UNIVERSITY OF ALABAMA
Department of Physics and Astronomy

PH 102 / LeClair   Summer II 2010

Problem Set 6: Induction

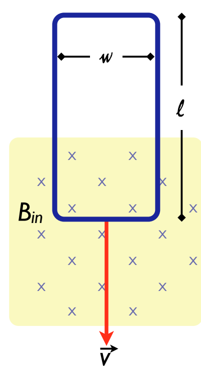
Instructions:

1. Answer all questions below. Show your work for full credit.
2. All problems are due Tues 27 July 2010 by the end of the day.
3. You may collaborate, but everyone must turn in their own work.

1. A conducting rectangular loop of mass M, resistance R, and dimensions w by l falls from rest into a magnetic field $\vec{\mathbf{B}}$, as shown at right. At some point before the top edge of the loop reaches the magnetic field, the loop attains a constant terminal velocity v_T. Show that the terminal velocity is:

$$v_T = \frac{MgR}{B^2 w^2}$$

NB – terminal velocity is reached when the net acceleration is zero. See the schematic figure below.



2. An ocean current flows at a speed of $1.4\,\mathrm{m/s}$ in a region where the vertical component of the earth's magnetic field is $3.5 \times 10^{-5}\,\mathrm{T}$. The resistivity of seawater in that region is about $\rho = 0.25\,\Omega\,\mathrm{m}$. On the assumption that there is no other horizontal component of $\vec{\mathbf{E}}$ other than the motional term $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$, find the horizontal current density J in $\mathrm{A/m^2}$? *NB – recall the general version of Ohm's law, viz.* $E = \rho J$. If you carried a bottle of seawater through the earth's field at this speed, would such a current be flowing in it?

3. Consider an MRI (magnetic resonance imaging) magnet that produces a magnetic field $B = 1.5\,\mathrm{T}$ at a current of $I = 140\,\mathrm{A}$. Assume the magnet is a solenoid with a radius of $0.30\,\mathrm{m}$ and a length of $2.0\,\mathrm{m}$. **(a)** What is the number of turns of the solenoid? **(b)** What is its inductance? **(c)** How much energy is stored in this magnet? **(d)** If all the energy in part (b) were converted to the kinetic energy of a car ($m = 1000\,\mathrm{kg}$), what would the speed of the car be?

4. Very large magnetic fields can be produced using a procedure called *flux compression.* A metallic cylindrical tube of radius R is placed coaxially in a long solenoid of somewhat larger radius. The space between the tube and the solenoid is filled with a highly explosive material. When the explosive is set off, it collapses the tube to a cylinder of radius $r < R$. If the collapse happens very rapidly, induced current in the tube maintains the magnetic flux nearly constant inside the tube, even though the area shrinks. If the initial magnetic field in the solenoid is $2.50\,\mathrm{T}$, and $R/r = 12.0$, what is the maximum field that can be reached?

5. In a mass spectrometer, a beam of ions is first made to pass through a *velocity selector* with perpendicular $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ fields. Here, the electric field $\vec{\mathbf{E}}$ is to the right, between parallel charged plates, and the magnetic field $\vec{\mathbf{B}}$ in the same region is into the page. The selected ions are then made to enter a region of different magnetic field $\vec{\mathbf{B}}'$, where they move in arcs of circles. The radii of these circles depend on the masses of the ions. Assume that each ion has a single charge e. Show that in terms of the given field values and the impact distance l the mass of the ion is

$$m = \frac{eBB'l}{2E}$$