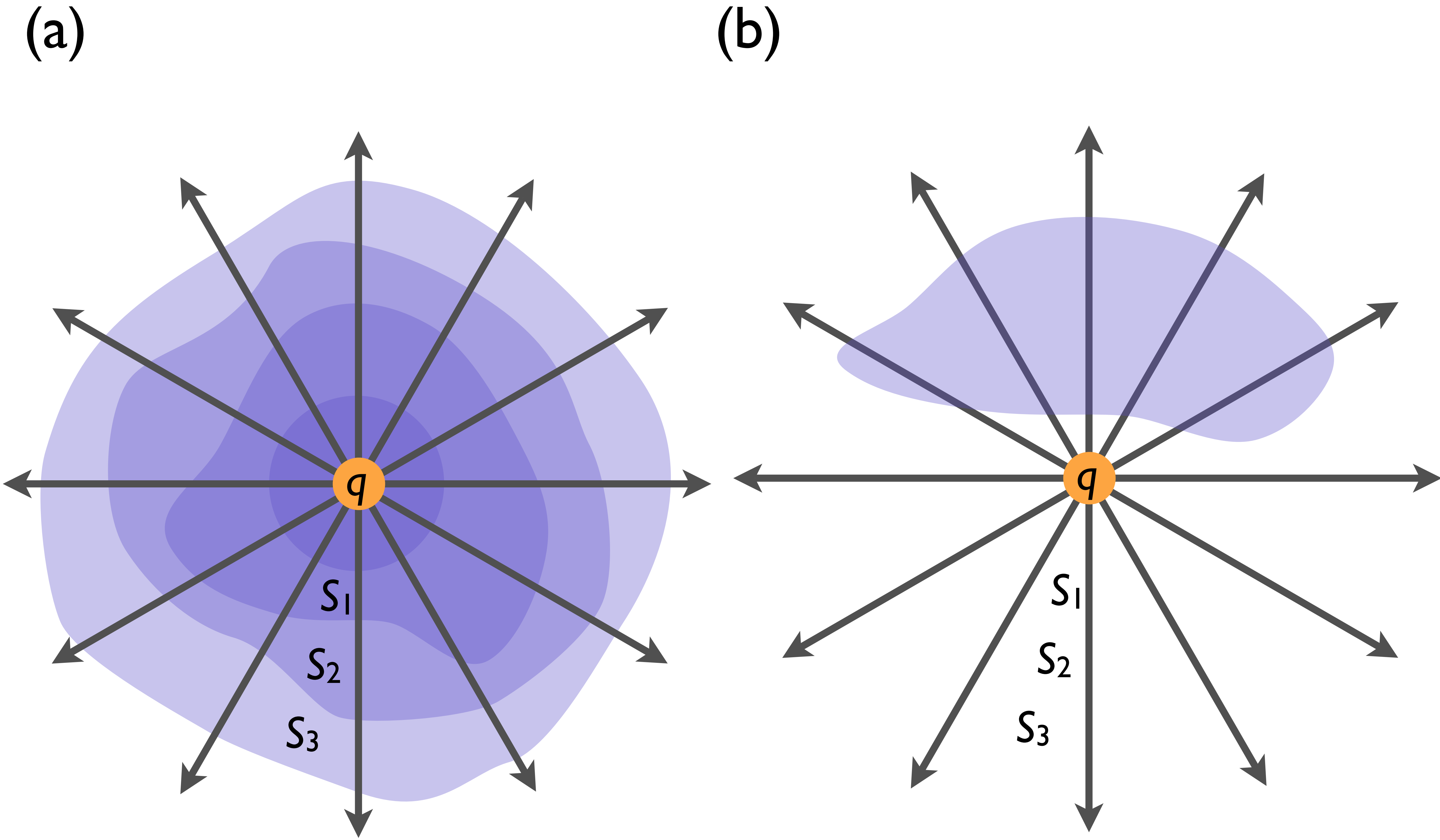
electrical energy & capacitance

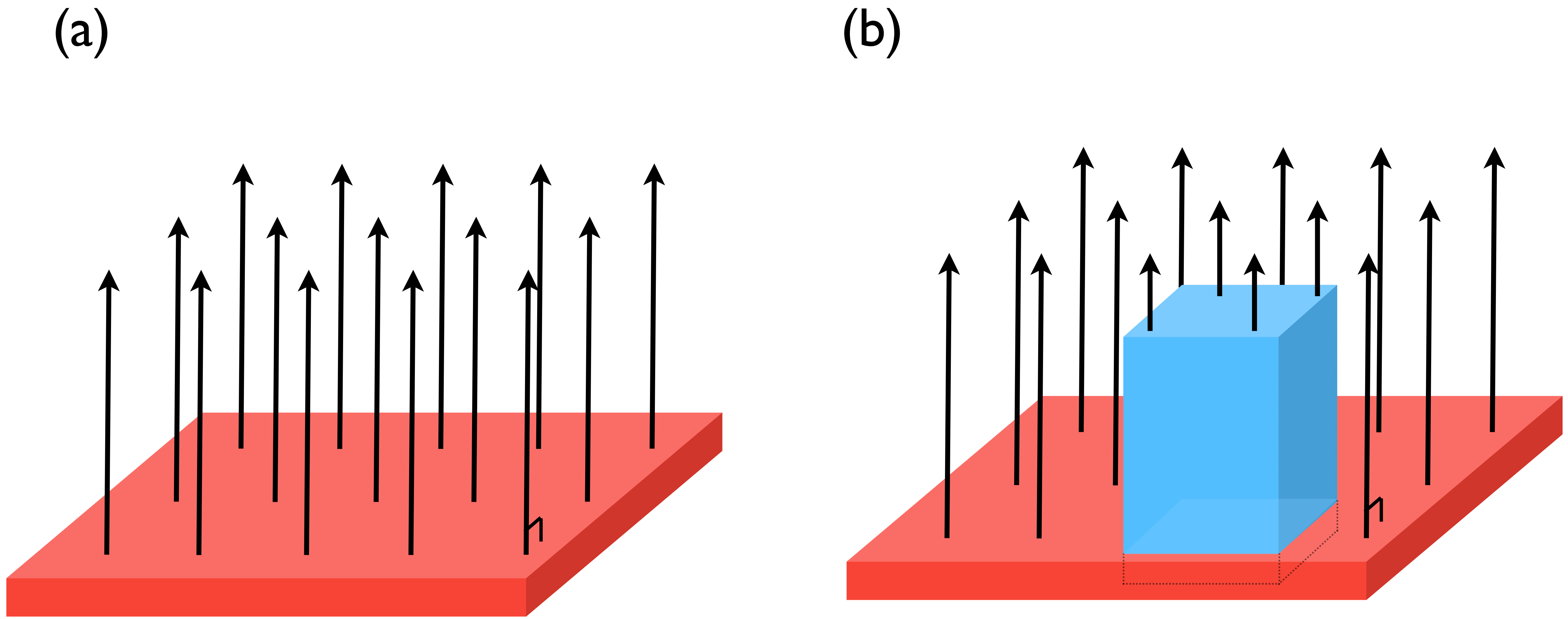
- today & tomorrow
- first: wrap up Gauss' law
- rest of the week: circuits/current/resistance
- NEXT MON: exam I

