PH102 Lab: Mutual Inductance

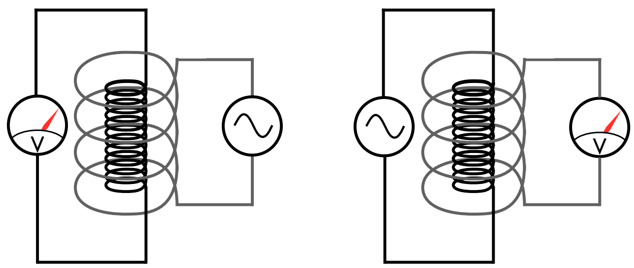
Group member's names:

For this lab, you will need:

- 1 - function generator
- 1 - hand-held multimeter
- 1 - pair of coaxial solenoids
- 4 - banana cables
- 1 - male BNC to banana plug adapter
- 1 - ruler or calipers

Introduction

Consider two coaxial solenoids, one inside the other, a situation we discussed recently in lecture:



In the arrangement shown at left, the outer coil is powered with an *ac* (time-varying) voltage. The resulting time-varying current creates a time varying magnetic field, which penetrates the inner coil uniformly. Thus, there is a time-varying magnetic flux in the inner coil, which induces a voltage on the inner coil. This voltage will also be time-varying, with the same frequency the voltage driving the outer coil.

We derived an expression relating the rate at which the current in the outer coil changes with time to the magnitude of the voltage induced across the inner coil:

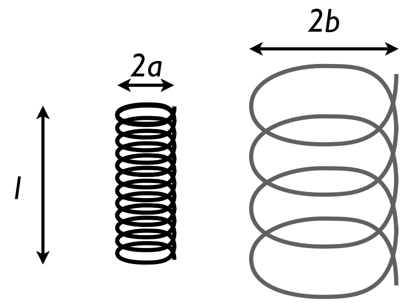
$$|V_{\text{outer}}| = -M\frac{\Delta I_{\text{inner}}}{\Delta t}$$

The constant of proportionality M is the **mutual inductance** of the two coils. If the current in the inner coil varies sinusoidally with frequency *f*, we can simplify this expression:

$$|V_{\text{outer}}| = -M \cdot 2\pi f I_{\text{inner}}$$

Remarkably, the situation is *exactly the same* if we instead power the inner coil and measure the voltage on the outer coil, as shown at right in the figure, even though the field produced by the shorter inner solenoid is terribly nonuniform. The relation between voltage in one coil and current in the other remains exactly the same, no matter which is powered and which is measured. Therefore: we choose the easier system to actually measure, namely, the right-hand one.

In the case of two solenoids, the mutual inductance M may be calculated exactly. Let the inner solenoid have length l and diameter 2a, the outer solenoid diameter 2b, as shown below



$$M = \mu_0 \pi a^2 n_{\mathrm{inner}} n_{\mathrm{outer}} l$$

In general, mutual inductance depends only on the geometry and number of turns in each coil, as in this example. If the outer coil completely encloses the inner, we can make one more simplification, as we did when discussing transformers: we know that the flux produced by the inner coil is completely captured by the outer coil, so the product of the number of turns and current is the same for both coils - $n_{inner} I_{inner} = n_{outer} I_{outer}$.

Using this relationship and the equations above, we can write the voltage on the outer coil as a function of the frequency driving the inner coil:

$$|V_{\mathrm{outer}}| = -\left[2\mu_0 \pi^2 a^2 l I_{\mathrm{outer}}\right] n_{\mathrm{outer}}^2 f$$

The quantity in brackets depends only on constants, the geometry of the inner coil, and the current in the outer coil, which you can determine from the measured voltage and resistance.[1] Thus, a plot of V versus *f* should allow us to measure the number of turns in the outer coil.

Preliminaries

1) Measure the diameter of both coils, and the length of the inner coil. Record these numbers below. Your coils should look something like this:

[1] Since $n_{inner} I_{inner} = n_{outer} I_{outer}$, we can determine the current at a single frequency, and presume it constant over the course of the measurement. The voltage on the outer coil will change, but its current will be roughly the same over the frequency range of interest.

