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Quiz 8: SOLUTION

1. White light, made up of all visible wavelengths of light, is incident on a prism in the form of an equilateral triangle. The index of refraction of red light for the prism is **less** than that for blue light. Which color of light will exit the prism at a **larger** angle relative to the incident angle?

- ■ blue
- □ red
- □ they will have the same angle
- □ cannot be determined without knowing the incident angle

Recall from the last homework that we actually *calculated* the difference in angle for red and blue light, where $n_{\text{red}} < n_{\text{blue}}$. Without going through all of that again, what we basically need to remember is Snell's law of refraction:

$$n_1 \sin\theta_i = n_2 \sin\theta_r$$

What this tells us is for a prism with index of refraction n_2 greater than that of its surrounding material (n_1), the refracted angle will be *smaller* than the incident angle. If the product $n \sin\theta$ must be the same for incident and refracted waves, then a larger refractive index means a smaller $\sin\theta$, which in turn means a smaller θ. Basically: the larger the refractive index, the more the refracted ray is "bent." Blue light will be refracted more at both the incident and exit interfaces of the prism, and will have a larger deviation angle than red light.

2. Two lenses are used in series to magnify an image. The first lens magnifies an object by a factor M_1, the second magnifies the resulting image by a factor M_2. What is the overall magnification of the object after **both** lenses?

- □ $M_{\text{eff}} = M_1 + M_2$
- ■ $M_{\text{eff}} = M_1 \cdot M_2$
- □ $M_{\text{eff}} = \sqrt{M_1^2 + M_2^2}$

Presume that the first lens magnifies the object such that the image formed differs in height by a factor M_1. The *image* of the first lens is the *object* of the second - the second lens takes the image formed by the first, and further magnifies it. If the object height is h, the image formed by the first lens has height $M_1 h$. This image will be further magnified by a factor M_2 by the second lens. An object of height h_2 for the second lens produces an image of height $M_2 h_2$. If the image of the first lens is the object for the second, then we simply have $h_2 = M_1 h$. Thus, the final height of the image formed by the combination of both lenses must be $M_1 M_2 h$, or a magnification factor $M_{\text{eff}} = M_1 \cdot M_2$. For the combination lens, we could simply write $h_{\text{final}} = M_{\text{eff}} h$.

3. Myopia, also called near- or short-sightedness, is a refractive defect of the eye in which collimated light produces image focus in front of the retina when accommodation is relaxed, rather than directly on the retina. What sort of lens(es) could be used to correct this condition?

- ■ convex
- □ it depends on the degree of myopia
- □ concave

From the wikipedia: "With myopia, the eyeball is too long, or the cornea is too steep, so images are focused in the vitreous inside the eye rather than on the retina at the back of the eye." Thus, we want to push the focal point back farther to the retina - we want to diverge the rays just a little bit to push the focal length back farther. For this we want a diverging lens, and a convex lens does nicely. See. `http://en.wikipedia.org/wiki/Myopia` for more information.

4. A contact lens is made of a plastic with an index of refraction of 1.60. The lens has an inner radius of curvature of 1.99 cm and an inner radius of curvature of 2.56 cm. What is the focal length of the contact lens?

- □ 53.1 cm

- ☐ 1.5 cm
- ☐ 10.7 cm
- ■ 14.9 cm

Qualitatively, we know that the lens must form a real image in order for contact lenses to function properly. Therefore, we know that in the end the lens must have a positive focal length. Further, we know the order of magnitude of the lens, based on the size of an average human head: it must be centimeters, clearly not meters, kilometers, or micrometers! In order to attack the problem quantitatively, we need the lensmaker's equation.

$$\frac{1}{f} = \left(\frac{n_2 - n_1}{n_1}\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \tag{1}$$

Since the outer surface of the lens is exposed to plain air, we may assume $n_1 = 1.00$ there. Since we just want the refractive index of the contact lens itself, not the contact lens in combination with the eye, we will assume the other surface is exposed to air as well. Given the refractive index of the lens material $n_2 = 1.60$ and the two radii, we need only solve for f. Since we know the answer has to be positive, we know that $R_2 = 2.56\,\text{cm}$ and $R_1 = 1.99\,\text{cm}$, not the other way around:

$$\frac{1}{f} = \left(\frac{1.60 - 1.00}{1.00}\right)\left[\frac{1}{1.99} - \frac{1}{2.56}\right] \tag{2}$$

$$\frac{1}{f} = (0.60)\,[0.112]\ \text{cm}^{-1} \tag{3}$$

$$\frac{1}{f} = 0.0617 \tag{4}$$

$$\implies \quad f = 14.9\,\text{cm} \tag{5}$$

5. An object is placed to the left of a converging lens. Which of the following statements are true and which are false?

1. The image is always to the right of the lens
2. The image can be upright or inverted
3. The image is always smaller or the same size as the object

- ☐ 1 and 2 are true, 3 is true
- ☐ 2 and 3 are false, 1 is true
- ■ 1 and 3 are false, 2 is true
- ☐ 2 and 3 are true, 1 is false