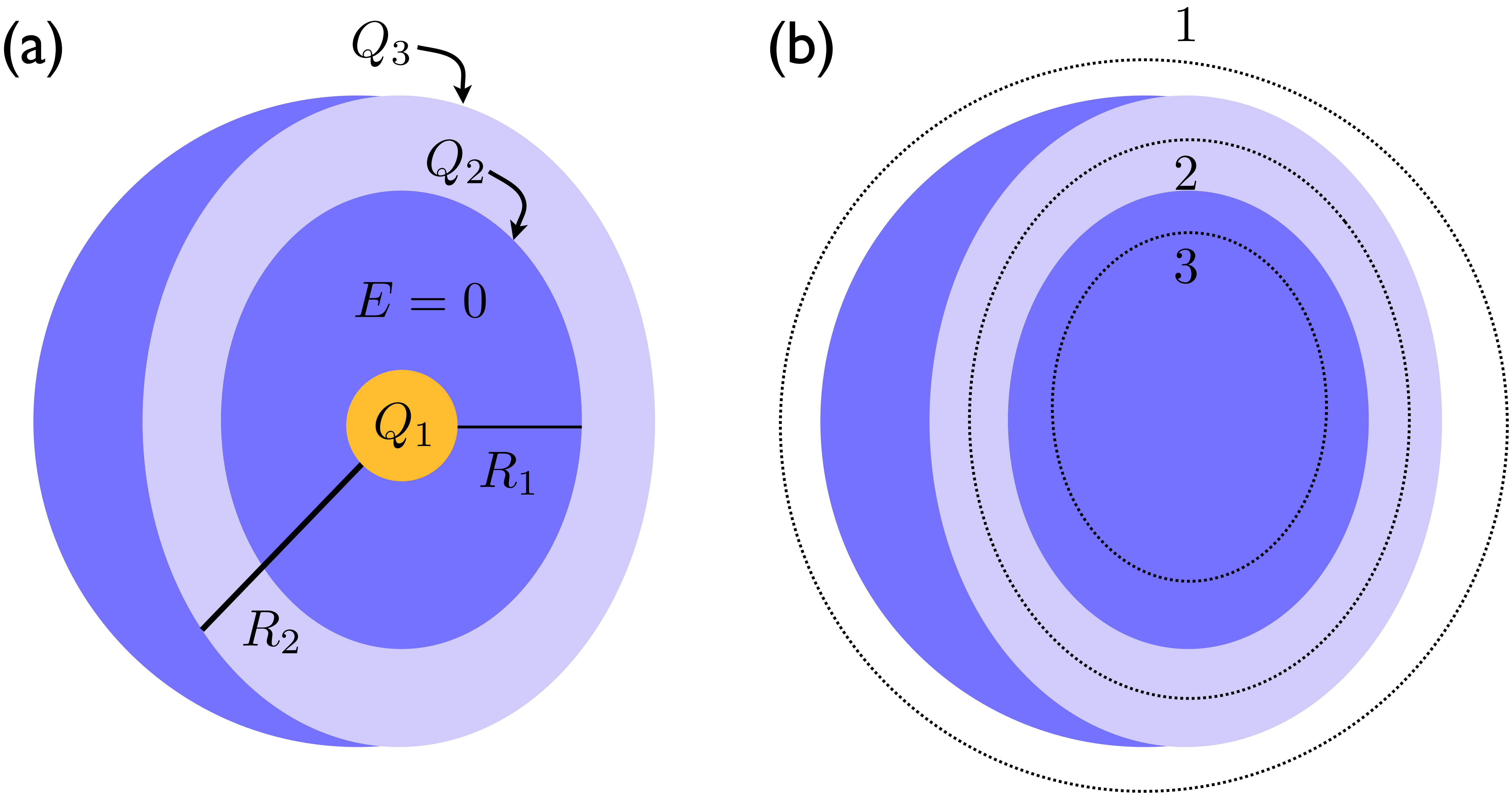
  multiple choice, cumulative

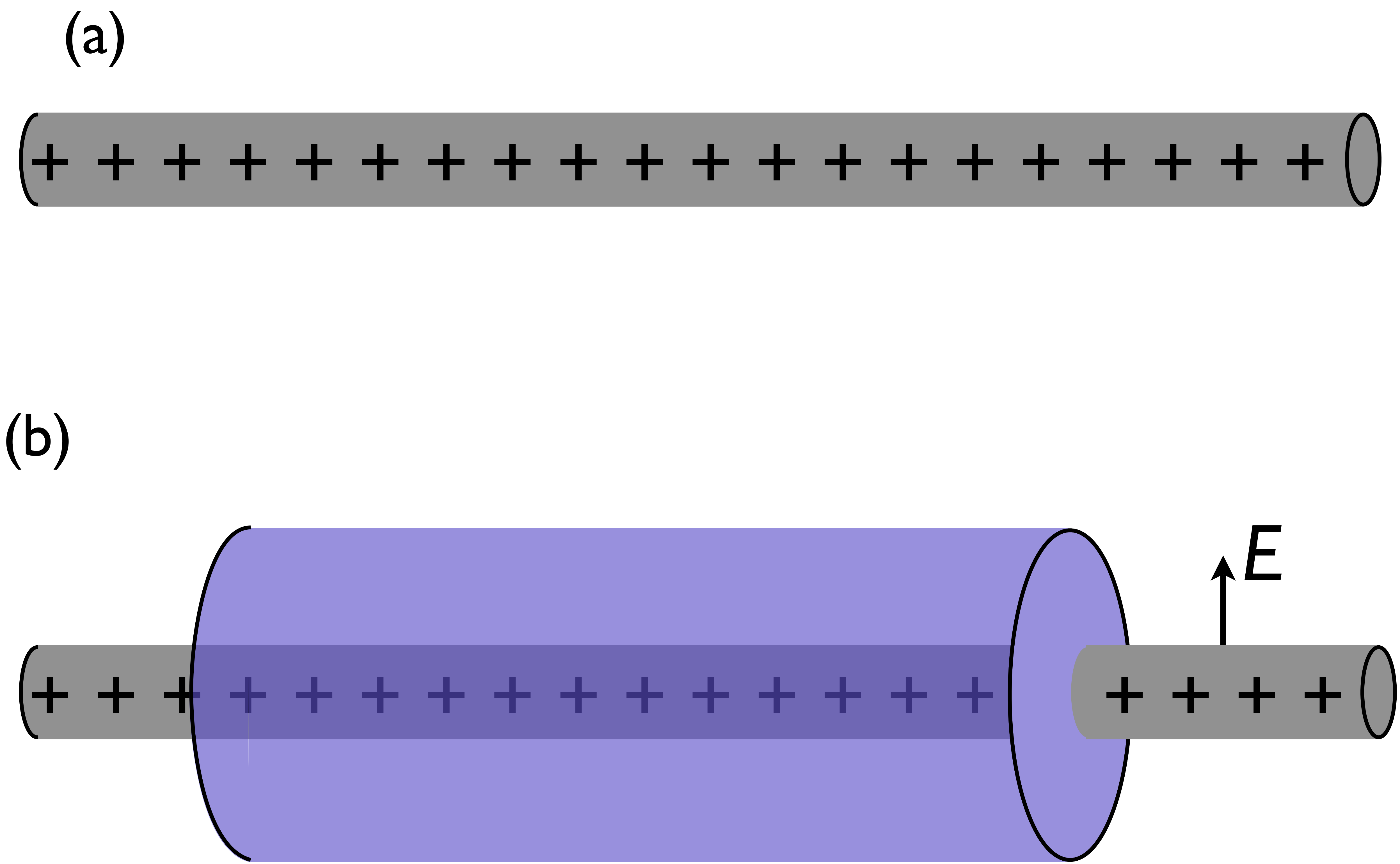
  more details throughout the week

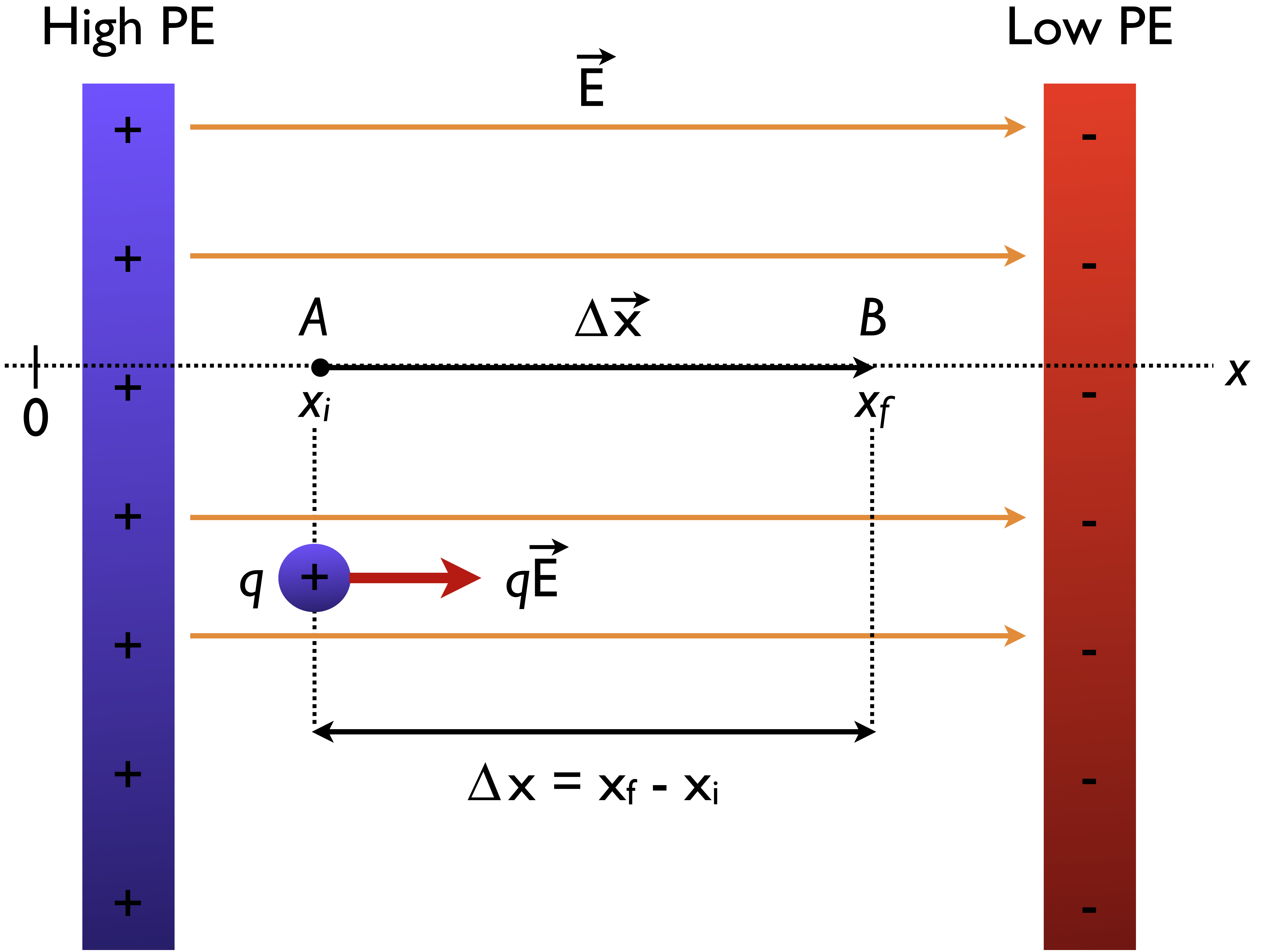


(a)
(b)
q
S1
S2
S3
q
S1
S2
S3

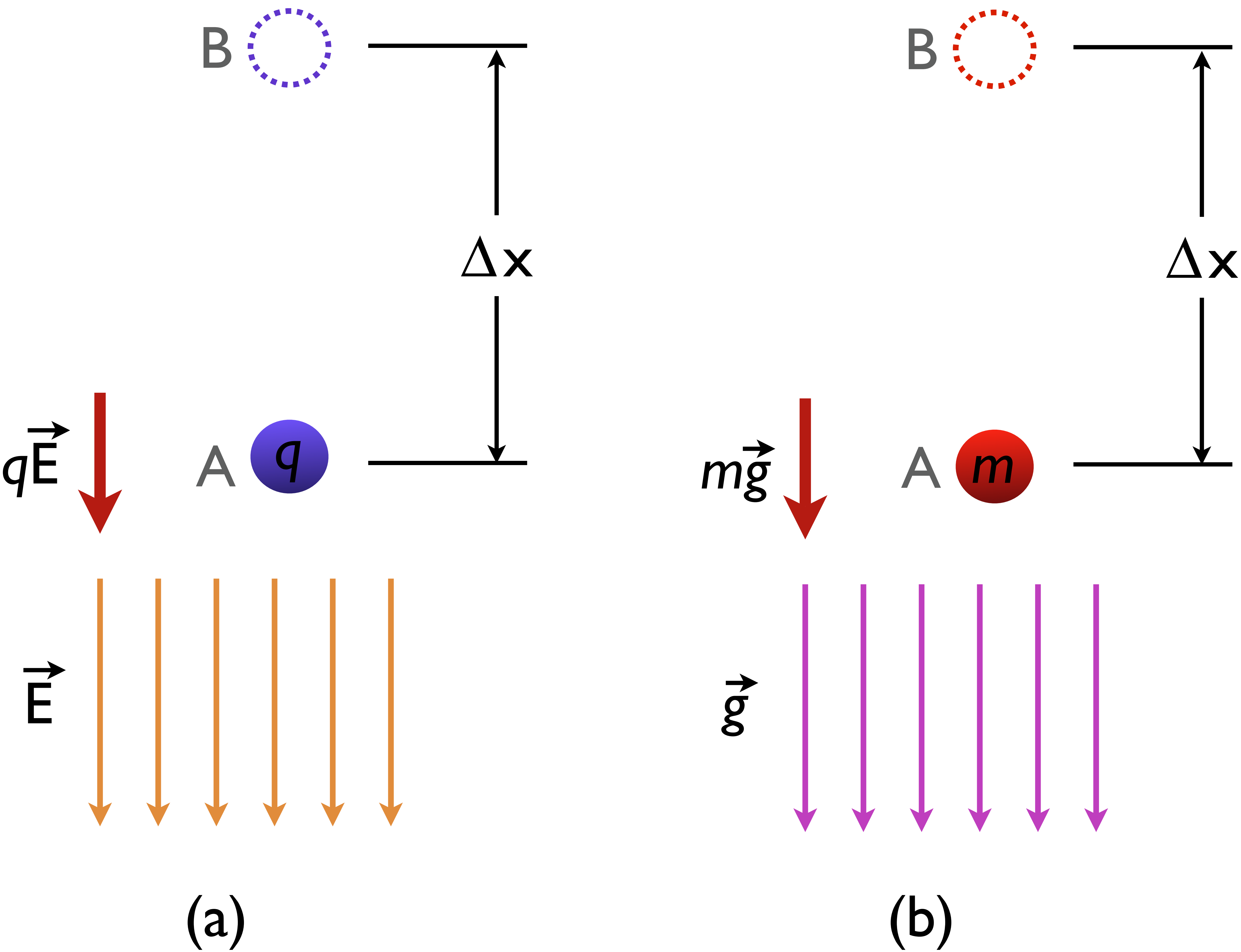

(a)
(b)


(a)
Q_3
Q_2
$E = 0$
Q_1
R_1
R_2
(b)
1
2
3

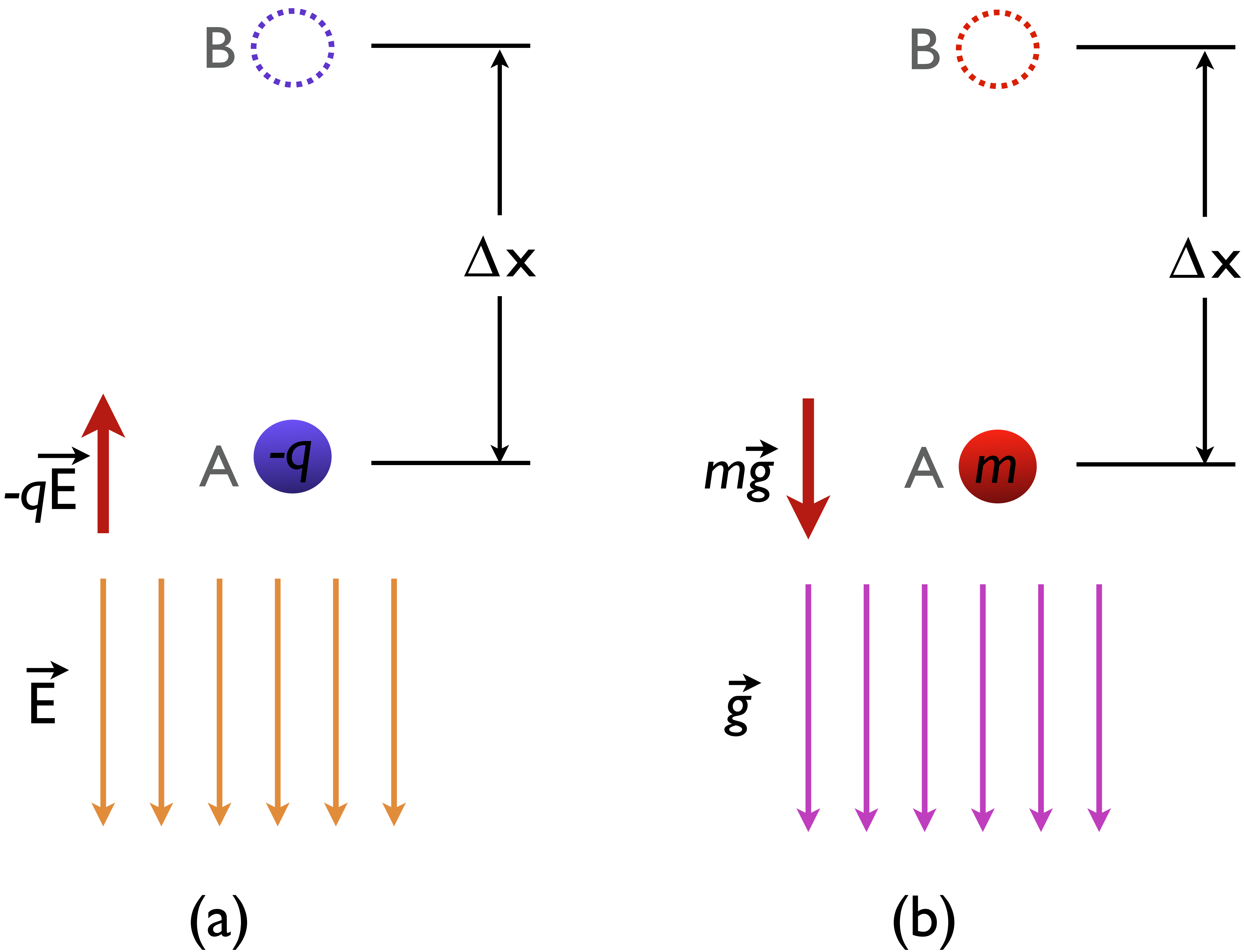

(a)
(b)
E



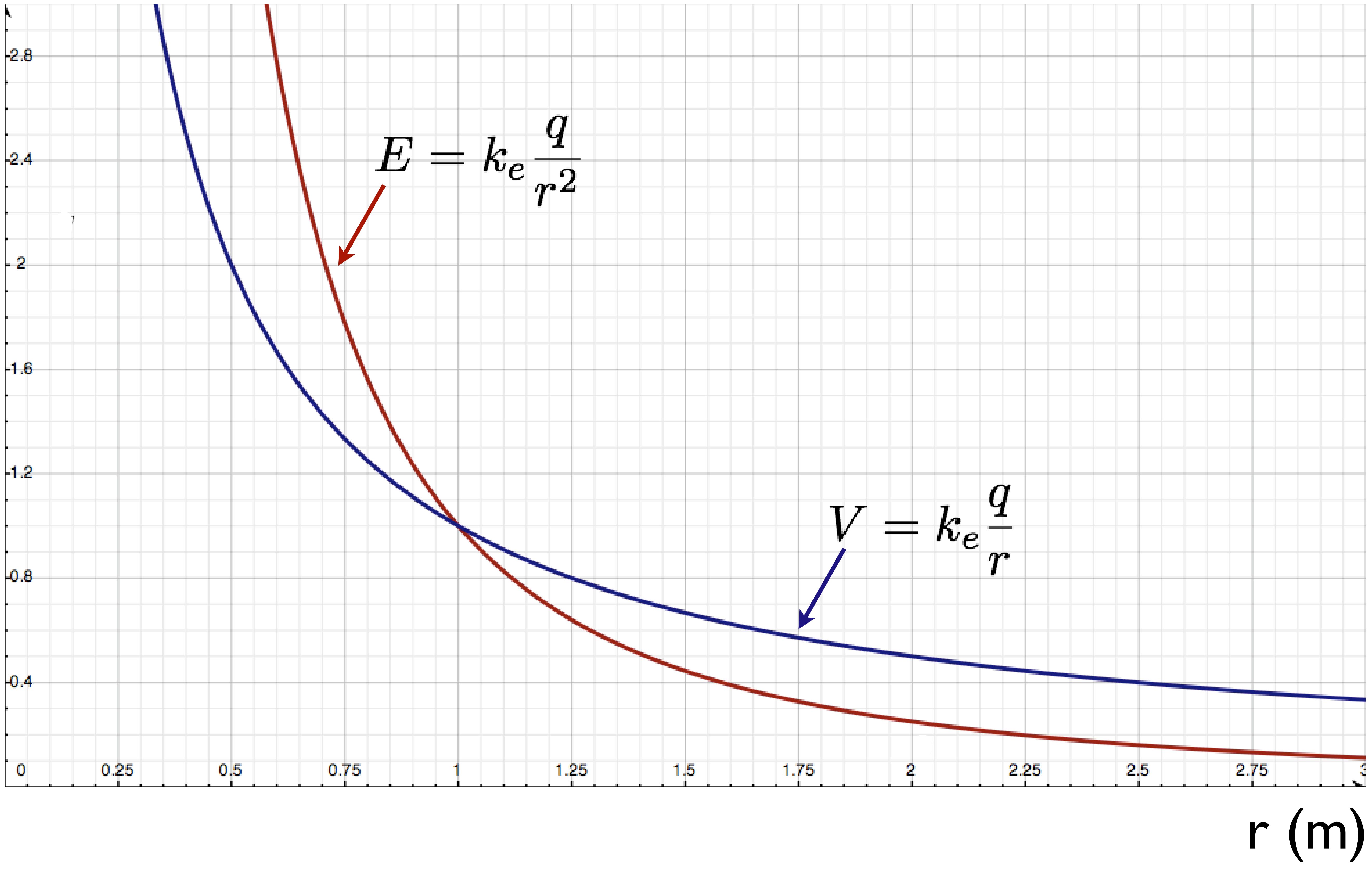
High PE
Low PE
E
A
Δx
B
x
0
x_i
x_f
q
qE
Δx = x_f - x_i



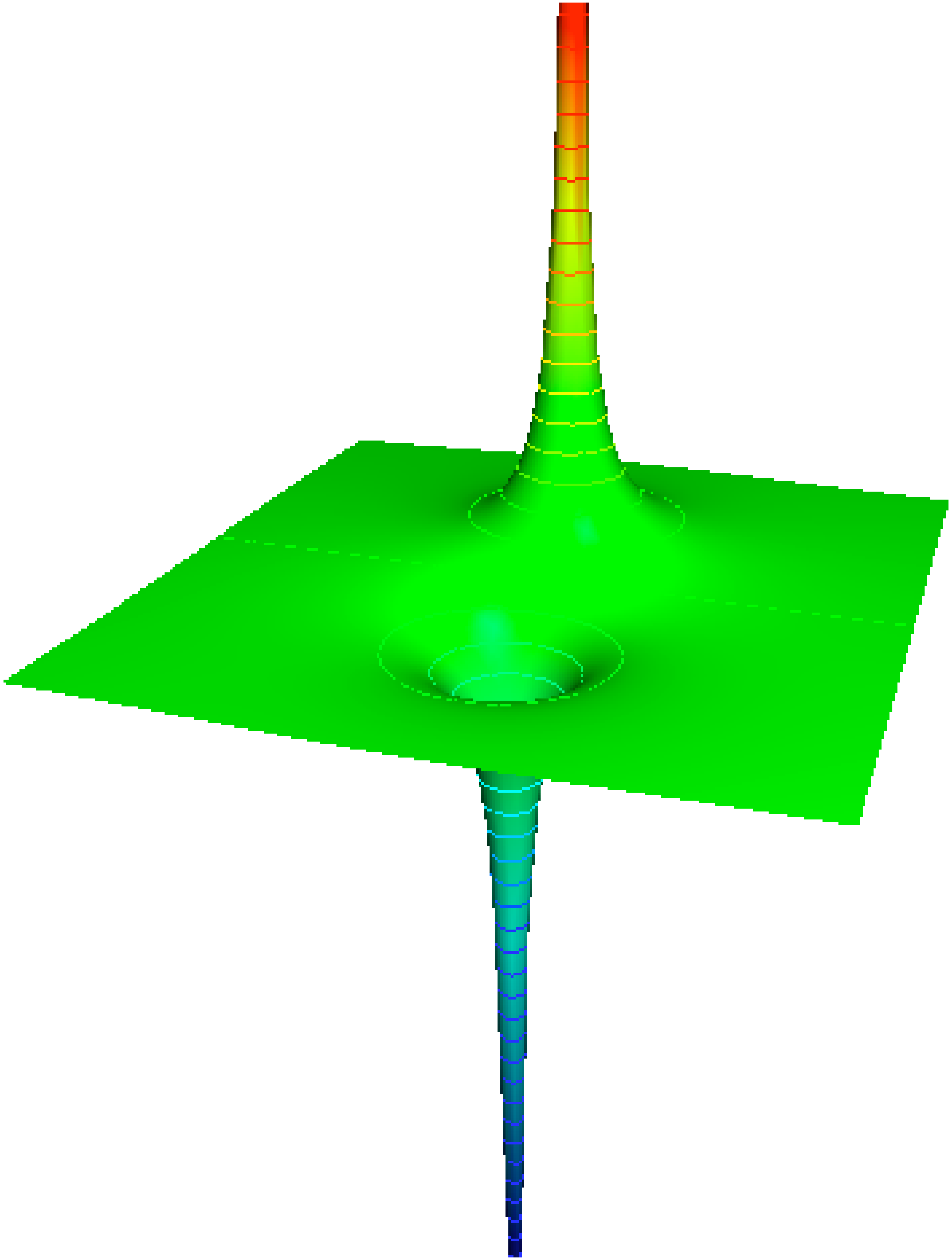
B
Δx
A
q
qE
E
(a)
B
Δx
A
m
mg
g
(b)



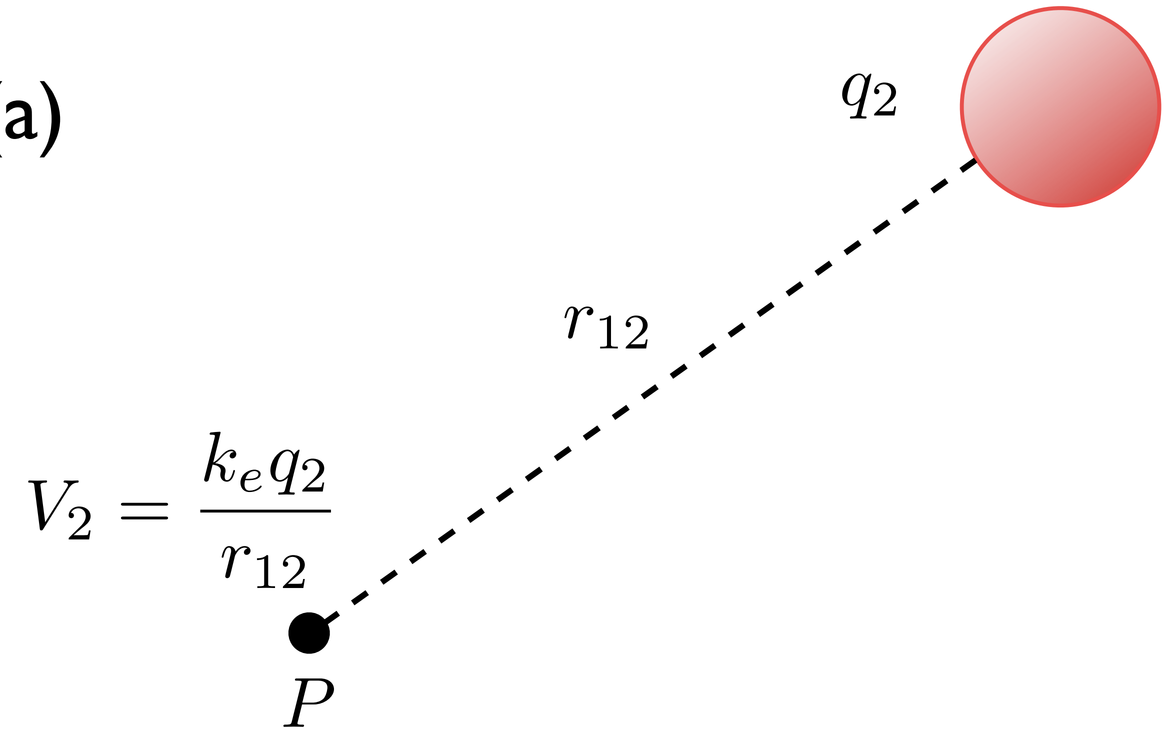
B
A
$-q$
$-q\vec{E}$
Δx
$\vec{E}$
(a)
B
A
m
$m\vec{g}$
Δx
$\vec{g}$
(b)

