University of Alabama
Department of Physics and Astronomy

Quiz 1: SOLUTION

1. An airplane 10.0 m long is flying at 300 m/s. How much shorter will this airplane appear to be to an observer on the ground?

- ⊗ $5 \times 10^{-6}\,\mu$m
- ○ 2×10^{-3} m
- ○ 0.1 m
- ○ 5 m

First of all: there was a typo in this problem I did not catch in time. The first choice should have read $10^{-6}\,\mu$m, not 10^{-6} m! Since this is my fault, you were all given credit for this question, no matter what you answered.

Anyway, how to solve it properly? From the point of view of an observer on the ground, the airplane is moving at a relative velocity of 300 m/s, and therefore will be shortened along the direction of motion. The proper length of the airplane is 10.0 m, as measured at rest, we can use the length contraction formula to find the apparent length as viewed from the ground. What we *want*, though, is the difference in the apparent and actual lengths - how much shorter the airplane appears to be. To get that, we want to subtract the proper and contracted lengths:

$$\begin{aligned}
L_P - L' &= L_p - \frac{L_p}{\gamma} \\
&= \left(1 - \frac{1}{\gamma}\right) L_p \\
&= \left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right) L_p \\
&= \left(1 - \sqrt{1 - \frac{\left(3 \times 10^2\,\mathrm{m/s}\right)^2}{\left(3 \times 10^8\,\mathrm{m/s}\right)^2}}\right) \cdot 10\,\mathrm{m} \\
&= \left(1 - \sqrt{1 - \left(10^{-6}\right)^2}\right) \cdot 10\,\mathrm{m}
\end{aligned}$$

Now, an average calculator does not necessarily have the precision to calculate the part under the square root - it will probably just give you "1." If this is the case, it is easiest to just approximate the answer - since all of the possible choices differ by quite a lot, we only have to get close. A good trick on things like the MCAT, for instance. The following approximation is incredibly useful in all sorts of cases, it is based on the binomial theorem

$$(1 \pm x)^n \approx 1 \pm nx$$

In this case, $n = -\frac{1}{2}$, and $x = 10^{-12}$, and we have:

$$\begin{aligned}
L_P - L' &= \left(1 - \sqrt{1 - 10^{-12}}\right) \cdot 10\,\text{m} \\
&\approx \left(1 - \left(1 + \frac{1}{2} \cdot 10^{-12}\right)\right) \cdot 10\,\text{m} \\
&= 5 \times 10^{-12}\,\text{m} \\
&= 5 \times 10^{-6}\,\mu\text{m}
\end{aligned}$$

If you didn't know how to approximate the answer, remember that it is multiple choice: based on the numbers you have, all but the first choice are way too large.

2. An electron in a television picture tube moves with $v = 0.250c$. What is its kinetic energy in electron volts? Note that the rest energy of an electron is $m_e c^2 = 0.511\,\text{MeV}$

- ◯ 0.528 MeV
- ◯ 0.511 MeV
- ⊗ 0.017 MeV
- ◯ 0.253 MeV

Relativistic kinetic energy can be most easily written as:

$$KE = (\gamma - 1)\,mc^2$$

Now recall that mc^2 is just the rest energy of the electron, which is quoted in the problem. Therefore, all we need to do is find $\gamma - 1$:

$$\gamma - 1 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 = \frac{1}{\sqrt{1 - 0.250^2}} - 1 \approx 0.0328$$

Now it is easy to find the kinetic energy:

$$KE = (\gamma - 1)\,mc^2 = (0.0328)\,(0.511\,\text{MeV}) \approx 0.017\,\text{MeV}$$

3. A crew watches a movie that is two hours long in a space-craft that is moving at high speed through space. Will an Earthbound observer, who is watching the movie through a powerful telescope, measure the duration of the movie to be:

- ⊗ Longer than two hours.
- ◯ Shorter than two hours.
- ◯ Equal to two hours.
- ◯ I'd tell you, but that would violate the temporal prime directive.

You can consider the beginning and end of the movie to be two separate events. The crew, at rest with respect to the movie, measures the proper time interval between start and end, 2 hr. The observer on earth is in motion with respect to the movie, and thus sees a longer (dilated) time interval between the start and and of the movie.

4. A proton has a mass of $1.67 \times 10^{-27}\,\text{kg}$. What is its rest energy in electron volts (eV)? Note $1\,\text{eV} = 1.60 \times 10^{-19}\,\text{J}$, and M is the metric prefix for 10^6.

- ◯ 42 MeV
- ◯ 313 MeV

⊗ 938 MeV
◯ 1320 MeV

This one is just plugging the numbers into the equation for rest energy, noting that $1\text{kg}\cdot\text{m/s}^2$ is by definition $1\,\text{J}$:

$$E_R = mc^2 = \left(1.67\times10^{-27}\,\text{kg}\right)\left(3\times10^{8}\,\text{m/s}\right)^2 = 1.50\times10^{-10}\,\text{J}\cdot\frac{1\,\text{eV}}{1.60\times10^{-19}\,\text{J}} \approx 9.38\times10^{8}\,\text{eV} = 938\,\text{MeV}$$

5. Which one of these things can two observers in different frames **not** agree on?

◯ Their relative speed of motion with respect to each other.
◯ The speed of light c.
◯ The simultaneity of two events taking place at the same position and same time in some frame.
⊗ The distance between two points that remain fixed in one of their frames.

Let's go through the choices one by one. First, the relative speed of motion is the same from either reference frame, so long as no one is accelerating. Both will agree on this. They will also agree on the speed of light - it is an invariant constant, independent of reference frame.

The third choice is trickier: observers in different frames cannot *generally* agree on simultaneity in an arbitrary sense. The one case in which observers in motion can agree on this is when the two events are not spatially separated - *i.e.,* the events take place at the same position in one frame. If the events take place in the same position in one frame, this will be true in all frames. You can see this from the formula for elapsed time:

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$$

For two events to be simultaneous for both observers, we need $\Delta t = \Delta t' = 0$. This says both observers see the events happen at the same time, there is no time interval between them. For this to be true, based on the equation above, we must also have $\Delta x = 0$, *i.e.,* the events happen at the same location according to one observer. If $\Delta x = 0$ in one frame, it is true in the other as well - length contracting zero just gives you zero again. Thus, as stated, the third choice is valid for both observers.

Finally, for the last choice, the distance between two points fixed with respect to one observer is just a *proper length* measured by that observer. An observer in relative motion cannot agree on this length, they would see a contracted length. If nothing else, by elimination, the third choice must be correct.