,\mathrm{N/C})\,(0.02\,\mathrm{m})}{\left(70\cdot1.66\times10^{-27}\,\mathrm{kg}\right)\left(1.8\times10^{4}\,\mathrm{m/s}\right)^2}\right] \\
&= \tan^{-1}\left[7.6\times10^{-3}\,\frac{\mathrm{N}}{\mathrm{kg\cdot m/s^2}}\right] \qquad \text{note } 1\,\mathrm{N} = 1\,\mathrm{kg\cdot m/s^2} \\
&= \tan^{-1} 7.6\times10^{-3} \approx 0.44^\circ
\end{aligned}$$

5. A charge of $1.8\,\mathrm{nC}$ sits at the center of a cube. What is the electric flux out of one face? Over the whole surface? Would your answer change if the charge is not at the center of the cube?

Solution: It is easier to answer the second question first. If the cube encloses the point charge, then the flux through the entire cube must be found in accordance with Gauss' law:[i]

$$\Phi_{E,\text{tot}} = 4\pi k_e q = 4\pi \left(9 \times 10^9 \left[\frac{\text{N m}^2}{\text{C}^2}\right]\right) \left(1.8 \times 10^{-9} \, [\text{C}]\right) = 204 \left[\frac{\text{N m}^2}{\text{C}}\right] \tag{10}$$

Back to the first question. If the charge sits *symmetrically* at the center of the cube, then the flux must be the same over every face of the cube. Thus, each side takes $1/6$ of the total flux, since there are six sides:

$$\Phi_{E,\text{side}} = \frac{1}{6}\Phi_{E,\text{tot}} = 33.9 \left[\frac{\text{N m}^2}{\text{C}}\right] \tag{11}$$

If the charge were not in the center of the cube, the flux through the *whole* cube would be the same, provided that the charge were still fully enclosed by the cube. If the charge were partially penetrated by one side of cube (say, half in and half out) then this would not be the case, and we would need to figure out how much of the charge was inside the cube and how much was outside. In either case, we could not say anything very clever about the flux through any given side – figuring that out relied crucially on the charge being symmetrically placed with respect to the six sides of the cube, so we could say they all had equal flux. If the charge were off-center, closer to one side than the others, this is no longer true, and our answer to the first part of the question no longer holds.

[i]Don't forget that "n" means 10^{-9}.