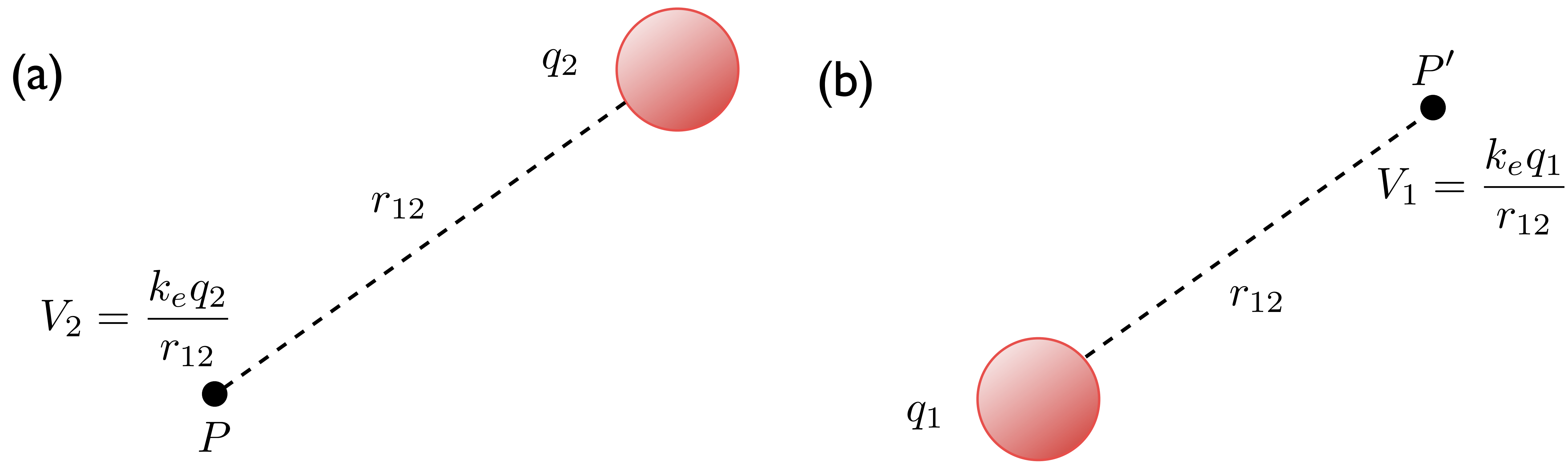


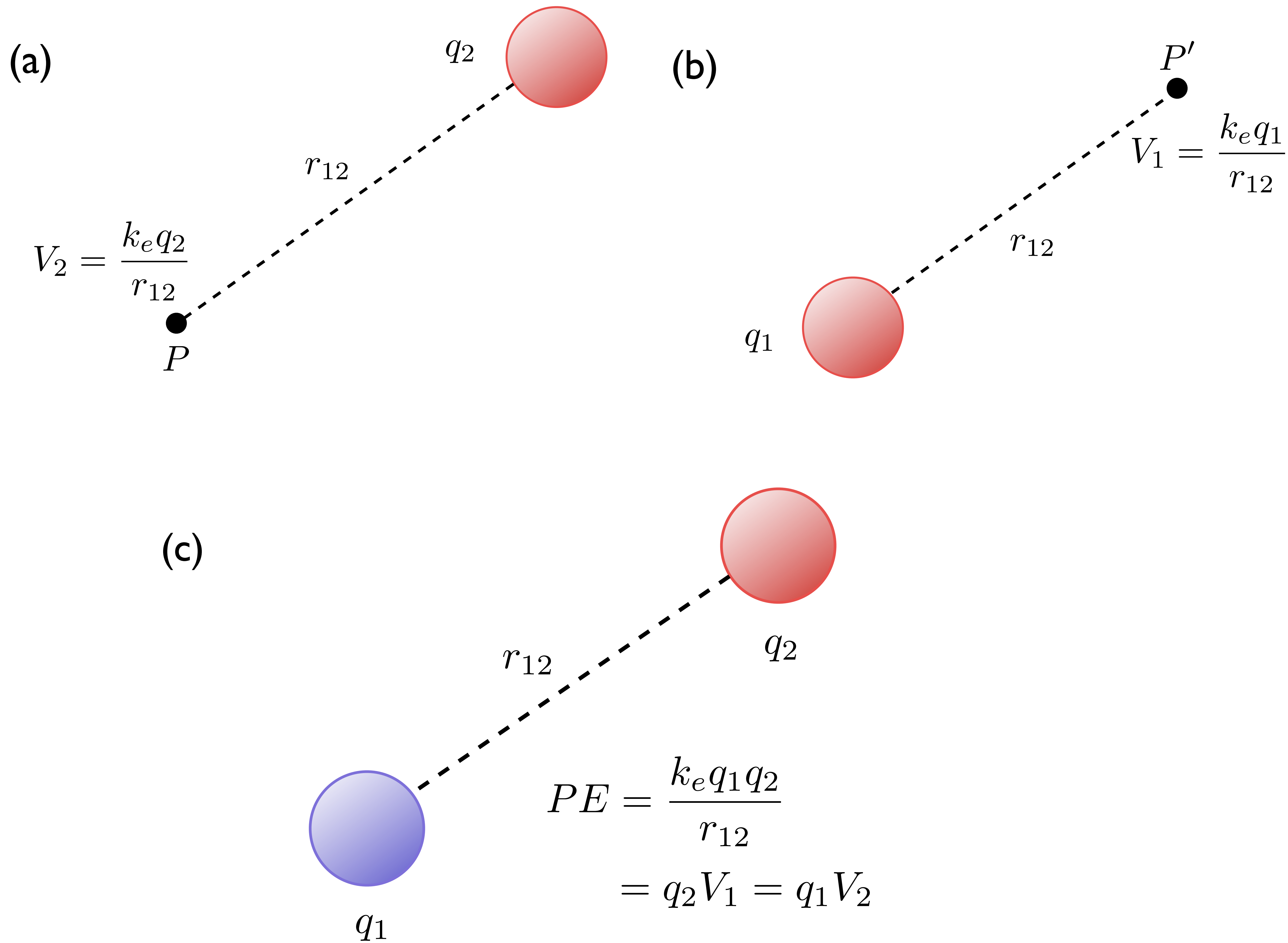
$E = k_e \frac{q}{r^2}$
$V = k_e \frac{q}{r}$
2.8
2.4
2
1.6
1.2
0.8
0.4
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3
r (m)



(a)




(a)
q_2
r_{12}
$V_2 = \frac{k_e q_2}{r_{12}}$
P
(b)
P'
$V_1 = \frac{k_e q_1}{r_{12}}$
r_{12}
q_1



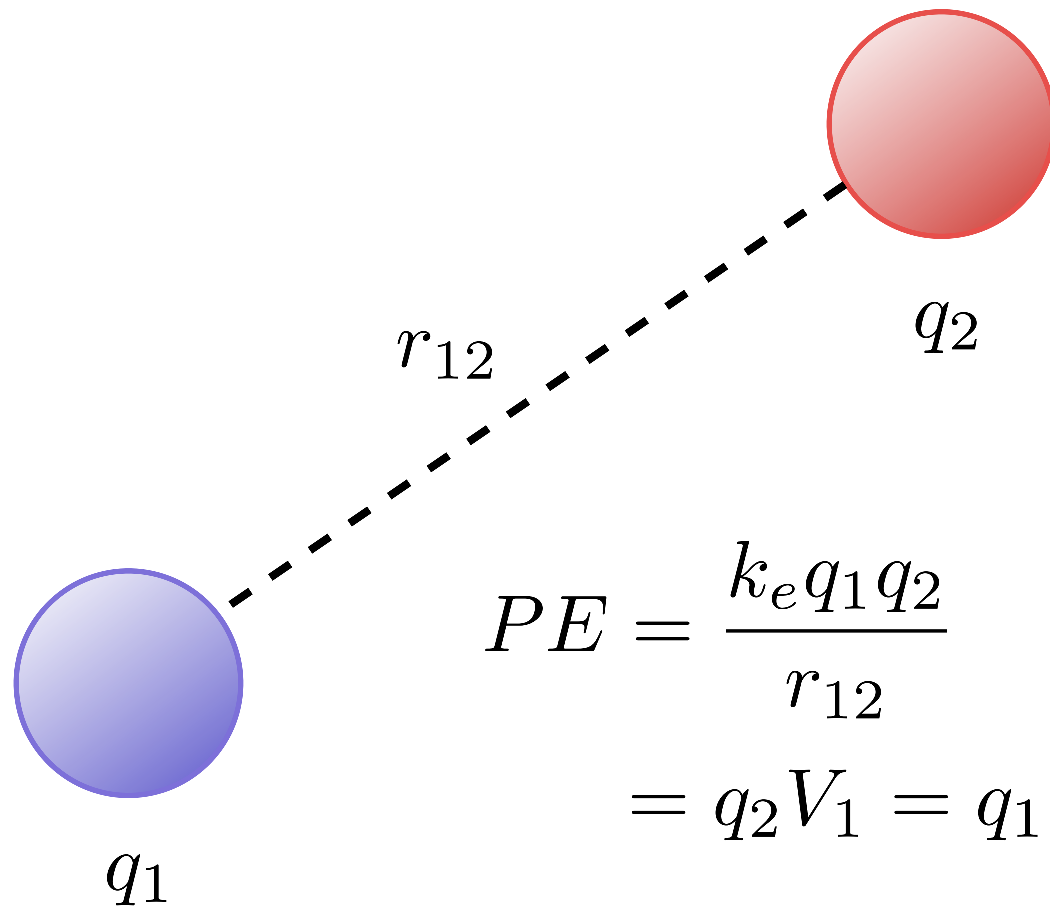
(a)
q_2
r_{12}
$V_2 = \frac{k_e q_2}{r_{12}}$
P
(b)
P'
$V_1 = \frac{k_e q_1}{r_{12}}$
r_{12}
q_1
(c)
r_{12}
q_2
q_1
$PE = \frac{k_e q_1 q_2}{r_{12}}$
$= q_2 V_1 = q_1 V_2$

PE = (1 due to 2) + (2 due to 1)

(E to bring 1 close to 2)

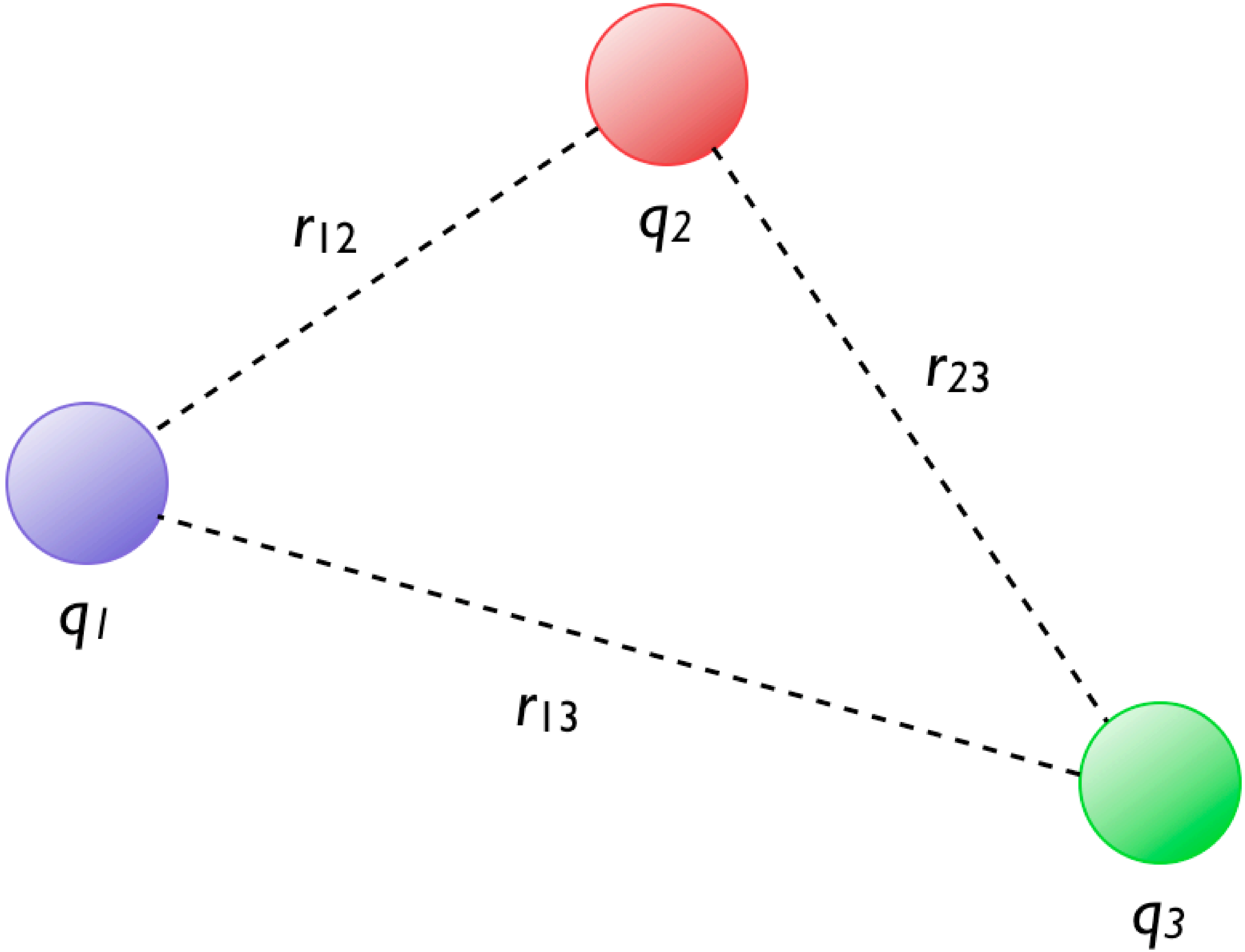
(E to bring 2 close to 1)

(c)



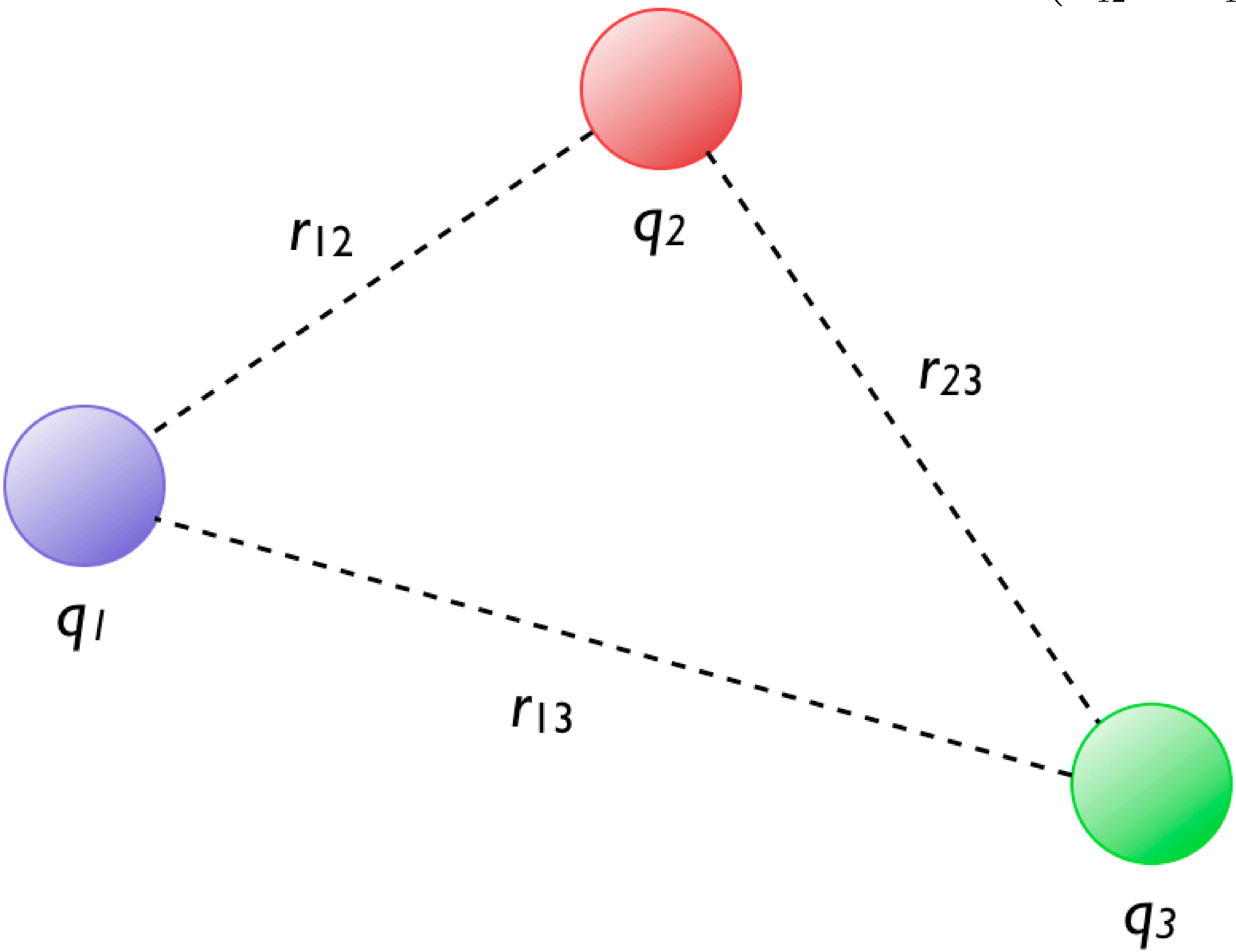
$$PE = \frac{k_e q_1 q_2}{r_{12}}$$
$$= q_2 V_1 = q_1 V_2$$

PE = (1 due to 2) + (2 due to 1)
(E to bring 1 close to 2)
(E to bring 2 close to 1)

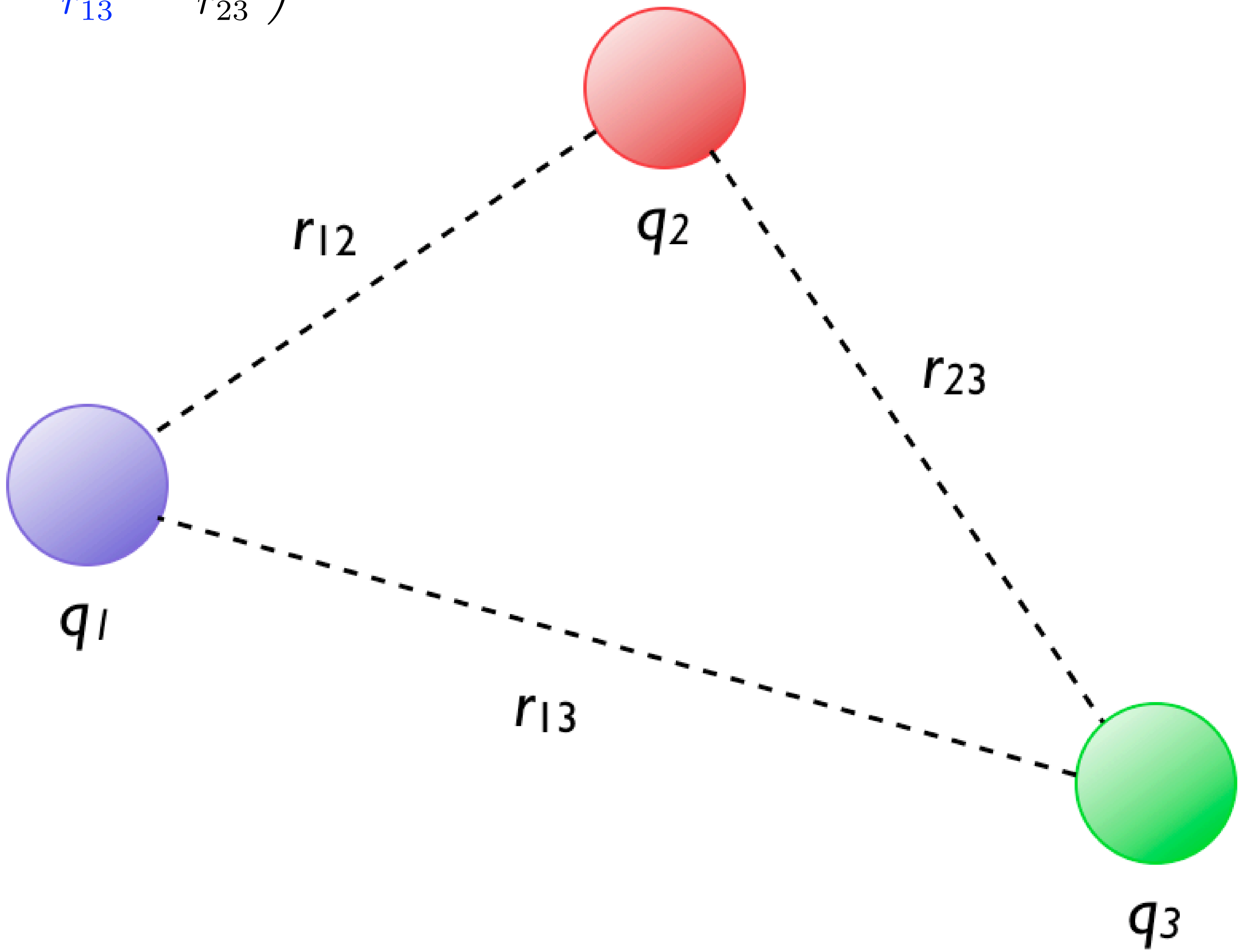


r12
q2
r23
q1
r13
q3

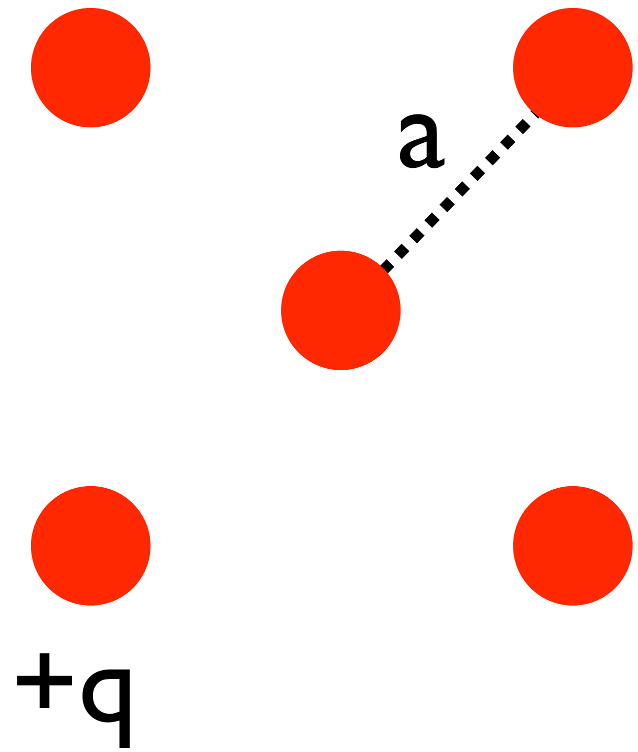
$$PE = PE_{1\&2} + PE_{2\&3} + PE_{1\&3} = PE_{2\&1} + PE_{3\&2} + PE_{3\&1} = k_e\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

