## 3.10 Quick Questions

**1.** Two charges of $+1\,\mu$C each are separated by 1 cm. What is the force between them?

- ◯ 0.89 N
- ◯ 90 N
- ◯ 173 N
- ◯ 15 N

**2.** The electric field *inside* an isolated conductor is

- ◯ determined by the size of the conductor
- ◯ determined by the electric field outside the conductor
- ◯ always zero
- ◯ always larger than an otherwise identical insulator

**3.** Which statement is false?

- ◯ Charge deposited on conductors stays localized
- ◯ Charge distributes itself evenly over a conductor
- ◯ Charge deposited on insulators stays localized
- ◯ Charges in a conductor are mobile, and move in response to an electric force

**4.** Which of the following is true for the electric force, but not the gravitational force?

- ◯ The force propagates at a speed of $c$
- ◯ The force acts at a distance without any intervening medium
- ◯ The force between two bodies depends on the square of the distance between them
- ◯ The force between two bodies can be repulsive as well as attractive.

**5.** Two charges of $+1\,\mu$C are separated by 1 cm. What is the magnitude of the electric field halfway between them?

- ◯ $9 \times 10^7$ N/C
- ◯ $4.5 \times 10^7$ N/C
- ◯ 0
- ◯ $1.8 \times 10^8$ N/C

**6.** A circular ring of charge of radius $b$ has a total charge of $q$ uniformly distributed around it. The magnitude of the electric field at the center of the ring is:

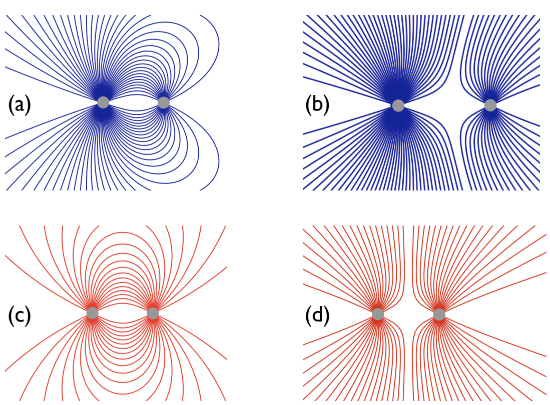
- ◯ 0
- ◯ $k_e q/b^2$
- ◯ $k_e q^2/b^2$
- ◯ $k_e q^2/b$
- ◯ none of these.

**7.** Two isolated identical conducting spheres have a charge of $q$ and $-3q$, respectively. They are connected by a conducting wire, and after equilibrium is reached, the wire is removed (such that both spheres are again isolated). What is the charge on each sphere?

- ◯ $q$, $-3q$
- ◯ $-q$, $-q$
- ◯ 0, $-2q$
- ◯ $2q$, $-2q$

**8.** A single point charge $+q$ is placed exactly at the center of a hollow conducting sphere of radius $R$. Before placing the point charge, the conducting sphere had zero net charge. What is the magnitude of the electric field *outside* the conducting sphere at a distance $r$ from the center of the conducting sphere? *I.e., the electric field for* $r > R$.

- ◯ $|\vec{\mathbf{E}}| = -\frac{k_e q}{r^2}$
- ◯ $|\vec{\mathbf{E}}| = \frac{k_e q}{(R+r)^2}$
- ◯ $|\vec{\mathbf{E}}| = \frac{k_e q}{R^2}$
- ◯ $|\vec{\mathbf{E}}| = \frac{k_e q}{r^2}$




**9.** Which set of electric field lines could represent the electric field near two charges of the *same sign*, but *different magnitudes*?

- ◯ a
- ◯ b
- ◯ c
- ◯ d

**10.** Referring again to the figure above, which set of electric field lines could represent the electric field near two charges of *opposite sign* and *different magnitudes*?