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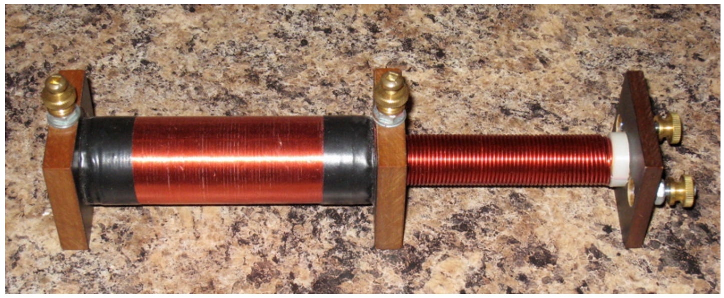


2) Count the number of turns on your inner coil, and record this number below.

3) Measure the resistance of the **outer coil** using the hand-held voltmeter in "Ohms" mode. Record this number below. It should be 50-100 Ohms in most cases.

Procedure

1) Turn on your function generator, and connect its output to the handheld voltmeter.

2) Set the function generator's frequency to 1kHz, its waveform to a sine wave, and maximize the amplitude. With the voltmeter in "ac volts" mode (symbolized by V~), measure the output of the function generator and record it.

3) Now connect the function generator output to the inner coil instead of the voltmeter. This will power the inner solenoid.

4) Connect the outer coil's terminals to the voltmeter, and place it in "ac volts" mode (symbolized by V~)

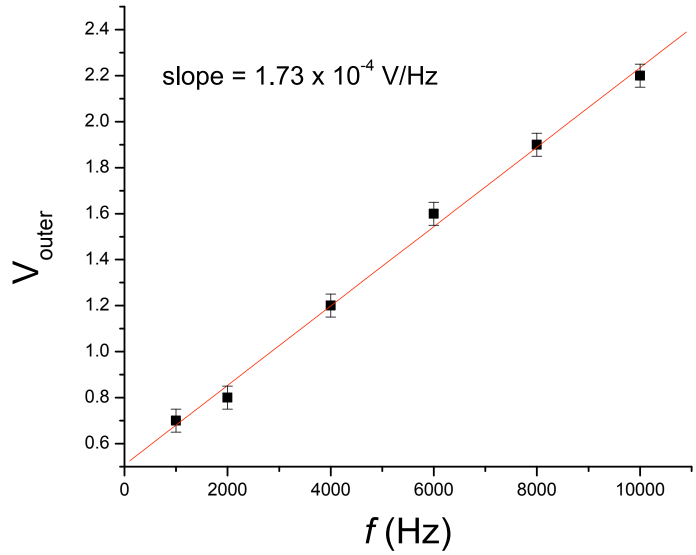
5) Place the inner solenoid completely inside the outer one, and note the voltage measured on the outer coil.

Question: What happens when you move the inner coil slowly out of the outer coil? Why?

6) Replace the inner coil such that is completely enclosed by the outer coil.

7) In steps of 1kHz, record the voltage on the outer coil versus the frequency of the voltage applied to the inner coil from 1-10kHz using the table below. Do not change the amplitude of the function generator output applied to the inner coil

8) Plot the voltage on the outer coil versus frequency of the inner coil's signal in excel, as shown in the example below. Using a linear trendline, determine the slope of the curve and record it below.



9) Using the relation below, determine the number of turns in the outer coil, and record this below. Determine I_{outer} from the measured resistance, and the measured voltage at 1kHz. Since we measure only the *magnitude* of the voltage, the minus sign is irrelevant here.

$$|V_{\text{outer}}| = -\left[2\mu_0\pi^2a^2lI_{\text{outer}}\right] n_{\text{outer}}^2 f$$

$$\implies \frac{\text{slope}}{\left[2\mu_0\pi^2a^2lI_{\text{outer}}\right]} = n_{\text{outer}}^2$$

10) Is your value reasonable? Estimate (roughly) the number of turns per layer and the number of layers in your outer coil.

When you are finished:

- *Power off the function generator and multimeter*
- *Straighten up your components and wires, return them to the cart*
- *Turn in a hard copy of your report and plots*

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Data

function generator output: V = _____________ Volts @ _________ kHz

outer coil resistance:R_o = ____________ Ohms

current in outer coil at frequency above: $I_0 = V_o/R_o$ = ___________ A

Number of turns on inner coil: n_i = _____________

inner coil diameter: 2a = ____________ m

inner coil length: l = _______________ m

outer coil diameter: 2b = ____________ m

f (kHz)	outer coil voltage (V)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Calculations

slope of V_{outer} versus frequency graph: ________________ V/Hz

number of turns in outer coil: _____________ +/ __________