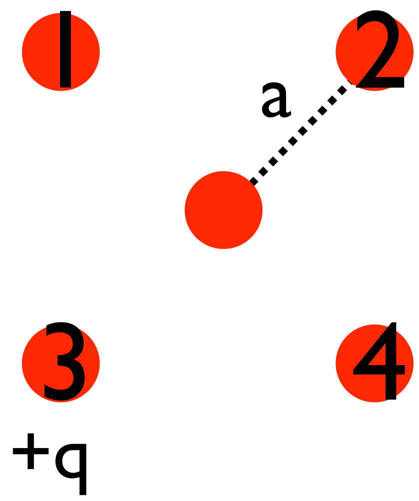


$$
\begin{aligned}
PE &= \frac{1}{2}\sum_{i=1}^{3}\sum_{\substack{j=1\\ j\neq i}}^{3}\frac{k_e q_i q_j}{r_i j} \\
&= \frac{1}{2}\left(\frac{k_e q_2 q_1}{r_{21}} + \frac{k_e q_3 q_1}{r_{31}} + \frac{k_e q_1 q_2}{r_{12}} + \frac{k_e q_3 q_2}{r_{32}} + \frac{k_e q_1 q_3}{r_{13}} + \frac{k_e q_2 q_3}{r_{23}}\right) \\
&= k_e\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)
\end{aligned}
$$



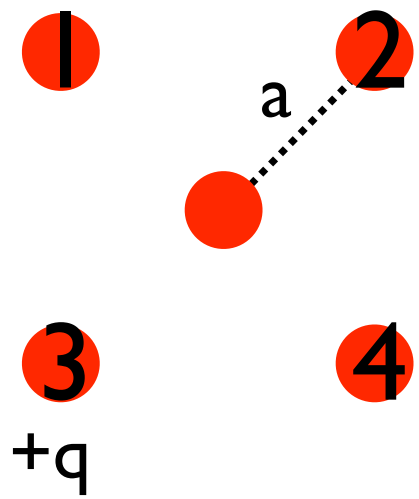
what is the potential energy of the “crystal”





we just have to sum the energy of all unique pairs of charges.

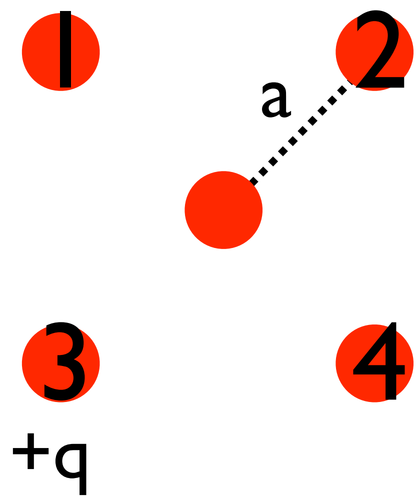
so how many are there?



we just have to sum the energy of all unique pairs of charges.

so how many are there?

$$\text{ways of choosing pairs from five charges} = \binom{5}{2} = {}^{5}C_{2} = \frac{5!}{2!\,(5-2)!} = \frac{5\cdot4\cdot3\cdot2\cdot1}{2\cdot1\cdot3\cdot2\cdot1} = 10$$



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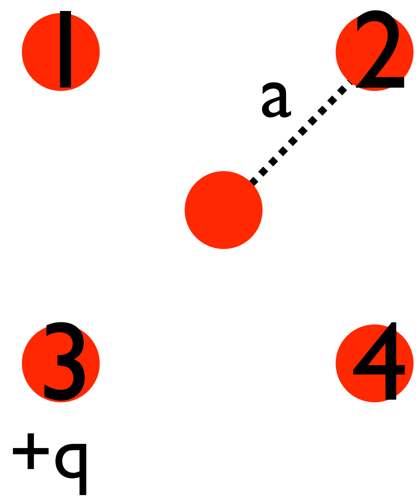
$$\begin{array}{cccc} (1,2) & (1,3) & (1,4) & (1,5) \\ (2,3) & (2,3) & (2,5) & \\ (3,4) & (3,5) & & \\ (4,5) & & & \end{array}$$



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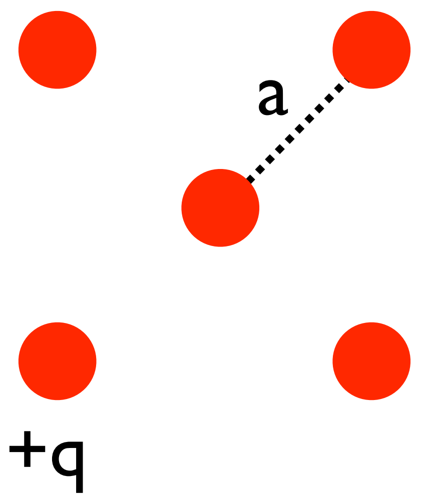
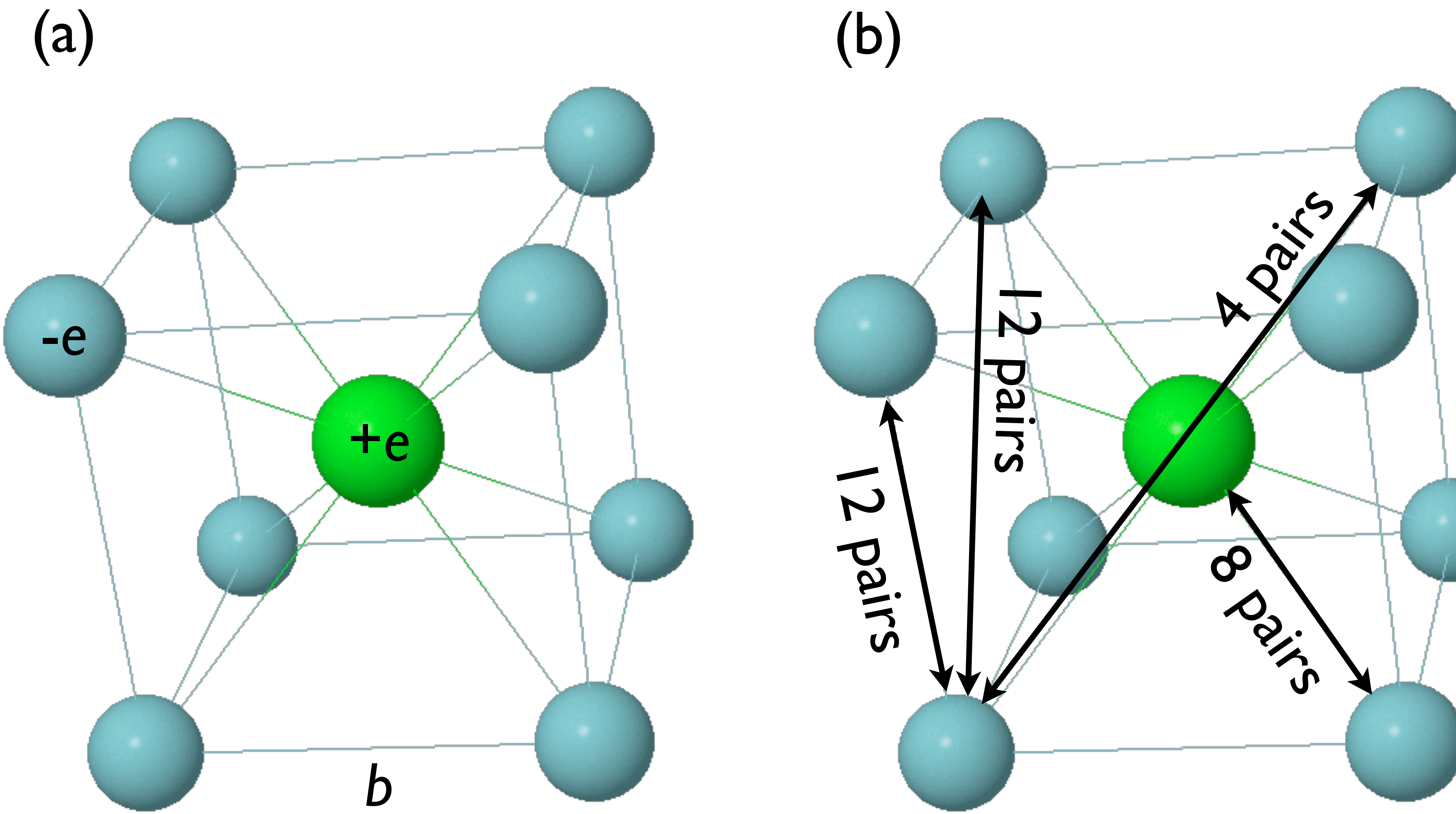
$$\begin{array}{llll} (1,2) & (1,3) & (1,4) & (1,5) \\ (2,3) & (2,3) & (2,5) & \\ (3,4) & (3,5) & & \\ (4,5) & & & \end{array}$$

#, pairing type	**separation**				**pairs**
4, center-corner	a	$(1,5)$	$(2,5)$	$(3,5)$	$(4,5)$
4, adjacent corners	$a\sqrt{2}$	$(1,4)$	$(3,4)$	$(2,3)$	$(1,2)$
2, far corner	$2a$			$(1,3)$	$(2,4)$

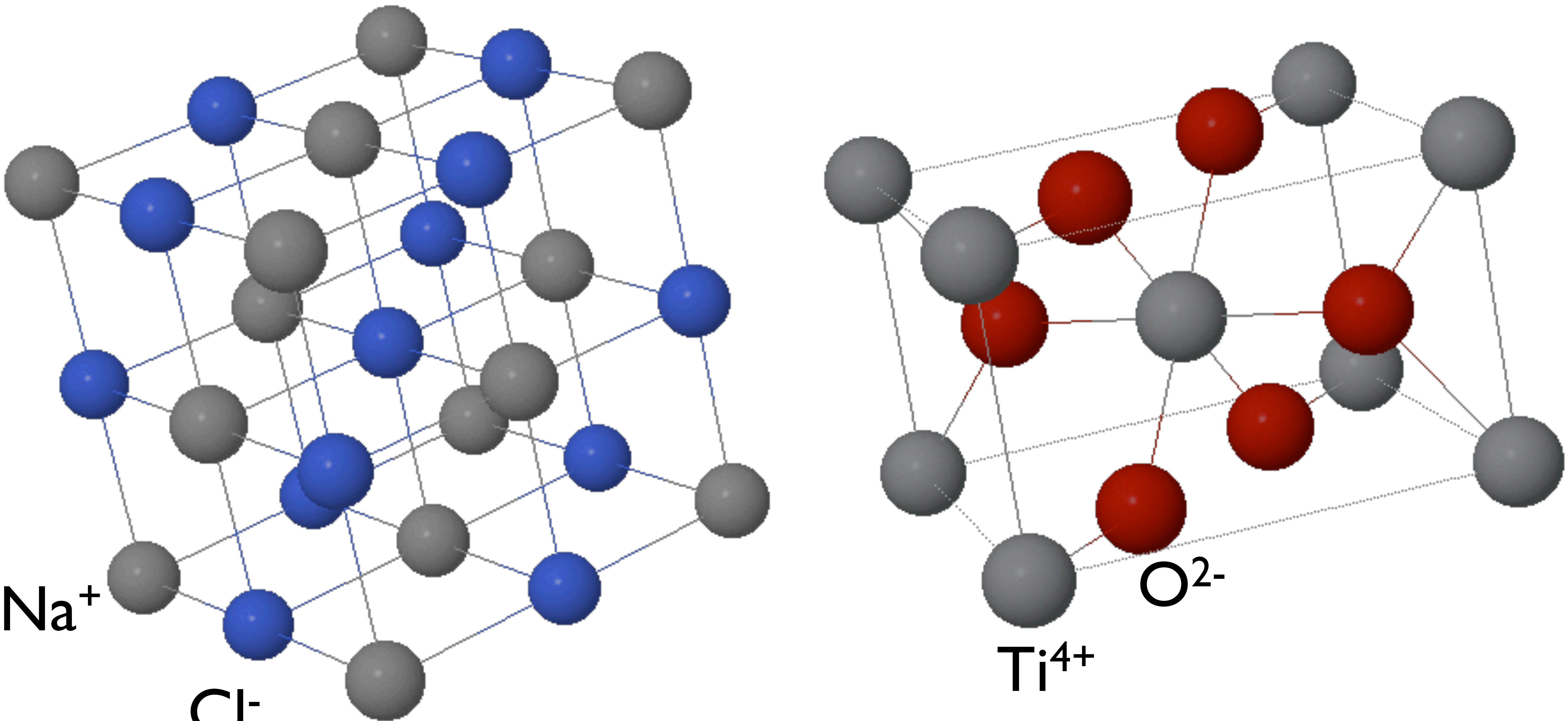


$$
\begin{aligned}
PE_{\text{square}} &= 4\,(\text{energy of center-corner pair}) + 2\,(\text{energy of far corner pair}) + 4\,(\text{energy of adjacent corner pair}) \\
&= 4\left[\frac{k_e q^2}{a}\right] + 2\left[\frac{k_e q^2}{2a}\right] + 4\left[\frac{k_e q^2}{a\sqrt{2}}\right] \\
&= \frac{k_e q^2}{a}\left[4 + 1 + \frac{4}{\sqrt{2}}\right] \\
&= \frac{k_e q^2}{a}\left[5 + 2\sqrt{2}\right] \approx 7.83\frac{kq^2}{a}
\end{aligned}
$$