- ◯ a
- ◯ b
- ◯ c
- ◯ d

**11.** A "free" electron and a "free" proton are placed in an identical electric field. Which of the following statements are true? *Check all that apply.*

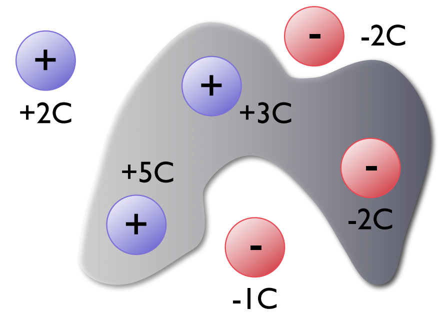
- ◯ Each particle is acted on by the same electric force and has the same acceleration.
- ◯ The electric force on the proton is greater in magnitude than the force on the electron, but in the opposite direction.
- ◯ The electric force on the proton is equal in magnitude to the force on the electron, but in the opposite direction.
- ◯ The magnitude of the acceleration of the electron is greater than that of the proton.
- ◯ Both particles have the same acceleration.

**12.** A point charge $q$ is located at the center of a (non-conducting) spherical shell of radius $a$ that has a charge $-q$ uniformly distributed on its surface. What is the electric field *for all points outside the spherical shell?*

- ◯ none of these
- ◯ $E=0$
- ◯ $E=q/4\pi r^2$
- ◯ $E=kq/r^2$
- ◯ $E=kq^2/r^2$

**13.** What is the electric field inside the same shell a distance $r<a$ from the center (*i.e.,* a point inside the spherical shell)?

- ◯ $E=kq/r^2$
- ◯ $E=kq^2/r^2$
- ◯ none of these
- ◯ $E=0$
- ◯ $E=q/4\pi r^2$

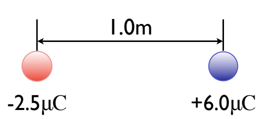



**14.** What is the electric flux through the surface at right?

- ◯ $+5\,\mathrm{C}/\epsilon_0$
- ◯ $-3\,\mathrm{C}/\epsilon_0$
- ◯ $0$
- ◯ $+6\,\mathrm{C}/\epsilon_0$

**15.** A spherical conducting object $A$ with a charge of $+Q$ is lowered through a hole into a metal (conducting) container $B$ that is initially uncharged (and is not grounded). When $A$ is at the center of $B$, but not touching it, the charge on the inner surface of $B$ is:

- ◯ $+Q$
- ◯ $-Q$
- ◯ $0$
- ◯ $+Q/2$
- ◯ $-Q/2$




**16.** Determine the point (other than infinity) at which the total electric field is zero. This point is not between the two charges.

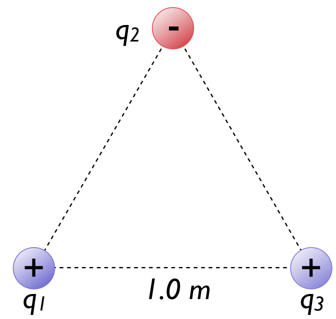
- ◯ 3.5 m to the left of the negative charge
- ◯ 2.1 m to the right of the positive charge
- ◯ 1.3 m to the right of the positive charge
- ◯ 1.8 m to the left of the negative charge

**17.** A flat surface having an area of $3.2\,\mathrm{m}^2$ is rotated in a uniform electric field of magnitude $E = 5.7 \times 10^5\,\mathrm{N/C}$. What is the electric flux when the electric field is parallel to the surface?

- ◯ $1.82 \times 10^6\,\mathrm{N \cdot m^2/C}$
- ◯ $0\,\mathrm{N \cdot m^2/C}$
- ◯ $3.64\,\mathrm{N \cdot m^2/C}$
- ◯ $0.91\,\mathrm{N \cdot m^2/C}$

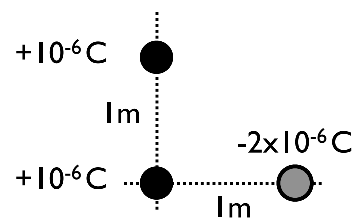
**18.** Three charges are arranged in an equilateral triangle, as shown at left. All three charges have the same *magnitude* of charge, $|q_1| = |q_2| = |q_3| = 10^{-9}\,\mathrm{C}$ (note that $q_2$ is negative though). What is the **force** on $q_2$, magnitude and direction?

