UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 102 / LeClair Summer II 2010

Problem Set 3: Solutions

1. A charge $5q$ and a charge $-q$ are separated by a distance d. Find all points along a line connecting the two charges where the electric *potential* is zero.

Solution: The $5q$ charge will give a positive potential everywhere in space, and the $-q$ charge a positive potential everywhere in space. In order to have net zero potential at any point, we will clearly have to be closer to the $-q$ charge than the $5q$ charge, since its potential is 5 times smaller in magnitude at the same distance.

Let the $5q$ charge be at the origin, with the $-q$ charge a distance d along the $+x$ axis. It is clear that the potential will be zero at some distance $x > d/2$, but we are not sure if it will be between the two charges or to the right of the $-q$ charge. No matter, let us say that our point of interest is at x, and if $x > d$ we know that the potential is zero to the right of the $-q$ charge, and if $x < d$ it is between the two charges. All we need to do now is write down the potentials from both charges at x and add them together. The $5q$ charge will be a distance x from the point of interest, and the $-q$ charge a distance $d-x$. Thus,

$$V_{tot} = \frac{k_e(5q)}{x} + \frac{k_e(-q)}{d-x} = \frac{5k_eq}{x} - \frac{k_eq}{d-x} = 0 \tag{1}$$

$$\frac{5k_eq}{x} = \frac{k_eq}{d-x} \tag{2}$$

$$\frac{5}{x} = \frac{1}{d-x} \tag{3}$$

$$5(d-x) = 5d - 5x = x \tag{4}$$

$$6x = 5d \qquad \Longrightarrow \qquad x = \frac{5}{6}d \tag{5}$$

Thus, the point where the potential vanishes is between the two charges, a distance $d/6$ from the $-q$ charge and $5d/6$ from the $5q$ charge.

2. A sphere the size of the earth has 1 C of charge distributed evenly over its surface. What is the electric field strength just outside the surface, in volts per meter? What is the potential of the sphere, in volts, if the zero potential is at an infinite distance from the sphere?

Solution: If the charge is distributed evenly over a sphere, then Gauss' law tells us that for points outside the sphere the electric field is exactly that of a point charge. Thus, the problem is equivalent to finding the electric field and potential from a $q = 1$ C charge a distance corresponding to the radius of the earth, $R_e = 6370$ km.

$$E = \frac{k_e q}{R_e^2} = \frac{\left(9\times10^9\,\mathrm{N\cdot m^2/C^2}\right)(1\,\mathrm{C})}{\left(6.370\times10^6\,\mathrm{m}\right)^2} \approx 2.2\times10^{-4}\,\frac{\mathrm{N}}{\mathrm{C}} \tag{6}$$

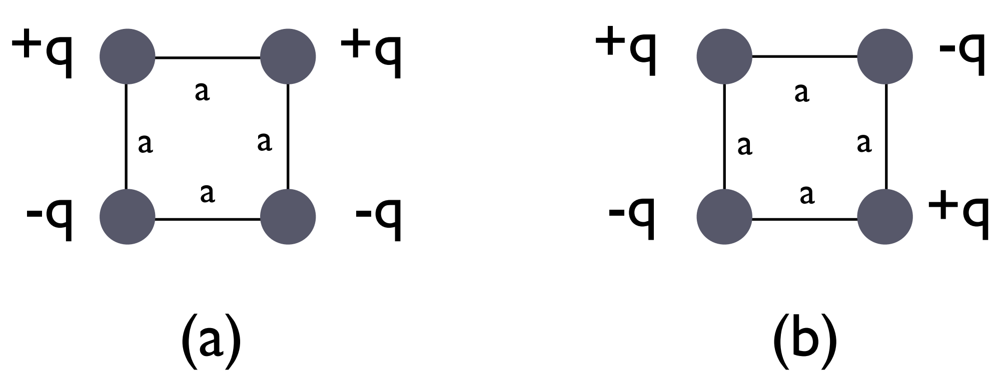
$$V = \frac{k_e q}{R_e} = \frac{\left(9\times10^9\,\mathrm{N\cdot m^2/C^2}\right)(1\,\mathrm{C})}{6.370\times10^6\,\mathrm{m}} \approx 1500\,\frac{\mathrm{N\cdot m}}{\mathrm{C}} = 1500\,\frac{\mathrm{J}}{\mathrm{C}} = 1500\,\mathrm{V} \tag{7}$$

3. What is wrong with the idea of a gravity screen, something that will "block" gravity the way a conducting sheet seems to "block" the electric field? Think about the difference between the gravitational source and electrical sources. Note that a closed conducting container does not block the field of outside sources, but merely allows the surface charges to set up a compensating field. Why can't something of this sort be contrived for gravity? What would you need to accomplish it?