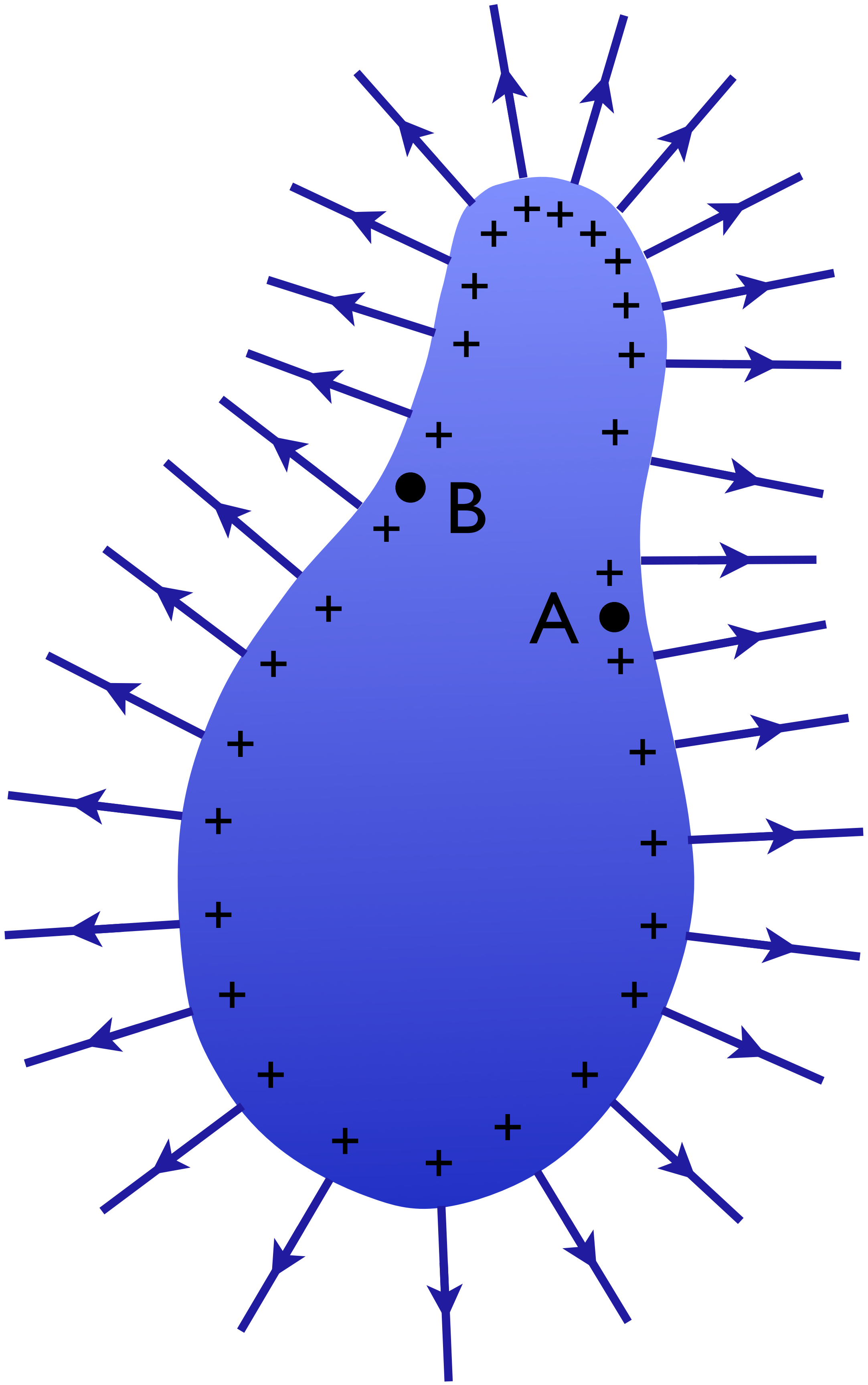
it works for more complicated stuff



(a) Rocksalt (b) Rutile



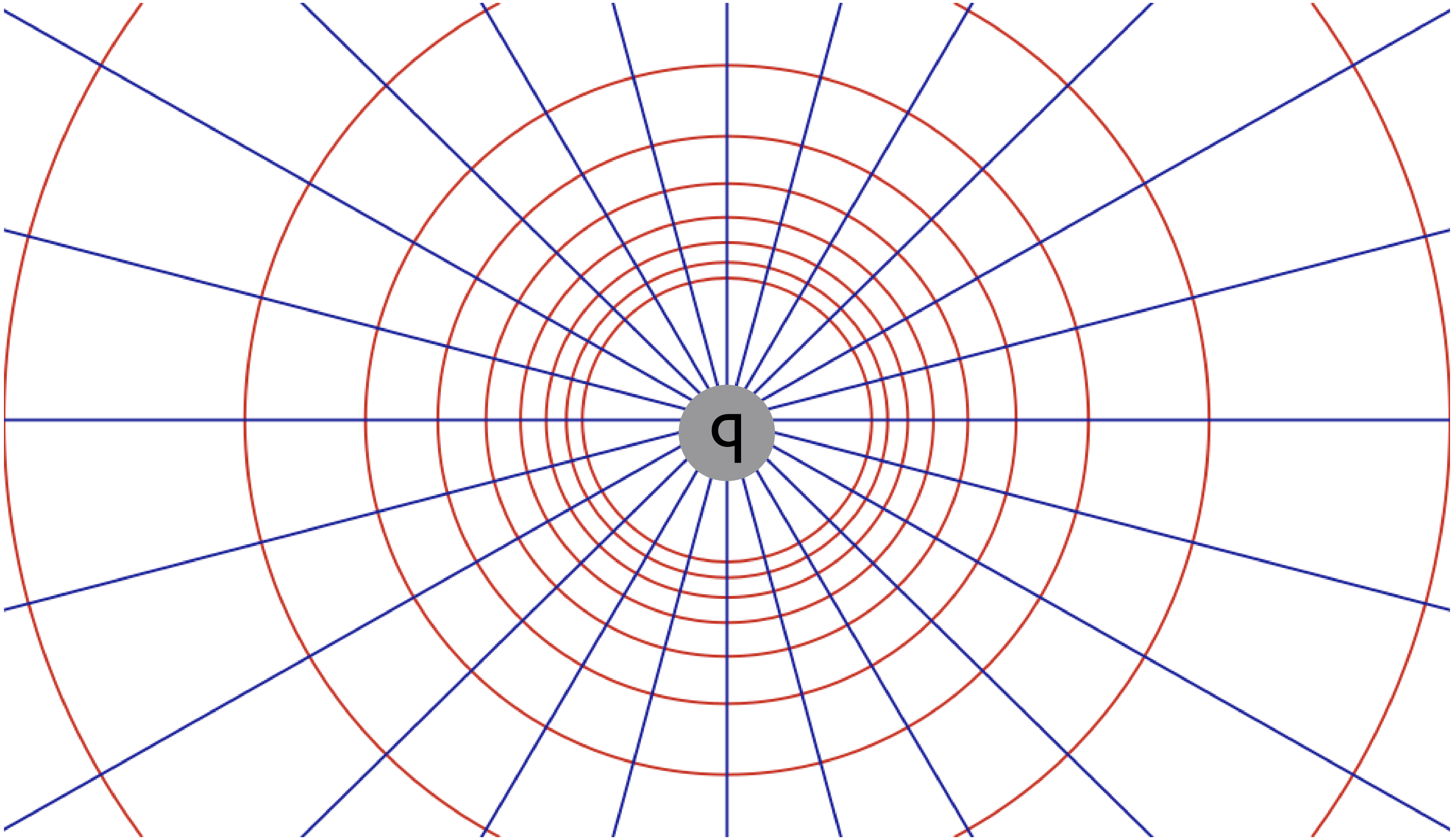
$M = -1.75$ $M = -4.82$

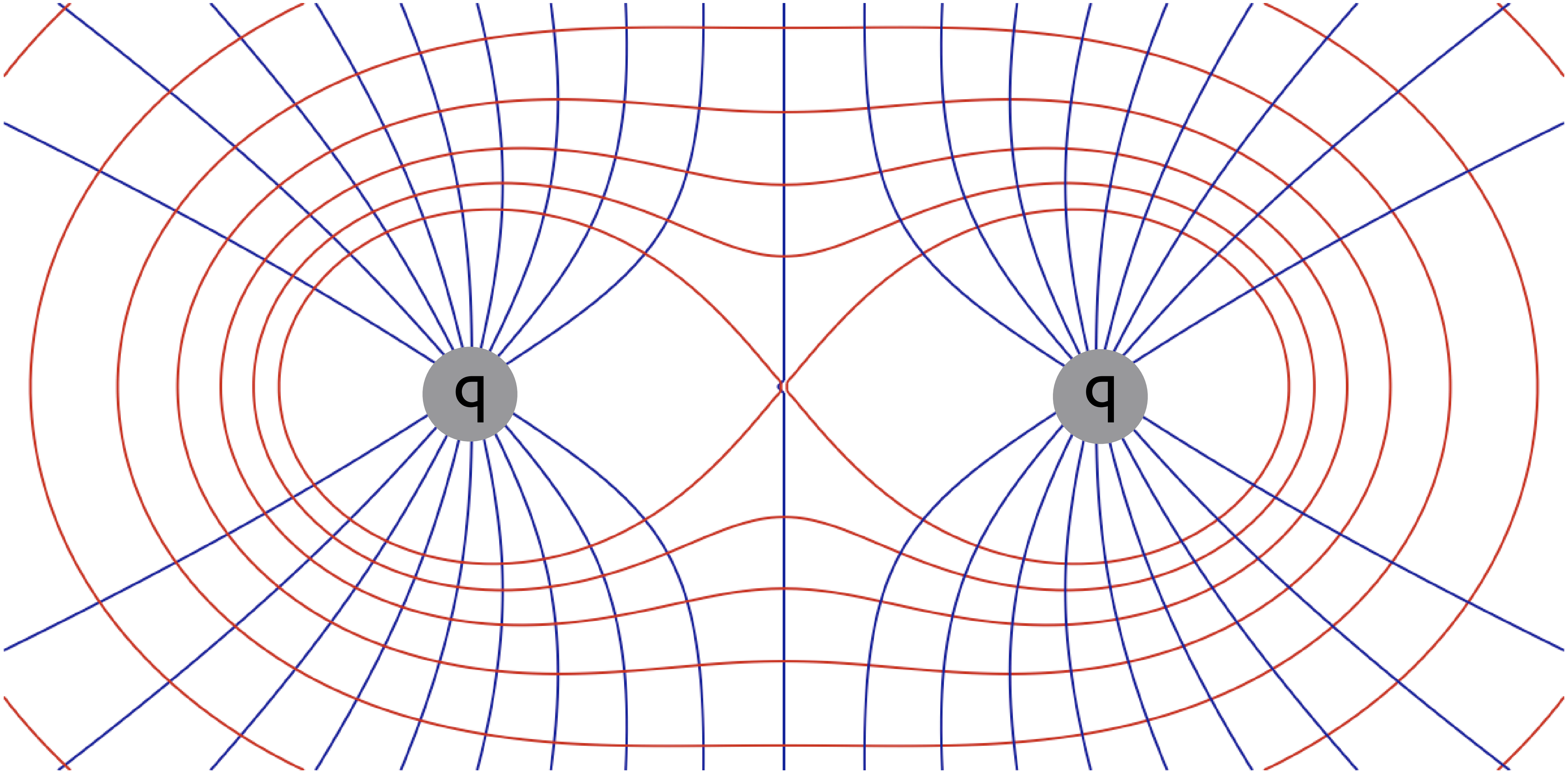


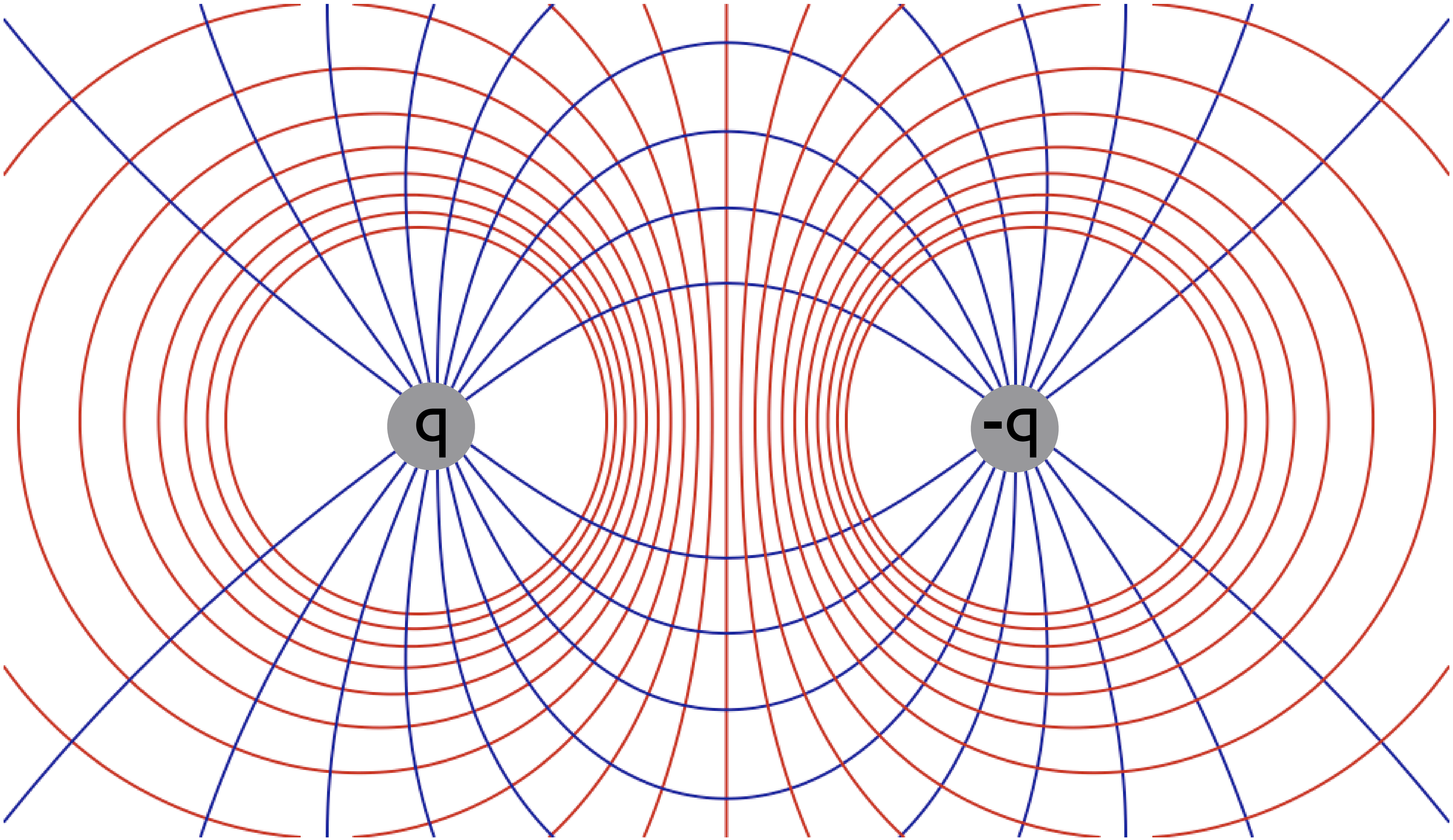
travel along surface:
E perpendicular to path everywhere

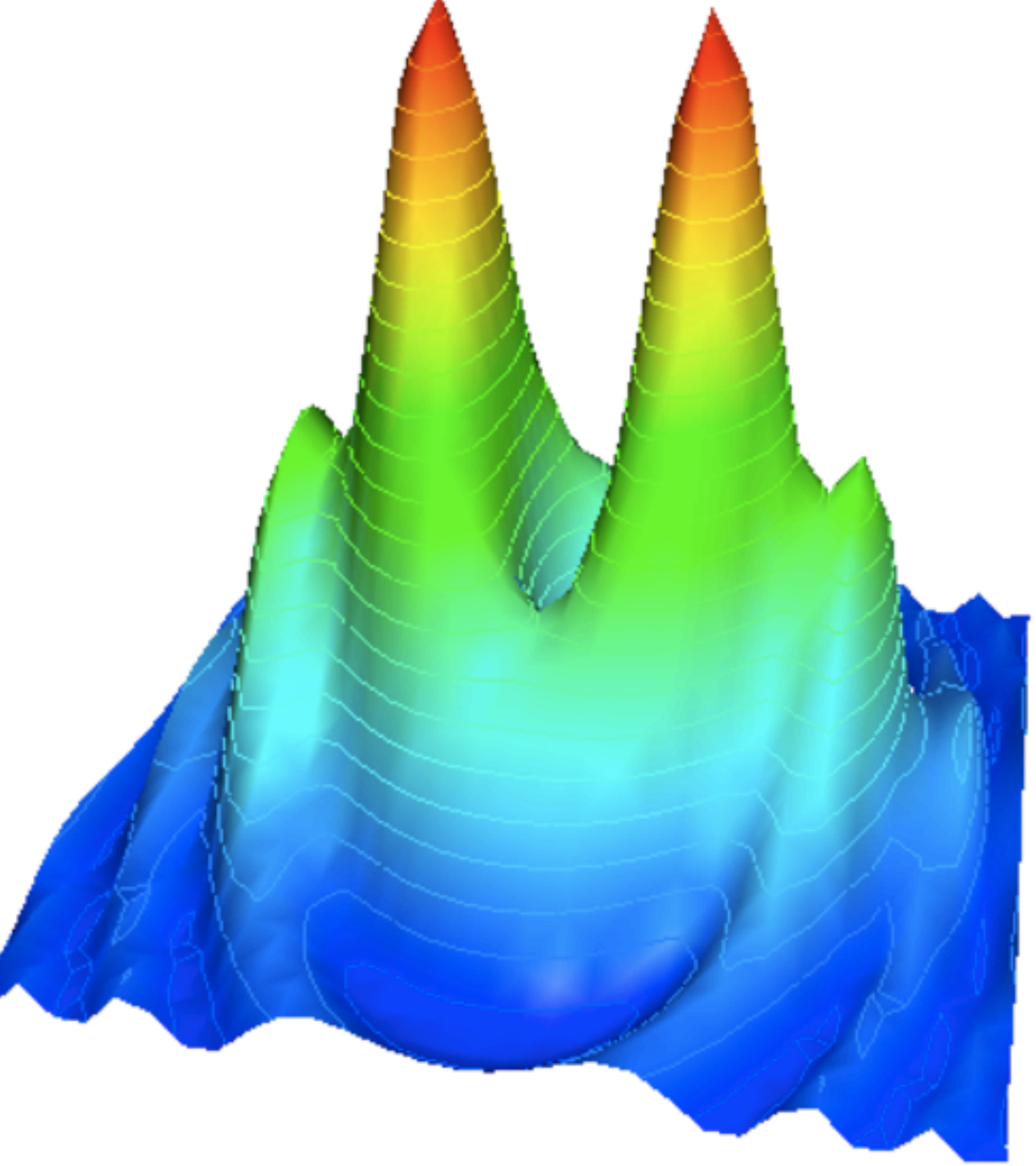
no work done!

electric force is conservative ...


q


q
q


q
-q

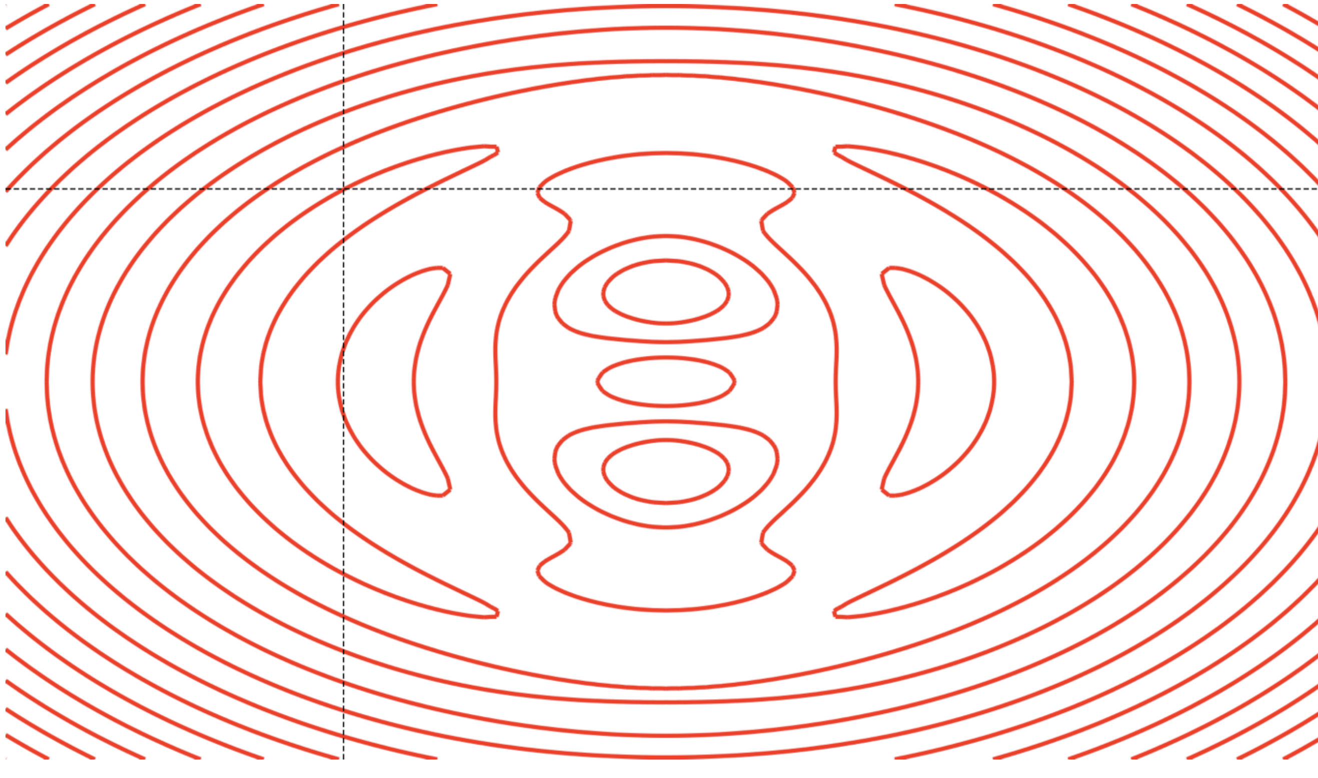


equipotential lines?
contours of constant V

no work to
move along them
(like gravity)