- ◯ $9.0\,\mu\mathrm{N}$, up (90°);
- ◯ $16\,\mu\mathrm{N}$, down (-90°);
- ◯ $18\,\mu\mathrm{N}$, down and left (225°);
- ◯ $8.0\,\mu\mathrm{N}$, up and right (-45°)

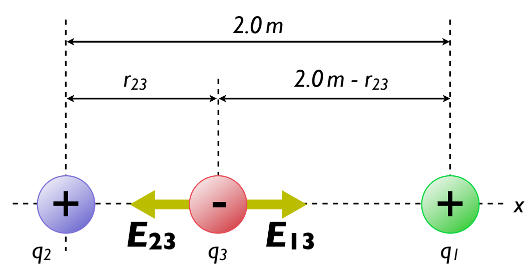



## 3.11 Problems

**1.** Two charges of $+10^{-6}\,\mathrm{C}$ are separated by 1 m along the vertical axis. What is the net **horizontal** force on a charge of $-2{\times}10^{-6}\,\mathrm{C}$ placed one meter to the right of the lower charge?




**2.** Three point charges lie along the $x$ axis, as shown at left. A positive charge $q_1 = 15\,\mu\mathrm{C}$ is at $x = 2\,\mathrm{m}$, and a positive charge of $q_2 = 6\,\mu\mathrm{C}$ is at the origin. Where must a *negative* charge $q_3$ be placed on the $x$-axis **between the two positive charges** such that the resulting electric force on it is zero?



## 3.12 Solutions to Quick Questions

**1. 90 N.** We just need to use Eq. 3.1 and plug in the numbers ... remembering that $\mu$ means $10^{-6}$:

$$\begin{aligned}
\vec{\mathbf{F}} &= k_e \frac{q_1 q_2}{r_{12}^2}\,\hat{\mathbf{r}}_{12} \\
|\vec{\mathbf{F}}| &= k_e \frac{q_1 q_2}{r_{12}^2} \\
&= 8.9875 \times 10^9\,\frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}} \left[\frac{\left(1 \times 10^{-6}\,\mathrm{C}\right)\left(1 \times 10^{-6}\,\mathrm{C}\right)}{\left(1 \times 10^{-2}\,\mathrm{m}\right)^2}\right] \\
&\approx 9 \times 10^9\,\frac{\mathrm{N} \cdot \cancel{\mathrm{m^2}}}{\cancel{\mathrm{C^2}}} \left[\frac{1 \times 10^{-12}\,\cancel{\mathrm{C^2}}}{1 \times 10^{-4}\,\cancel{\mathrm{m^2}}}\right] \\
&= 9 \times 10^1\,\mathrm{N} \\
|\vec{\mathbf{F}}| &= 90\,\mathrm{N}
\end{aligned}$$

**2. Always zero.** Re-read Sect. 3.5 to remind yourself *why* this must be true.

**3. Charge deposited on conductors stays localized.** See Sect. 3.2.

**4. The force between two bodies can be repulsive as well as attractive.** Both the electric and gravitational forces propagate at the speed of light, both act through empty space, and both are inverse-square laws. The only difference is that gravity can only be attractive, since there is no such thing as negative mass.

**5. 0.** Halfway between, the magnitude of the field from each individual charge is the same, but *they act in opposite directions*. Therefore, exactly in the middle, they cancel, and the field is zero. This is the same as the field exactly at the midpoint of an electric dipole. It might be easier to convince yourself the field is zero if you draw a picture including the electric field lines.

**6. 0.** The field at the center from a point on the ring is always canceled by the field from another point 180° away.

**7. $-q$, $-q$.** The thing to remember is that any charge on a conductor spreads out evenly over its surface. When we have the conducting spheres isolated, they have $q$ and $-3q$ respectively, and this charge is spread evenly over each sphere. When we connect them with a conducting wire, suddenly charges are free to move from one conductor, across the wire, into the other conductor. Its just the same as if we had one big conductor, and all the *total net charge of the two conductors combined* will spread out evenly over *both* spheres and the wire.

If the charge from each sphere is allowed to spread out evenly over both spheres, then the $-3q$ and $+q$ will both be spread out evenly everywhere. The $+q$ will cancel part of the $-3q$, leaving a total net charge of $-2q$ spread over evenly over both spheres, or $-q$ on each sphere. Once we disconnect the two spheres again, the charge remains equally distributed between the two.