Solution: A conductor does not "block" electric fields. Rather, because its charges are extremely mobile, the electric field causes positive and negative charges to align on the surface of the conductor in such a way to set up its own internal electric field opposing the external field. Inside the conductor, the field is zero because the surface charge distribution resulting from the external field exactly compensates the external electric field. For example: you can make a conducting spherical shell which has $E=0$ everywhere inside, *independent* of what is going on outside.

With gravity, this is not possible. There are no "negative charges" in gravity (i.e., no "negative mass"), so no such compensating field can be set up. You can't make a spherical shell of material that results in no gravitational force inside, any mass inside the shell would still feel the gravitational influence of the matter outside. The fact that there is only regular matter, and no equivalent of negative charge, for the gravitational force prohibits the construction of a gravity screen.

4. Two positive and two negative charges are arranged on a square lattice of side a in two different ways, shown below. Calculate the potential energy of each configuration. Which configuration of charges is more stable? Why?



Solution: Using the principle of superposition, we know that the potential energy of a system of charges is just the sum of the potential energies for all the unique pairs of charges. The problem is then reduced to figuring out how many different possible pairings of charges there are, and what the energy of each pairing is. The potential energy for a single pair of charges, both of magnitude q, separated by a distance d is just:

$$PE_{\text{pair}} = \frac{k_e q^2}{a}$$

We need figure out how many pairs there are, and for each pair, how far apart the charges are. Once we've done that, we need to figure out the two different arrangements of charges and run the numbers.

In this case, there are not many possibilities. Label the upper left charge in each diagram "1" and number the rest clockwise. The possible pairings are then only

$$q_1q_2,\, q_1q_3,\, q_1q_4$$
$$q_2q_3,\, q_2q_4$$
$$q_3q_4$$

Since there are the same number of possibilities for either crystal, the total potential energy in either case is just adding all of these pairs' contributions together. Except for pairs q_2q_4 and q_1q_3, which are separated by a distance $a\sqrt{2}$, all others are separated by a distance a. Thus,

$$PE = \frac{k_e q_1 q_2}{a} + \frac{k_e q_1 q_3}{a\sqrt{2}} + \frac{k_e q_1 q_4}{a} + \frac{k_e q_2 q_3}{a} + \frac{k_e q_2 q_4}{a\sqrt{2}} + \frac{k_e q_3 q_4}{a} \tag{8}$$

First, consider configuration (a). All we need to do now is plug in $+q$ for q_1 and q_2, and $-q$ for q_3 and q_4:

$$PE_a = \frac{k_e q^2}{a} + \frac{k_e\left(-q^2\right)}{a\sqrt{2}} + \frac{k_e\left(-q^2\right)}{a} + \frac{k_e\left(-q^2\right)}{a} + \frac{k_e\left(-q^2\right)}{a\sqrt{2}} + \frac{k_e q^2}{a} \tag{9}$$
$$= \frac{k_e q^2}{a}\left(-\frac{2}{\sqrt{2}}\right) = -\sqrt{2}\frac{k_e q^2}{a} \approx -1.414\frac{k_e q^2}{a} \tag{10}$$

For configuration (b), we need $+q$ for q_1 and q_3, and $-q$ for q_2 and q_4:

$$PE = \frac{k_e\left(-q^2\right)}{a} + \frac{k_e q^2}{a\sqrt{2}} + \frac{k_e\left(-q^2\right)}{a} + \frac{k_e\left(-q^2\right)}{a} + \frac{k_e q^2}{a\sqrt{2}} + \frac{k_e\left(-q^2\right)}{a} \tag{11}$$
$$= \frac{k_e q^2}{a}\left(-4 + \frac{2}{\sqrt{2}}\right) = \frac{k_e q^2}{a}\left(-4 + \sqrt{2}\right) \approx -2.586\frac{k_e q^2}{a} \tag{12}$$

Configuration (b) has a lower potential energy, and is therefore more stable. Qualitatively, this makes sense: configuration (b) keeps the like charges as far away as possible, which also maximizes the number of favorable opposite pairings at close distance.

5. An electron is accelerated from rest through a $5000\,\mathrm{V}$ potential difference. What velocity does the electron attain?

Solution: If no other forces are acting, we can apply conservation of energy: the electrical potential energy of the electron is converted into kinetic energy. (We also know that the potential difference must be negative, since electrons will want to go from low potential to high potential.)

$$
\begin{aligned}
q\Delta V &= -e\Delta V = \frac{1}{2}mv^2 \\
v &= \sqrt{\frac{-2e\Delta V}{m}} = \sqrt{\frac{-2\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(-5000\,\mathrm{V}\right)}{9.11\times10^{-31}\,\mathrm{kg}}} \approx 4.19\times10^{7}\,\mathrm{m/s} \approx 0.14c
\end{aligned}
$$

Note that the electron's velocity is a significant fraction of c, meaning that we are close to requiring relativity. Luckily, at this speed $\gamma \approx 1.009$, so our error in ignoring relativity is less than 1%.