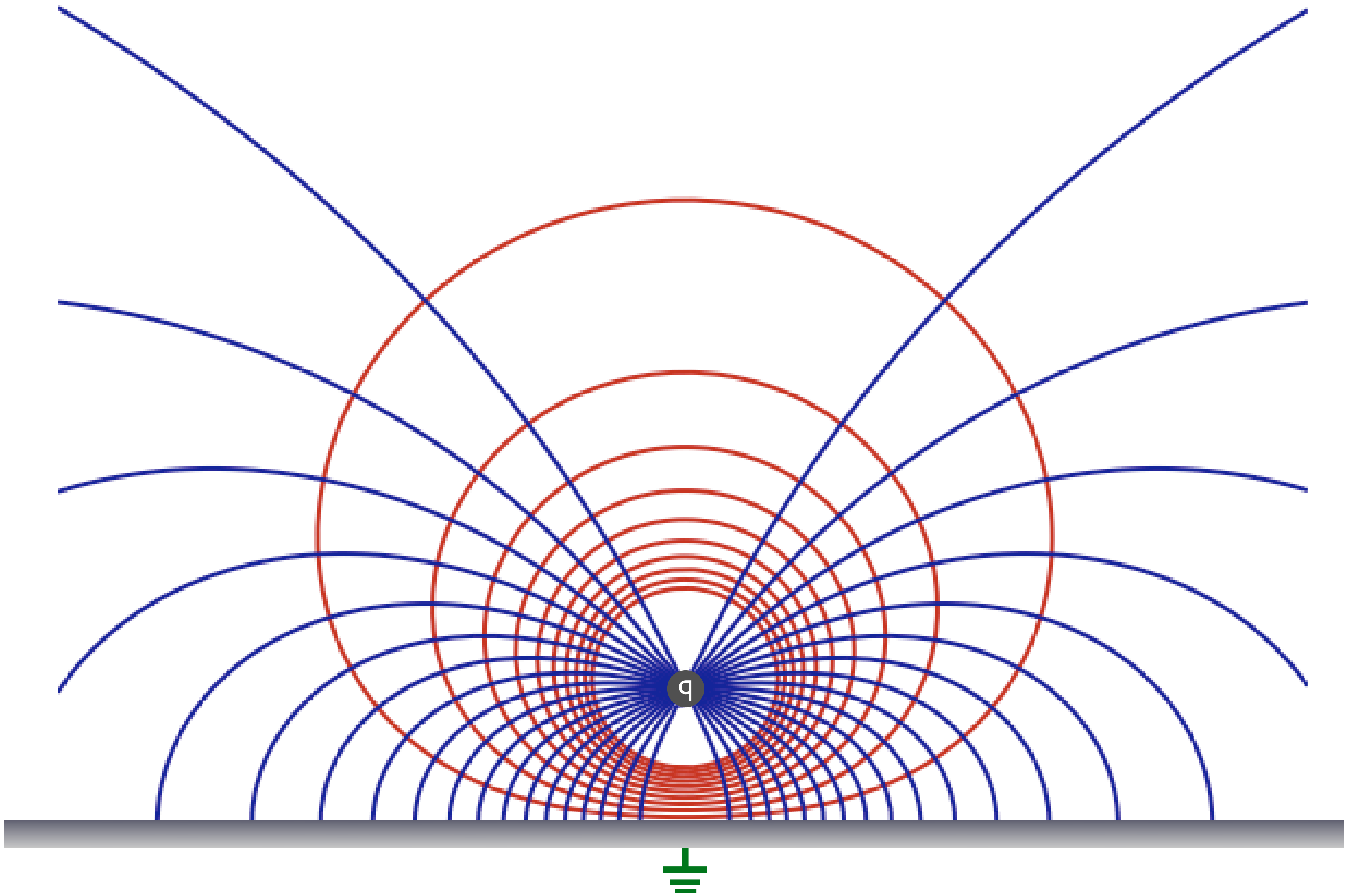
x, y = spatial coordinates
potential constant on lines

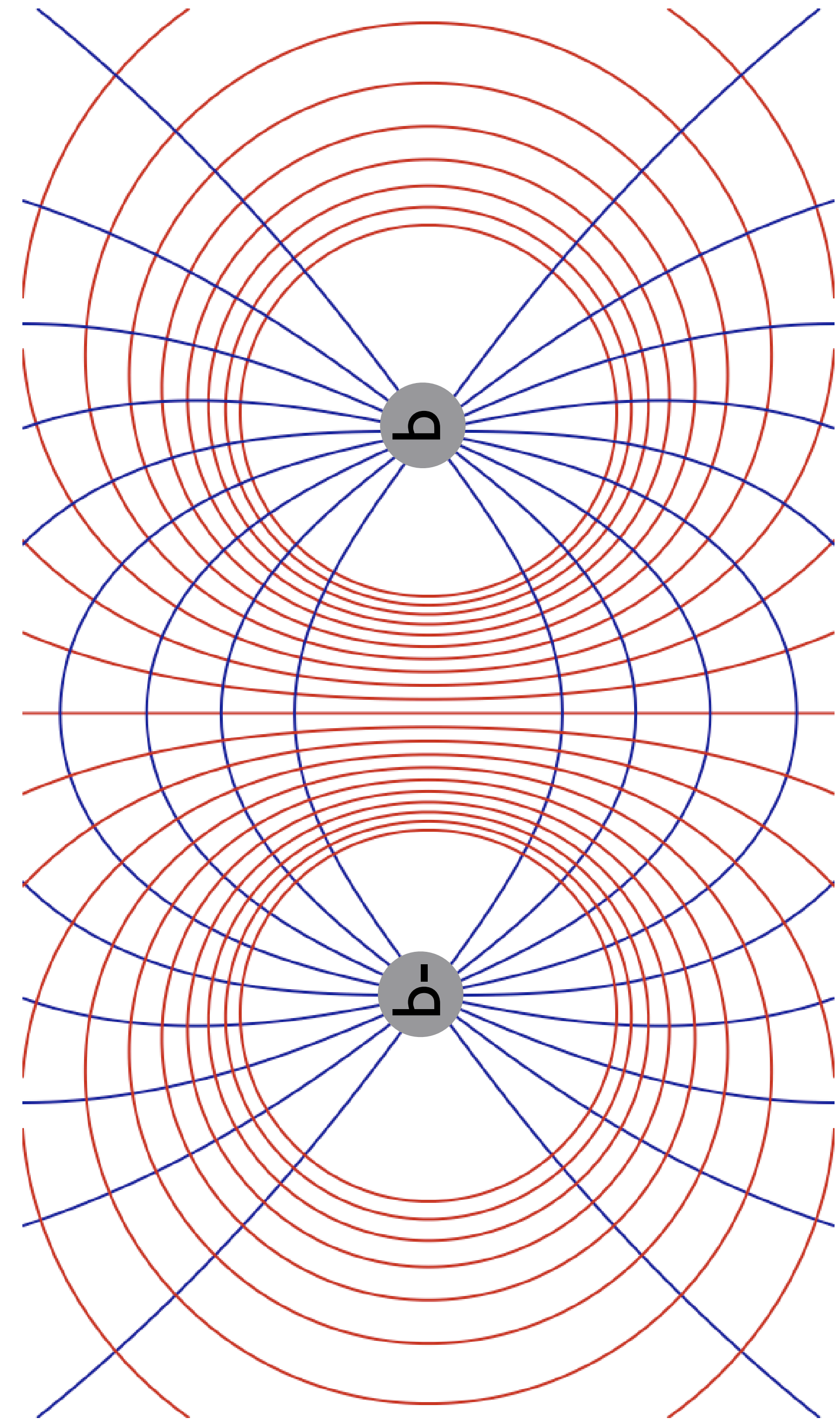
2d



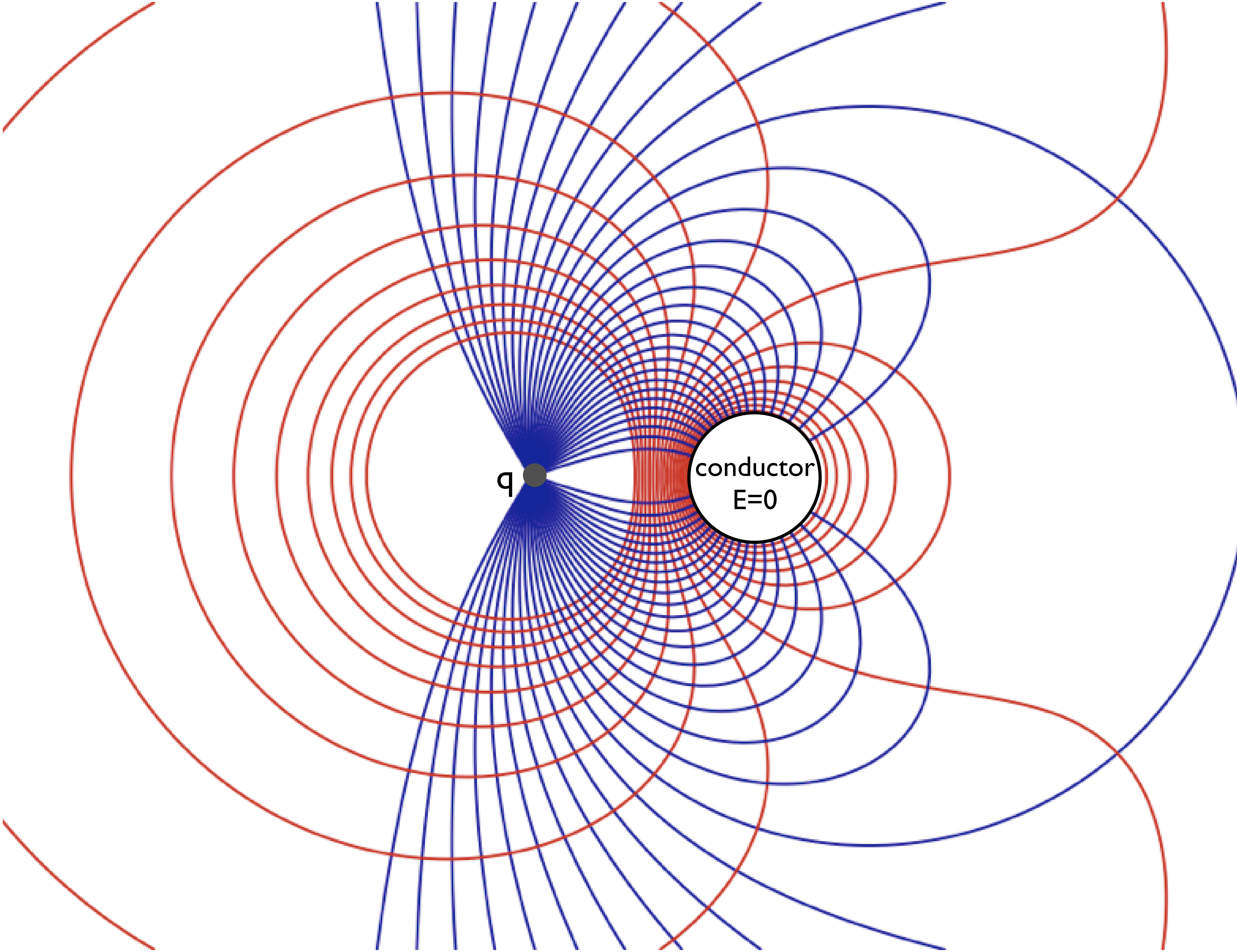
x, y = spatial coordinates
z = electric potential

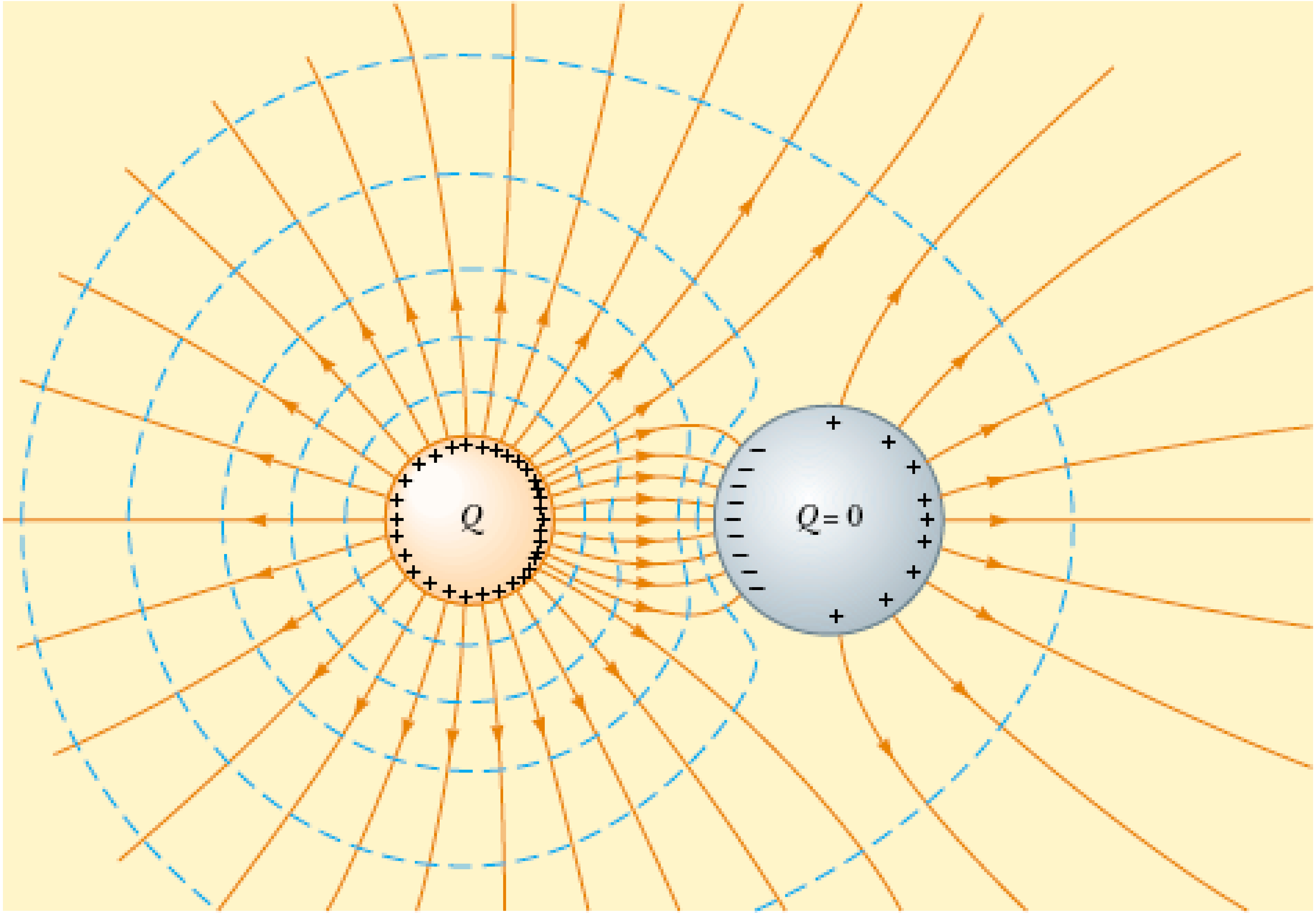
3d



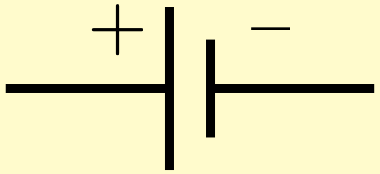
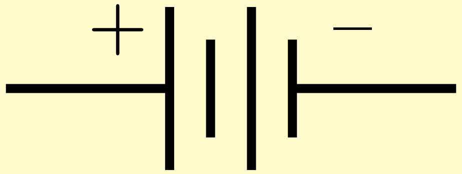


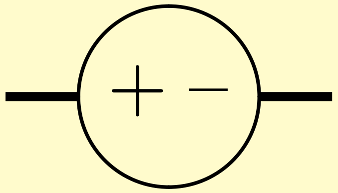
conductor = mirror for field & potential lines