**8.** $|\vec{\mathbf{E}}|=\frac{k_e q}{r^2}$**.** The easiest way out of this one is Gauss' law. First, Gauss' law told us that any spherically symmetric charge distribution behaves as a point charge. Second, Gauss' law tells us that the electric flux out of some surface depends only on the enclosed charge. If we draw a spherical surface of radius $r$ and area $A$ around the shell and point charge, centered on the center of the conducting sphere, Gauss' law gives:

$$\begin{aligned}\Phi_E &= \frac{q_{encl}}{\epsilon_0} = 4\pi k_e q_{encl} \\ EA &= 4\pi k_e q_{encl} \\ E &= \frac{4\pi k_e q_{encl}}{A}\end{aligned}$$

The surface area of a sphere is $A=4\pi r^2$. In this case, the enclosed charge is just $q$, since the hollow conducting sphere itself has no charge of its own. Gauss' law only cares about the *total net charge* inside the surface of interest. This gives us:

$$E = \frac{4\pi k_e q}{4\pi r^2} = \frac{\cancel{4\pi} k_e q}{\cancel{4\pi} r^2} = \frac{k_e q}{r^2}$$

There we have it, it is just the field of a point charge $q$ at a distance $r$.

If we want to get formal, we should point out that the point charge $q$ induces a negative charge $-q$ on the inner surface of the hollow conducting sphere. Since the sphere is overall neutral, the outer surface must therefore have a net positive charge $+q$ on it. This makes no difference in the result – the total *enclosed* charge, for radii larger than that of the hollow conducting sphere ($r>R$), is still just $q$. If we start with an uncharged conducting sphere, and keep it physically isolated, any induced charges have to cancel each other over all.

If this is still a bit confusing, go back and think about induction charging again. A charged rod was used to induce a positive charge on one side of a conductor, and a negative charge on the other. *Overall*, the 'induced charge' was just a rearrangement of existing charges, so if the conductor started out neutral, no amount of 'inducing' will change that. We only ended up with a *net charge* on the conductor when we used a ground connection to 'drain away' some of the induced charges. Or, if you like, when we used a charged rod to repel some of the conductor's charges through the ground connection, leaving it with a net imbalance.

**9. (b).** If the charges are of the *opposite* sign, then the field lines would have to run from one charge directly to the other. Field lines start on a positive charge and end on a negative one, and there should be many lines which run from one charge to the other. Since opposite charges attract, the field between them is extremely strong, the lines should be densest right between the charges. This is the case in (a) and (b), so they are not the right ones.

By the same token, for charges of the *same* sign, the force is repulsive, and the electric field midway between them cancels. The field lines should "push away" from each other, and no field line from a given charge should reach the other charge – field lines cannot start and end on the same sign charge. This means that only (b) and (d) could possibly correspond to two charges of the same sign.

Next, the field lines leaving or entering a charge has to be proportional to the magnitude of the charge. In (d) there are the same number of lines entering and leaving each charge, so the charges are of the same magnitude. One can also see this from the fact that the lines are symmetric about a vertical line drawn midway between the charges. In (b) there are clearly many more lines near the left-most charge.

Or, right off the bat, you could notice that only (a) and (b) are asymmetric, and only (b) and (d) look like two like charges. No sense in over-thinking this one.

**10. (a).** By similar reasoning as above, only figure **a** could represent two opposite charges of different magnitude.

**11.** The electric force on the proton is equal in magnitude to the force on the electron, but in the opposite direction. The magnitude of the acceleration of the electron is greater than that of the proton.

**12.** $E=0$. The simplest way to solve this one is with Gauss' law. First, Gauss law told us that any spherically symmetric charge distribution behaves as a point charge. Second, Gauss law tells us that the electric flux out of some surface depends only on the enclosed charge. If we draw a spherical surface of radius r and area A enclosing the shell *and* the point charge, centered on the center of the conducting sphere, the total enclosed charge is that of the shell plus that of the point charge: $q_{\text{encl}}=q+(-q)=0$. If the enclosed charge is zero for any sphere drawn outside of and enclosing the spherical shell, then the electric field for all points outside the spherical shell.

**13.** $E=k_eq/r^2$. Just like the last question, we need Gauss' law. This time, we have to draw a sphere surrounding the point charge, but *inside* of the spherical shell. Gauss' law tells us that the electric field depends only on the *enclosed* charge within our sphere. The only charge enclosed is the point charge at the center of the shell, $q$ – the charge on the spherical shell is outside of our spherical surface, so it is not enclosed and does not contribute to the electric field inside. Now we just apply Gauss' law, knowing that the enclosed charge is $q$, and the surface area of the sphere is $4\pi r^2$:

$$\Phi_E = \frac{q_{\text{encl}}}{\epsilon_0} = 4\pi k_e q \tag{3.29}$$
$$EA = 4\pi k_e q \tag{3.30}$$
$$E = \frac{4\pi k_e q}{4\pi r^2} = \frac{k_e q}{r^2} \tag{3.31}$$

**14.** $+6\,\text{C}/\epsilon_0$. Again, this question requires Gauss' law. We know that the electric flux through this surface only depends on the total amount of enclosed charge. All we need to do is add up the *net* charge inside the surface, since any charges outside the surface do not contribute to the flux. There are only three charges enclosed by the surface ... so:

$$\text{net charge} = 3\,\text{C} + 5\,\text{C} - 2\,\text{C} = 6\,\text{C} \tag{3.32}$$

The electric flux $\Phi_E$ is then just the enclosed charge divided by $\epsilon_0$, or $+6\,\text{C}/\epsilon_0$.

**15.** $-Q$. The charge $+Q$ on object $A$ induces a negative charge $-Q$ on the inner surface of the conducting container $B$.

**16.** $1.8$ **m to the left of the negative charge.** By symmetry, we can figure out on which side the field should be zero. In between the two charges, the field from the positive and negative charges *add together*. The force on a fictitious positive test charge placed in between the two would experience a force to the left due to the positive charge, and another force to the left due to the negative charge. There is no way the fields can cancel here.

If we place a positive charge to the *right of the positive charge*, it will feel a force to the right from the positive charge, and a force to the left from the negative charge. The directions are opposite, but the fields still cannot cancel because the test charge is closest to the larger charge.

This leaves us with points to the left of the negative charge. The forces on a positive test charge will be in opposite directions here, and we are closer to the smaller charge. What position gives zero field? First, we will call the position of the negative charge $x = 0$, which means the positive charge is at $x = 1\,\text{m}$. We will call the position where electric field is zero $x$. The distance from this point to the negative charge is just $x$, and the distance to the positive charge is $1 + x$. Now write down the electric field due to each charge:

$$\begin{aligned} E_{\text{neg}} &= \frac{k_e(-2.5\,\mu\text{C})}{x^2} \\ E_{\text{pos}} &= \frac{k_e(6\,\mu\text{C})}{(1+x)^2} \end{aligned}$$

The field will be zero when $E_{\text{neg}} + E_{\text{pos}} = 0$:

$$\begin{aligned} E_{\text{neg}} + E_{\text{pos}} &= 0 \\ \frac{k_e(-2.5\,\mu\text{C})}{x^2} + \frac{k_e(6\,\mu\text{C})}{(1+x)^2} &= 0 \\ \frac{\cancel{k_e}(-2.5\,\cancel{\mu\text{C}})}{x^2} + \frac{\cancel{k_e}(6\,\cancel{\mu\text{C}})}{(1+x)^2} &= 0 \\ \frac{-2.5}{x^2} + \frac{6}{(1+x)^2} &= 0 \\ \Rightarrow \qquad \frac{2.5}{x^2} &= \frac{6}{(1+x)^2} \end{aligned}$$

Cross multiply, apply the quadratic formula:

$$\begin{aligned}
2.5(1+x)^2 &= 6x^2 \\
2.5+5x+2.5x^2 &= 6x^2 \\
3.5x^2-5x-2.5 &= 0 \\
\Rightarrow \qquad x &= \frac{-(-5)\pm\sqrt{5^2-4(-2.5)(3.5)}}{2(3.5)} \\
x &= \frac{5\pm\sqrt{25+35}}{7} \\
x &= \frac{5\pm 7.75}{7} = 1.82, -0.39
\end{aligned}$$

Which root do we want? We wrote down the distance $x$ the distance to the *left* of the negative charge. A negative value of $x$ is then in the wrong direction, in between the two charges, which we already ruled out. The positive root, $x = 1.82$, means a distance 1.82 m to the *left* of the negative charge. This is what we want.