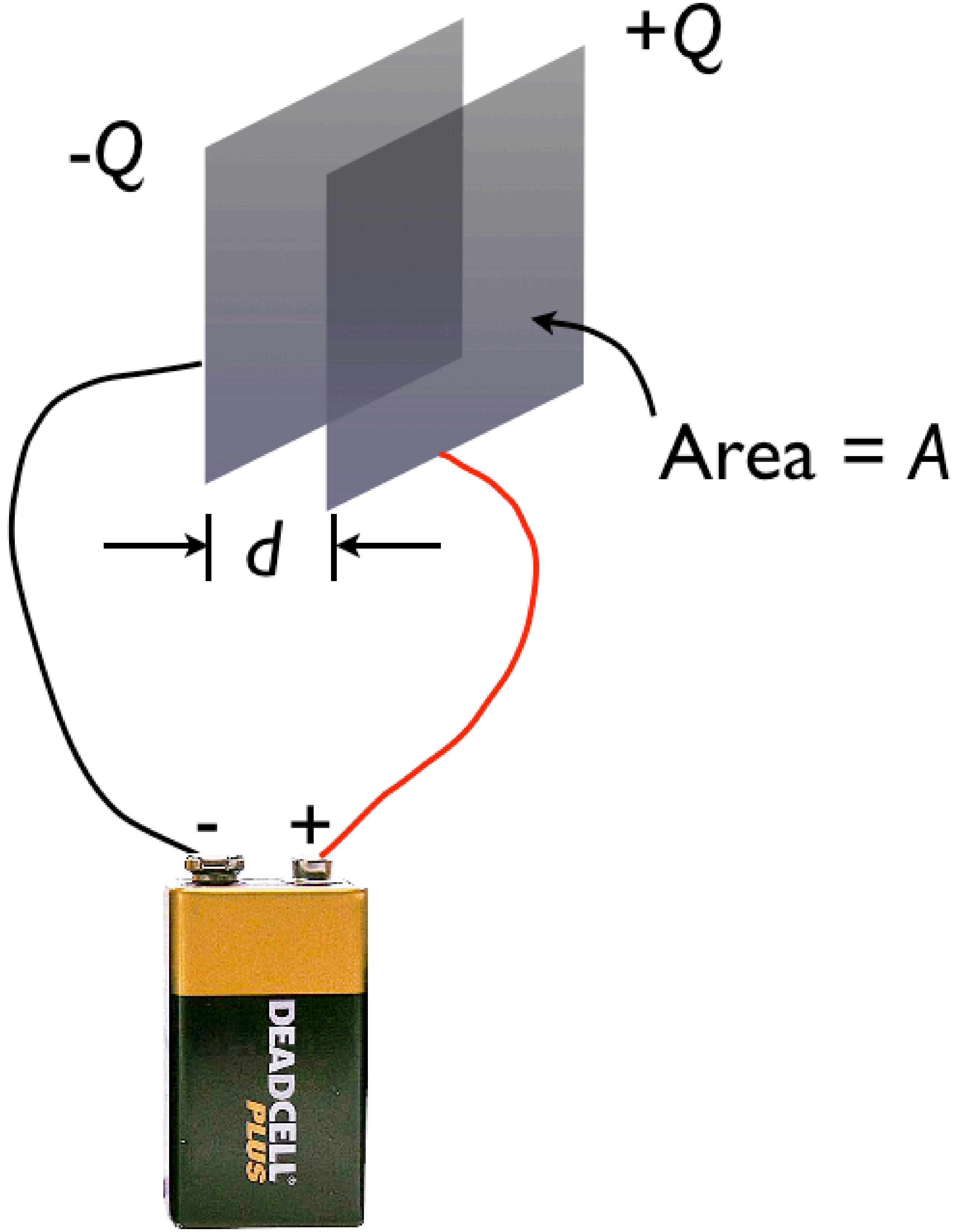

q
conductor
E=0

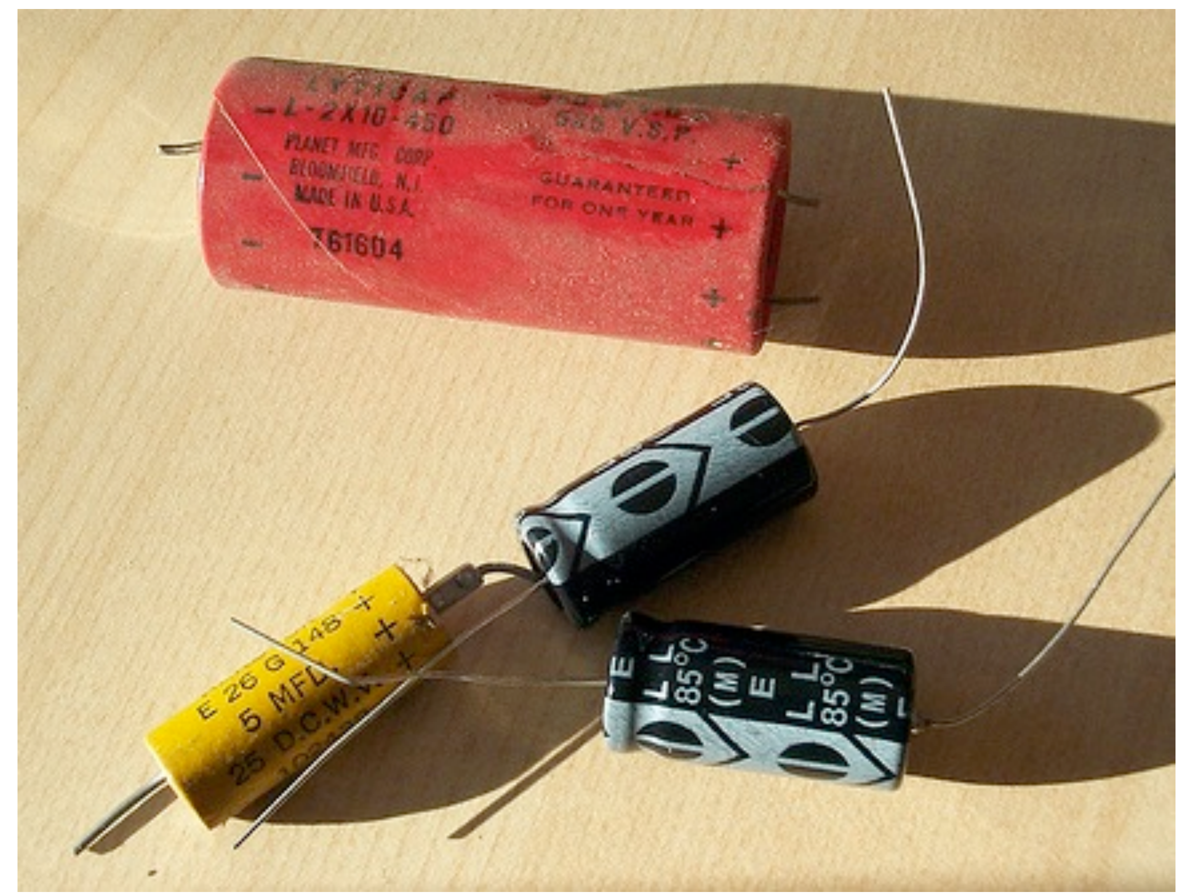

Q
Q = 0

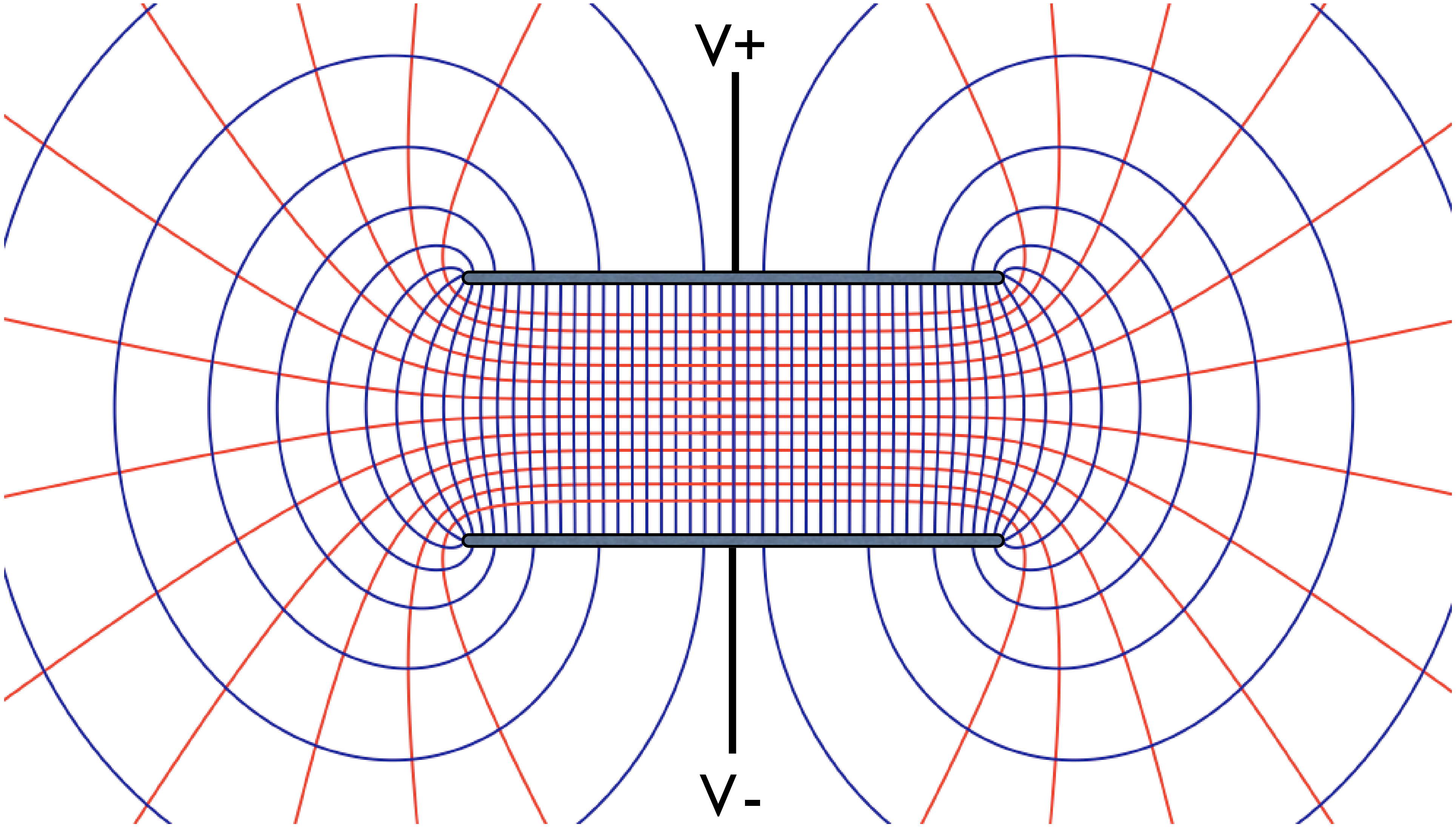
Circuit diagram symbol for voltage sources:

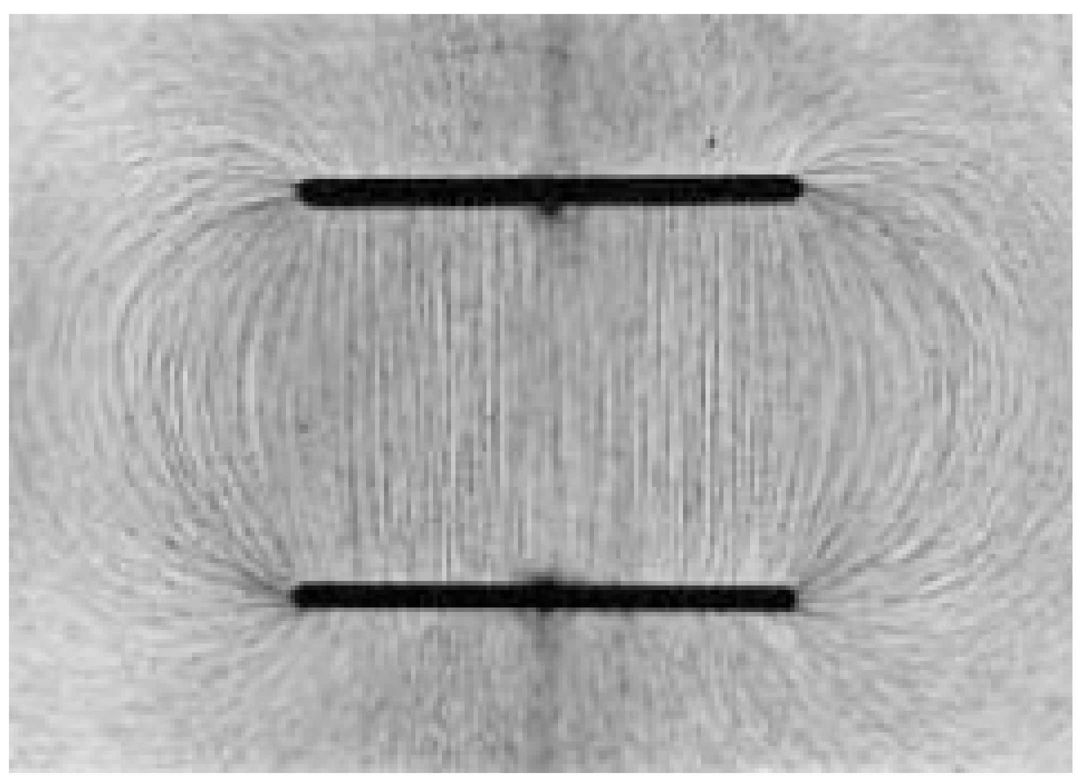
Batteries:  

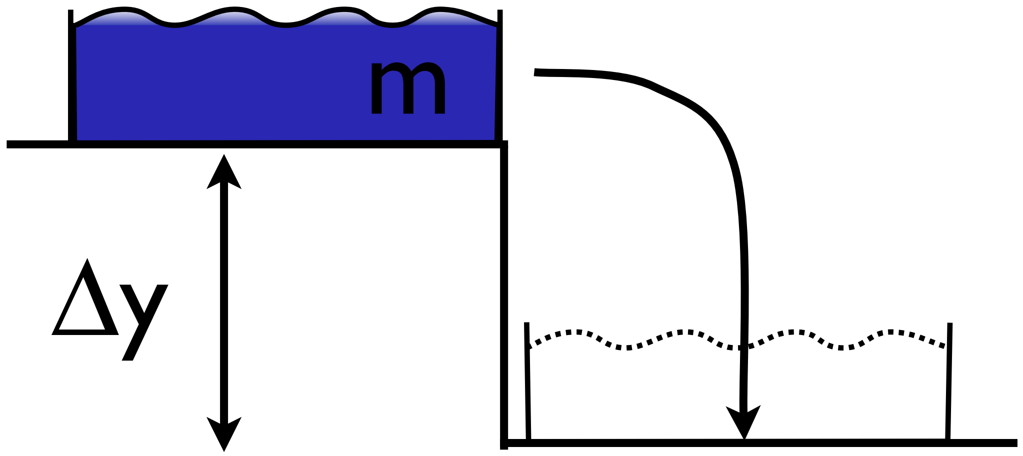
General constant voltage source: 


+Q
-Q
Area = A
d
-
+
DEADCELL
PLUS


L-2X10-450
PLANET MFG. CORP.
BLOOMFIELD, N.J.
MADE IN U.S.A.
T61604
GUARANTEED
FOR ONE YEAR
E 26 G 148
5 MFD
25 D.C.W.
85°C
(M)

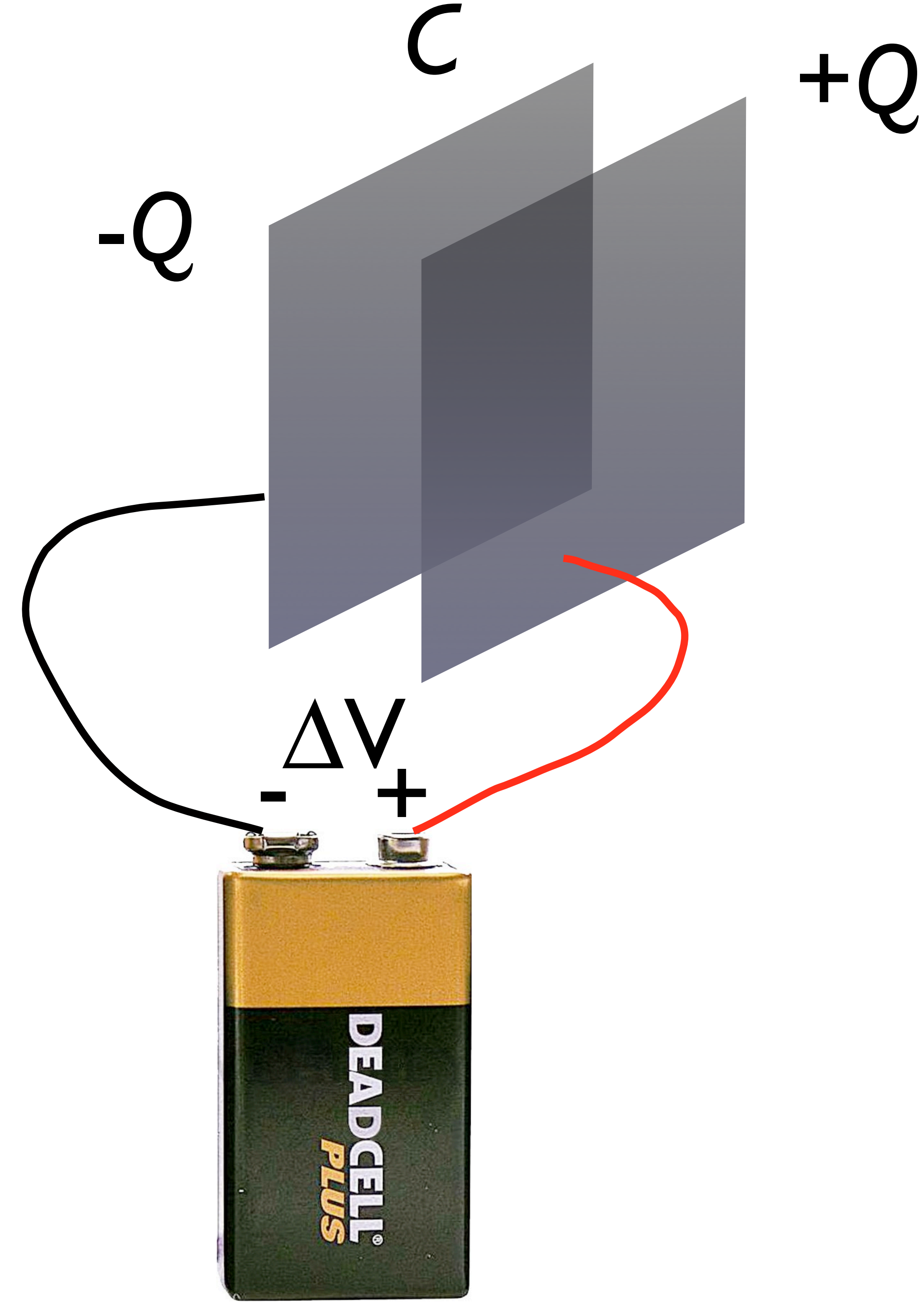

V+
V-





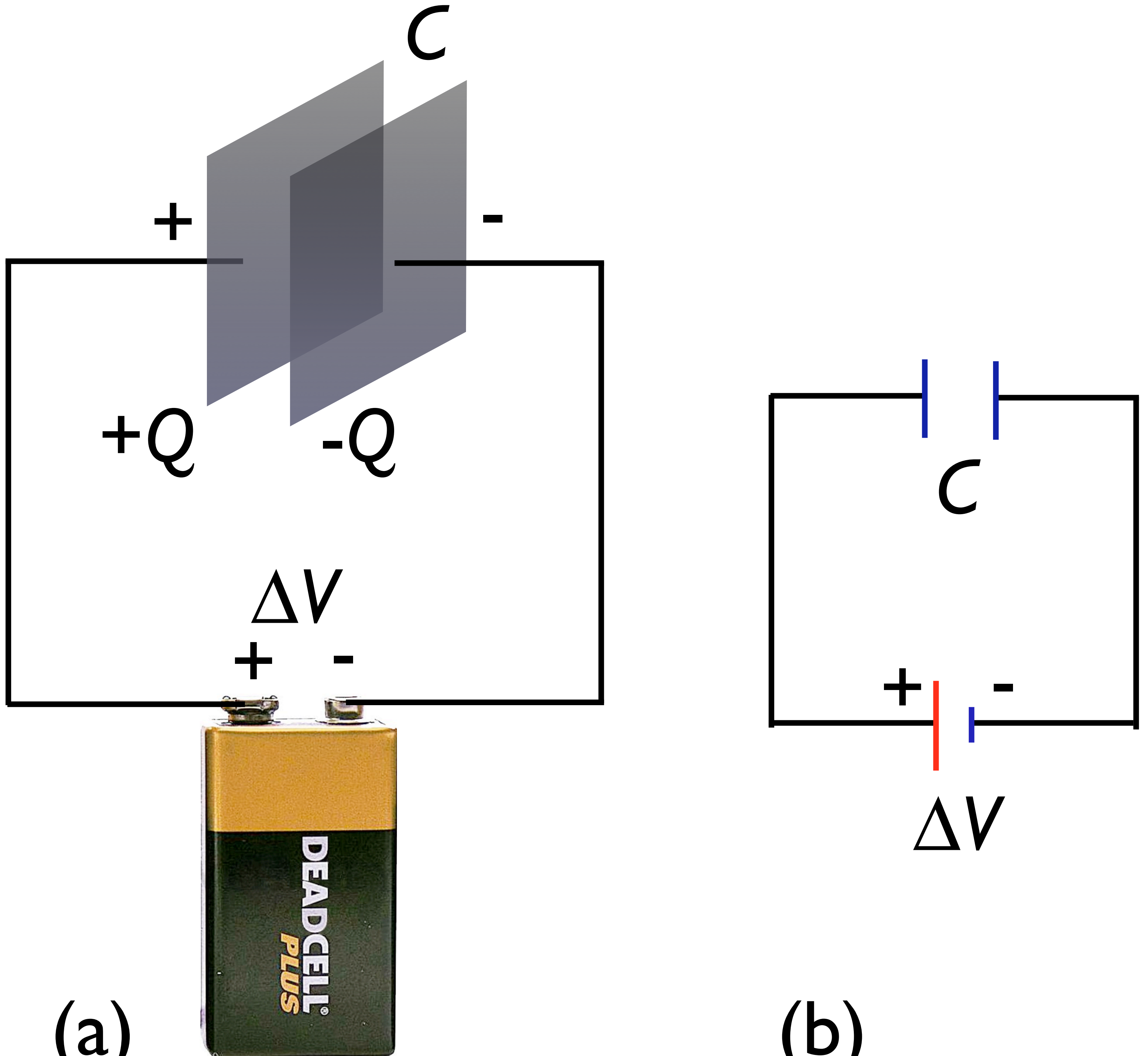
$$W = mg\Delta y$$

(a)