**17.** $0\,\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}$**.** Remember that electric flux is $\Phi_E = EA\cos\theta$, where $\theta$ is the angle between a line perpendicular to the surface and the electric field. If $E$ is *parallel* to the surface, then $\theta = 90$ and $\Phi_E = 0$.

Put more simply, there is only an electric flux if field lines penetrate the surface. If the field is parallel to the surface, no field lines penetrate, and there is no flux.

**18.** $16\,\mu$**N, down (-90°).** The easiest way to solve this one is by symmetry and elimination. The negative charge $q_2$ feels an attractive force from both $q_1$ and $q_2$. Since both charges are the same vertical distance away and below $q_2$, both will give a force in the vertical downward direction of equal magnitude and direction. Since both charges are horizontally the same direction away but *on opposite sides*, the horizontal forces will be equal in magnitude but *opposite* in direction – the horizontal forces will cancel. Therefore, the net force has to be purely in the vertical direction and downward, so the second choice is the only option! Of course, you can calculate all of the forces by components and add them up ... you will arrive at the same answer.

## 3.13 Solutions to Problems

**1.** $-0.0244$ **N.** We are only interested in the $x$ component of the force, which makes things easier. First, we are trying to find the force on a negative charge due to two positive charges. Both positive charges are to the left of the negative charge, and both forces will be attractive. We will adopt the usual convention that the positive horizontal direction is to the right and called $+x$, and the negative horizontal direction is to the left and called $-x$.

First, we will find the force on the negative charge due to the positive charge in the lower left, which we will call "1" to keep things straight. We will call the negative charge "2." This is easy, since the force is purely in the $-x$ direction:

$$\begin{aligned}
F_{x,1} &= k_e \frac{q_1 q_2}{r_{12}^2} \\
&= \left(9\times 10^9\,\mathrm{N\cdot m^2/C^2}\right) \frac{\left(10^{-6}\,\mathrm{C}\right)\cdot\left(-2\times 10^{-6}\,\mathrm{C}\right)}{\left(1\,\mathrm{m}\right)^2} \\
&= \left(9\times 10^9\,\mathrm{N}\cdot \cancel{\mathrm{m}^2}/\cancel{\mathrm{C}^2}\right)\left(-2\times 10^{-12}\,\cancel{\mathrm{C}^2}/\cancel{\mathrm{m}^2}\right) \\
&= -18\times 10^{-3}
\end{aligned}$$

So far so good, but now we have to include the force from the upper left-hand positive charge, which we'll call "3." We calculate the force in exactly the same way, with two little difference: the separation distance is slightly larger, and now the force has both a horizontal and vertical component. First, let's calculate the magnitude of the net force, we'll find the horizontal component after that.

Plane geometry tells us that the separation between charges 3 and 2 has to be $\sqrt{2}\cdot 1\,\mathrm{m}$, or $\sqrt{2}\,\mathrm{m}$ – connecting the charges with straight lines forms a 1-1-$\sqrt{2}$ right triangle, with $45°$ angles.

$$\begin{aligned}
F_{net,3} &= k_e \frac{q_2 q_3}{r_{23}^2} \\
&= \left(9\times 10^9\,\mathrm{N\cdot m^2/C^2}\right) \frac{\left(10^{-6}\,\mathrm{C}\right)\cdot\left(-2\times 10^{-6}\,\mathrm{C}\right)}{\left(\sqrt{2}\,\mathrm{m}\right)^2} \\
&= \left(9\times 10^9\,\mathrm{N}\cdot \cancel{\mathrm{m}^2}/\cancel{\mathrm{C}^2}\right)\frac{-2\times 10^{-12}\,\cancel{\mathrm{C}^2}}{2\,\cancel{\mathrm{m}^2}} \\
&= -9\times 10^{-3}\,\mathrm{N}
\end{aligned}$$

So the *net* force from the upper left charge is just half as much, since it is a factor $\sqrt{2}$ farther away. We only want the horizontal component though! Since we are dealing with a 45-45-90 triangle here, the horizontal component is just the net force times $\cos 45°$:

$$\begin{aligned}
F_{x,3} &= F_{net,3}\cos 45° \\
&= -9\times 10^{-3}\cdot\frac{\sqrt{2}}{2}\,\mathrm{N} = -9\times 10^{-3}\cdot 0.707\,\mathrm{N} \\
&\approx -6.4\times 10^{-3}\,\mathrm{N}
\end{aligned}$$

The total horizontal force is just the sum of the horizontal forces from the two positive charges:

$$\begin{aligned}
F_{x,\text{total}} &= F_{x,1} + F_{x,3} \\
&= \left(-18 \times 10^{-3}\right) + \left(-6.4 \times 10^{-3}\right)\,\text{N} \\
&= -24.4 \times 10^{-3}\,\text{N} = -0.0244\,\text{N}
\end{aligned}$$

**2.** $x\!=\!0.77\,$**m.** We have two positive charges and one negative charge along a straight line. If we want there to be no net force on the negative charge, the electric forces from both of the positive charges on it must cancel. For that to happen, there is only one possibility: the negative charge has to be between the two positive charges. Outside that middle region, both positive charges will exert an attractive force on the negative charge in the same direction, and there is no way they can cancel each other. Only in the middle region do the forces from both positive charges act in opposite directions on a negative charge, and only there can they cancel each other. We want to find the position $r_{23}$ such that both forces are equal in magnitude. All charges are on the $x$ axis, so the problem is one-dimensional and does not require vectors.

Intuitively, we know that the negative charge $q_3$ must be closer to the smaller of the positive charges. Since electric forces get larger as separation decreases, the only way the force due to the larger charge can be the same as that due to the smaller charge is if *if the negative charge is farther away from the larger charge.*

Let $F_{32}$ be the force on $q_3$ due to $q_2$, and $F_{31}$ be the force on $q_3$ due to $q_1$, and we will take the positive $x$ direction to be to the right. Since both forces are repulsive, $F_{32}$ acts in the $-x$ direction *and must therefore be negative*, while $F_{31}$ acts in the $+x$ direction and is positive. This is only true for the region between the two positive charges! Elsewhere, both positive charges would give an attractive force, and there is no way they could cancel each other. We are not told about any other forces acting, so our force balance is this:

$$-F_{32} + F_{31} = 0 \qquad \Longrightarrow \qquad F_{32} = F_{31}$$

It didn't really matter which one we called negative and which one we called positive, just that they have different signs. The separation between $q_2$ and $q_3$ is $r_{23}$, and the separation between $q_1$ and $q_3$ is then $2\!-\!r_{23}$. Now we just need to down the electric forces. We will keep everything perfectly general, and plug in actual numbers at the end ... this is always safer.

$$\begin{aligned}
F_{32} &= F_{31} \\
\frac{k_e q_3 q_2}{r_{23}^2} &= \frac{k_3 q_3 q_2}{\left(2 - r_{23}\right)^2} \\
\frac{\cancel{k_e}\cancel{q_3} q_2}{r_{23}^2} &= \frac{\cancel{k_3}\cancel{q_3} q_1}{\left(2 - r_{23}\right)^2} \\
\frac{q_2}{r_{23}^2} &= \frac{q_1}{\left(2 - r_{23}\right)^2}
\end{aligned}$$

Note how this doesn't depend at all on the actual magnitude *or sign* of the charge in the middle! From here, there are two ways to proceed. We could cross-multiply, use the quadratic formula, and that would be that. On the other hand, since we know that $q_3$ is supposed to be between the other two charges, then $r_{23}$ must be positive, and less than 2. That means that we can just take the square root of both sides of the equation above without problem, since neither side would be negative afterward.[xvi] Using this approach first:

$$\begin{aligned}
\frac{q_2}{r_{23}^2} &= \frac{q_1}{(2-r_{23})^2} \\
\Longrightarrow \frac{\sqrt{q_2}}{r_{23}} &= \frac{\sqrt{q_1}}{2-r_{23}}
\end{aligned}$$