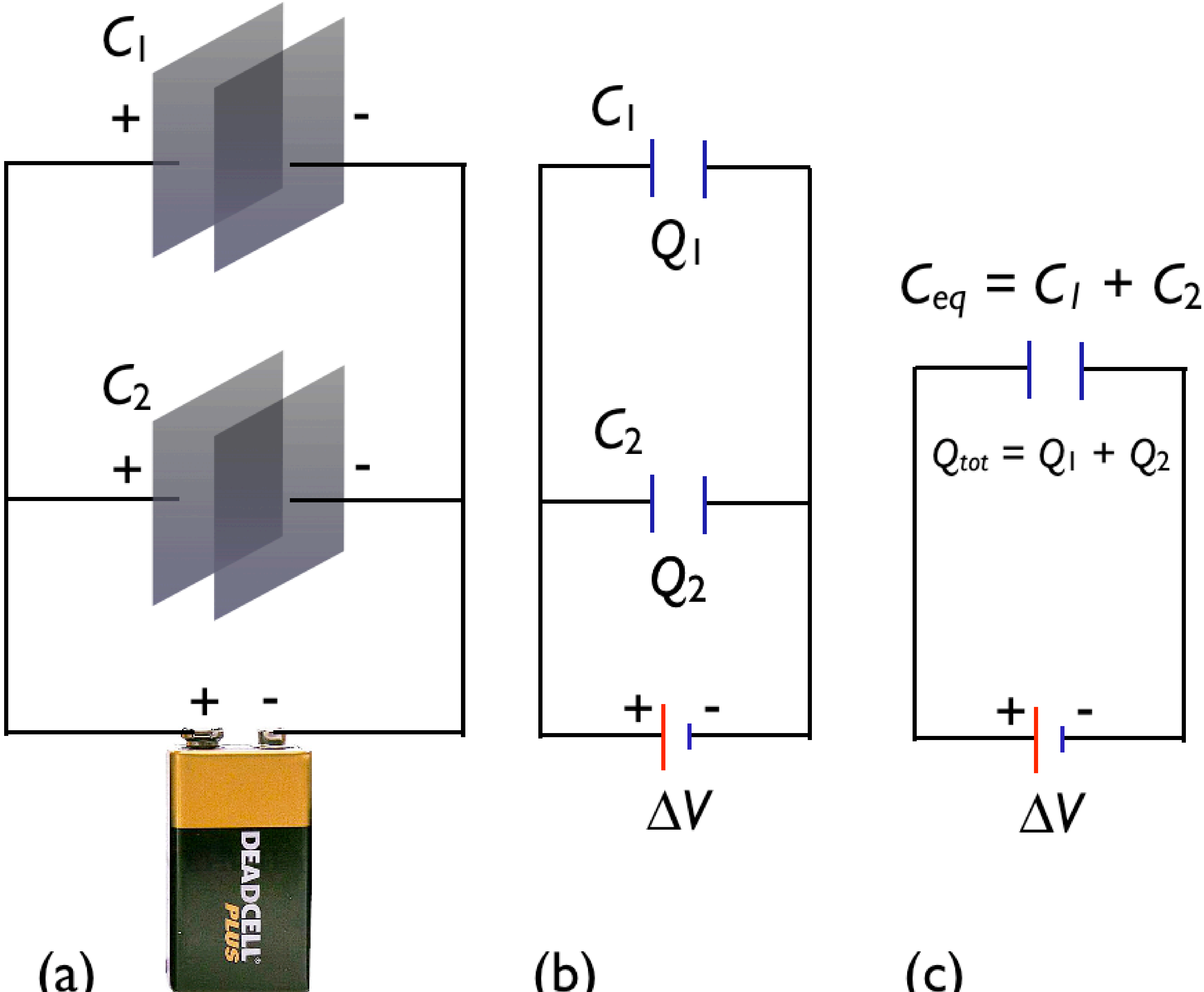


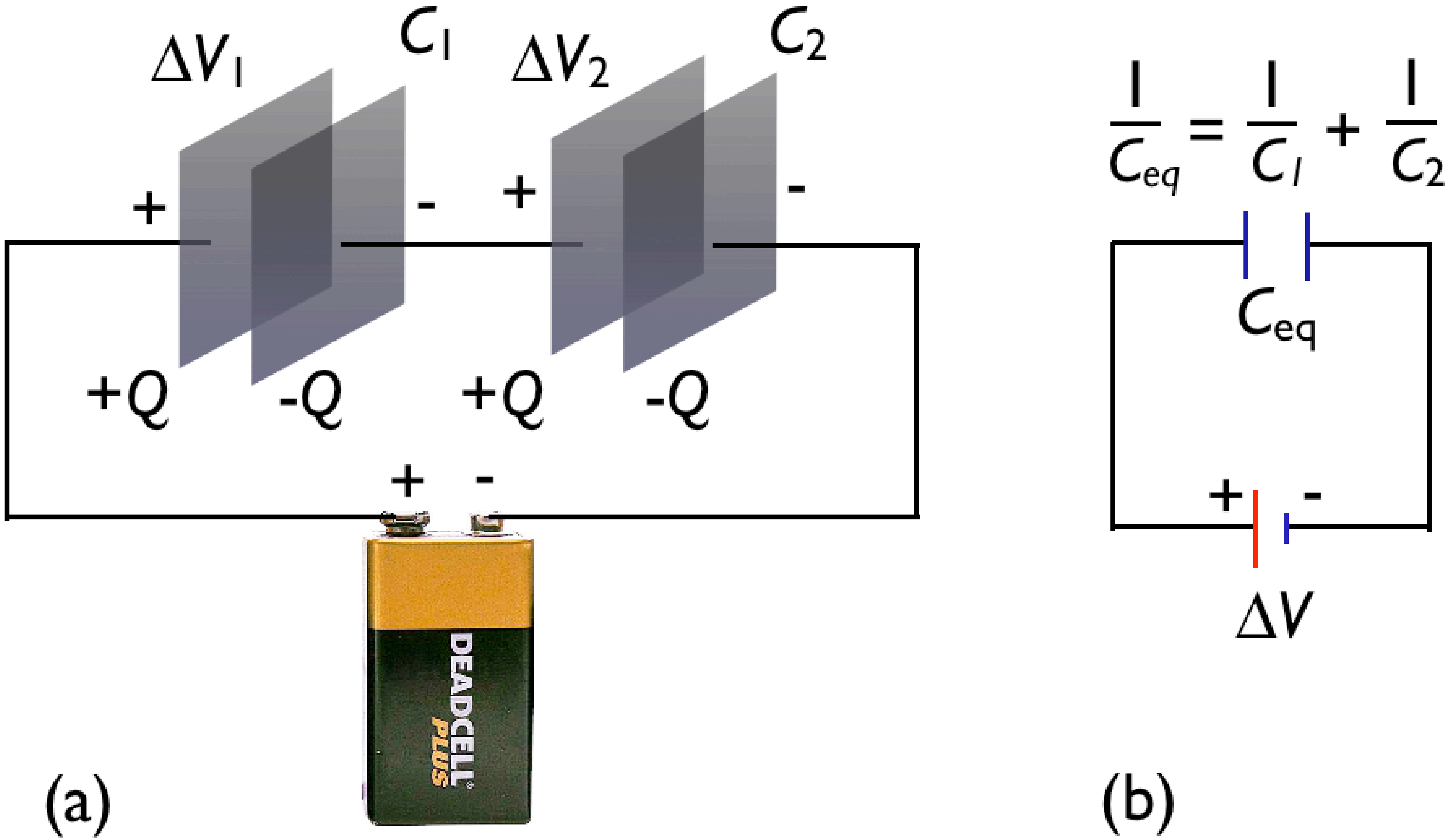
$$W = \tfrac{1}{2}\, Q\Delta V = \tfrac{1}{2}\, Q^2/C$$

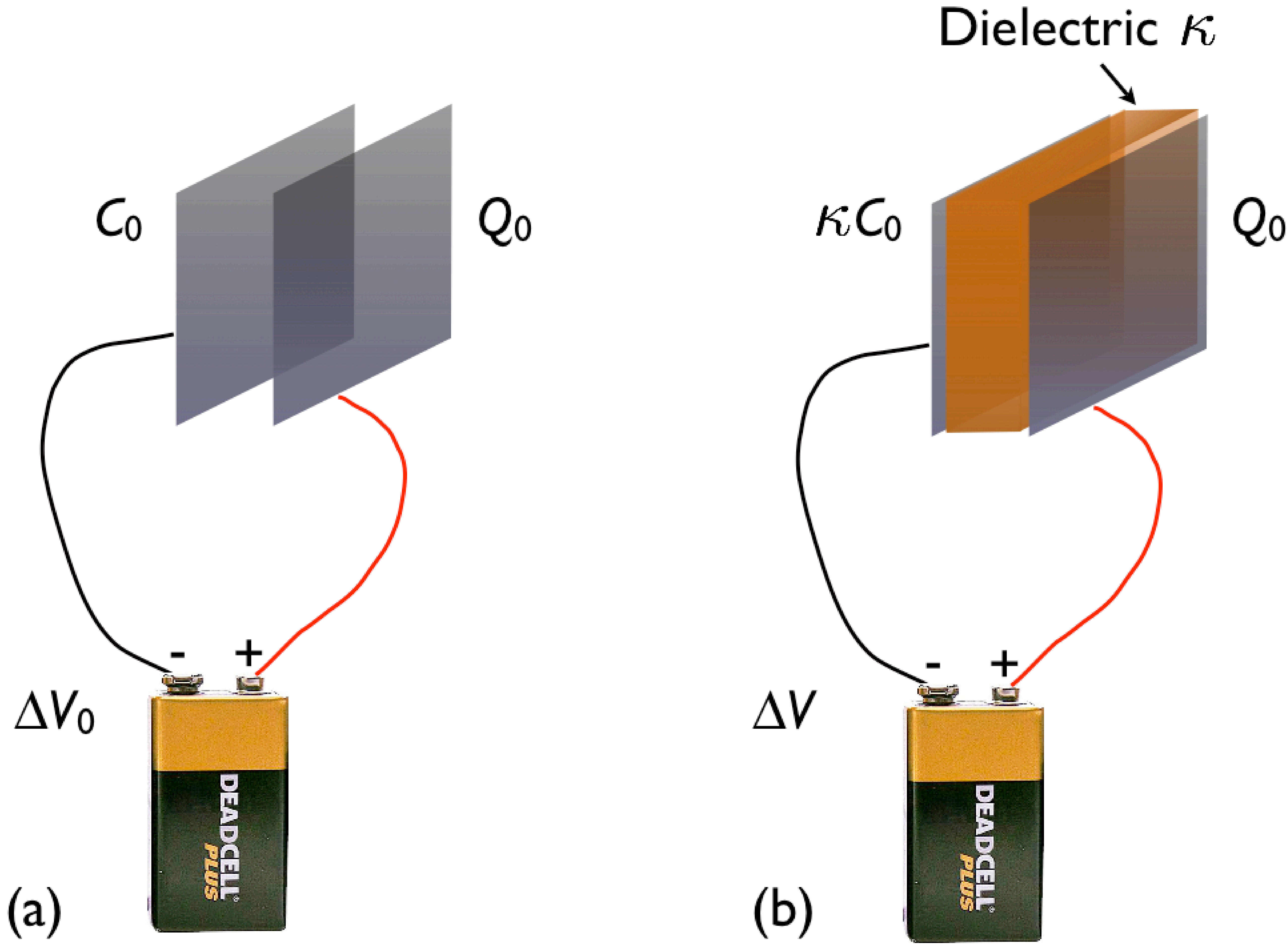
(b)



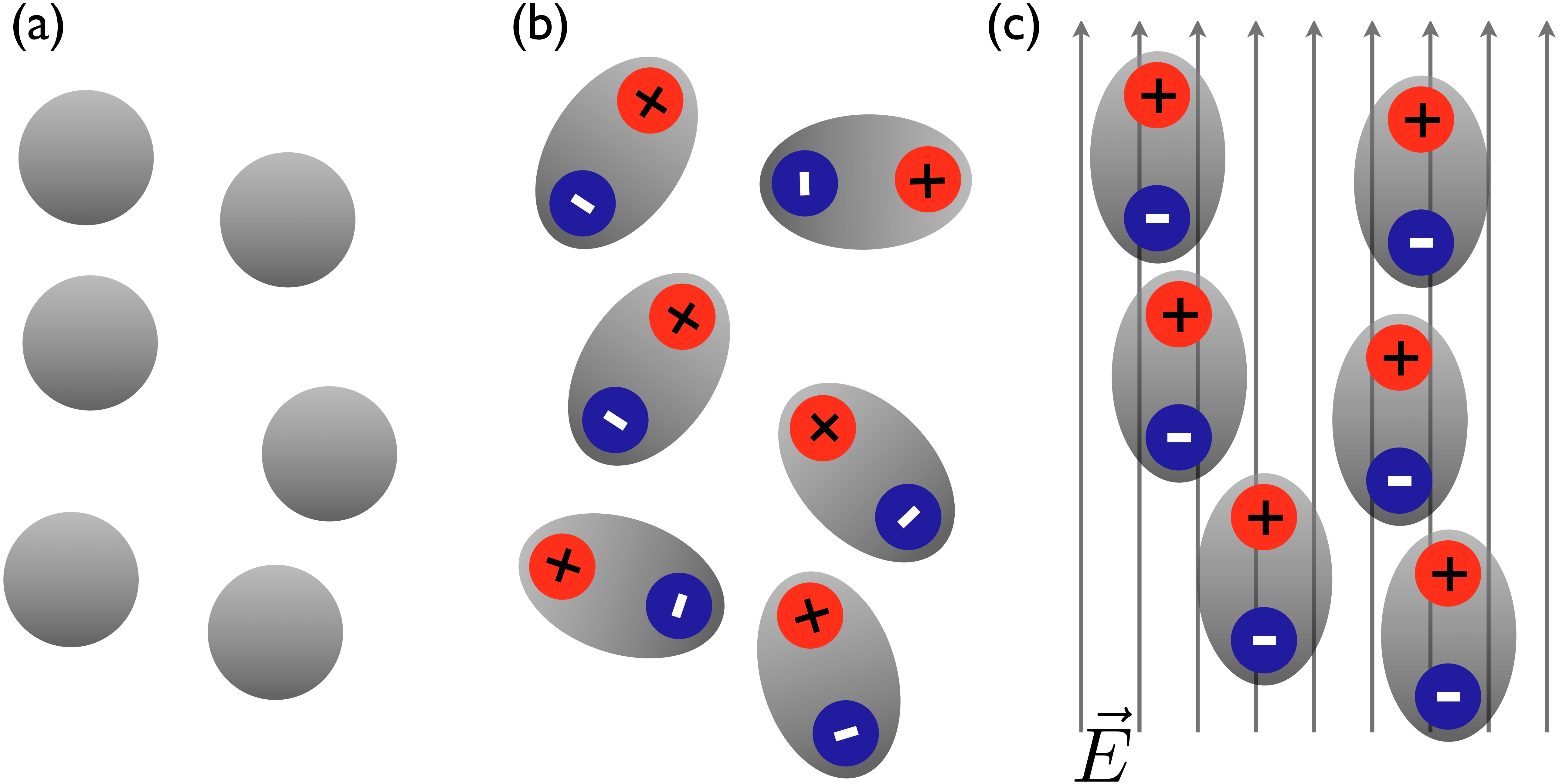
(a) (b)

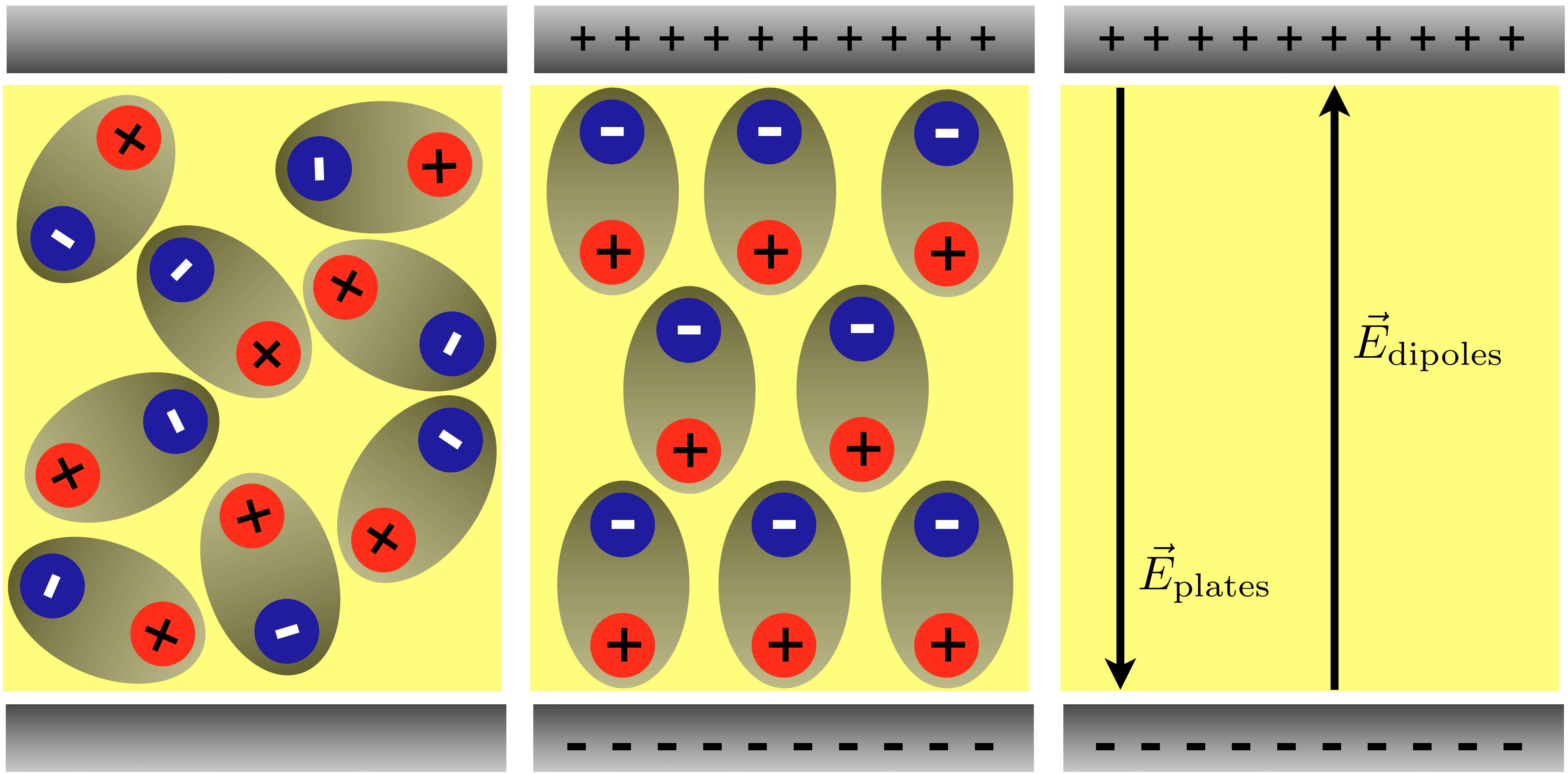

C_1
+
-
C_2
+
-
+
-
DEADCELL
PLUS
(a)
C_1
Q_1
C_2
Q_2
+
-
ΔV
(b)
$C_{eq} = C_1 + C_2$
$Q_{tot} = Q_1 + Q_2$
+
-
ΔV
(c)


ΔV_1
C_1
ΔV_2
C_2
+
-
+
-
$+Q$
$-Q$
$+Q$
$-Q$
+
-
DEADCELL
PLUS
(a)
$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$
C_{eq}
+
-
ΔV
(b)



C_0
Q_0
-
+
ΔV_0
DEADCELL PLUS
(a)
Dielectric κ
κC_0
Q_0
-
+
ΔV
DEADCELL PLUS
(b)


(a)
(b)
(c)
$\vec{E}$



$\vec{E}_{\text{dipoles}}$
$\vec{E}_{\text{plates}}$