Now we can cross-multiply, and solve the resulting linear equation:

$$\begin{aligned}
\sqrt{q_2}\,(2-r_{23}) &= \sqrt{q_1}r_{23} \\
2\sqrt{q_2} - \sqrt{q_2}r_{23} &= \sqrt{q_1}r_{23} \\
2\sqrt{q_2} &= (\sqrt{q_2}+\sqrt{q_1})\,r_{23} \\
r_{23} &= \frac{2\sqrt{q_2}}{\sqrt{q_2}+\sqrt{q_1}}
\end{aligned}$$

Plugging in the numbers we were given (and noting that all the units cancel):

$$r_{23} = \frac{2\sqrt{q_2}}{\sqrt{q_2}+\sqrt{q_1}} = \frac{2\sqrt{6\,\mu\text{C}}}{\sqrt{6\,\mu\text{C}}+\sqrt{15\,\mu\text{C}}} = \frac{2\sqrt{6}}{\sqrt{6}+\sqrt{15}} = \frac{2\sqrt{2}}{\sqrt{2}+\sqrt{5}} \approx 0.77\,\text{m}$$

For that very last step, we factored out $\sqrt{3}$ from the top and the bottom. An unnecessary step if you are using a calculator anyway, but we prefer to stay in practice.

The more general solution is to go back before we took the square root of both sides of the equation and solve it completely:

$$\begin{aligned}
\frac{q_2}{r_{23}^2} &= \frac{q_1}{(2-r_{23})^2} \\
q_2\,(2-r_{23})^2 &= q_1 r_{23}^2 \\
q_2\left(4-4r_{23}+r_{23}^2\right) &= q_1 r_{23}^2 \\
(q_2-q_1)\,r_{23}^2 - 4q_2 r_{23} + 4q_2 &= 0
\end{aligned}$$

Now we just have to solve the quadratic ...

$$\begin{aligned}
r_{23} &= \frac{4q_2 \pm \sqrt{(-4q_2)^2 - 4\,(q_2-q_1)\cdot 4q_2}}{2\,(q_1-q_2)}\,\text{m} \\
&= \frac{4\cdot 6\,\mu\text{C} \pm \sqrt{(-4\cdot 6\,\mu\text{C})^2 - 4\,(6\,\mu\text{C}-15\,\mu\text{C})\cdot 4\cdot 6\,\mu\text{C}}}{2\,(6\,\mu\text{C}-15\,\mu\text{C})}\,\text{m}
\end{aligned}$$

We can cancel all of the $\mu C$ ...

[xvi] This would not work if we wanted the point to the left of $q_2$.

$$\begin{aligned}
r_{23} &= \frac{24 \pm \sqrt{24^2 - 4(-9)(4)(6)}}{2(-9)}\,\text{m} \\
&= \frac{24 \pm \sqrt{24^2 + 36(24)}}{-18}\,\text{m} \\
&= \frac{-24 \mp \sqrt{1440}}{18}\,\text{m} \\
&= (0.775, -3.44)\ \text{m}
\end{aligned}$$

Just as we expected: one solution ($r_{23} = 0.775\,\text{m}$) is right between the two charges, a little bit closer to the smaller charge. What about the positive solution? This corresponds to a position far away from both charges 3.44 m to the left of $q_2$. As stated above, the forces act in the same direction outside of the middle region, and cannot cancel! This solution is physically impossible, just an artifact of the mathematics. We specified originally that the equations were only good for the middle region, so if we get an answer that falls outside we must discard it as outside the scope of our equations.

Our equations as we have written them do not take into account the fact that the fields change direction on one side of a charge versus the other. Properly speaking, outside the middle region between the positive charges, we should write $F_{32} = -F_{31}$ since the forces act in the same direction. Try repeating the problem starting there, and you will find that there are no real (non-imaginary) solutions outside the middle region - two positive forces cannot add up to zero.

Remember: in the end, we always need to make sure that the solutions are physically sensible in addition to being mathematically correct.