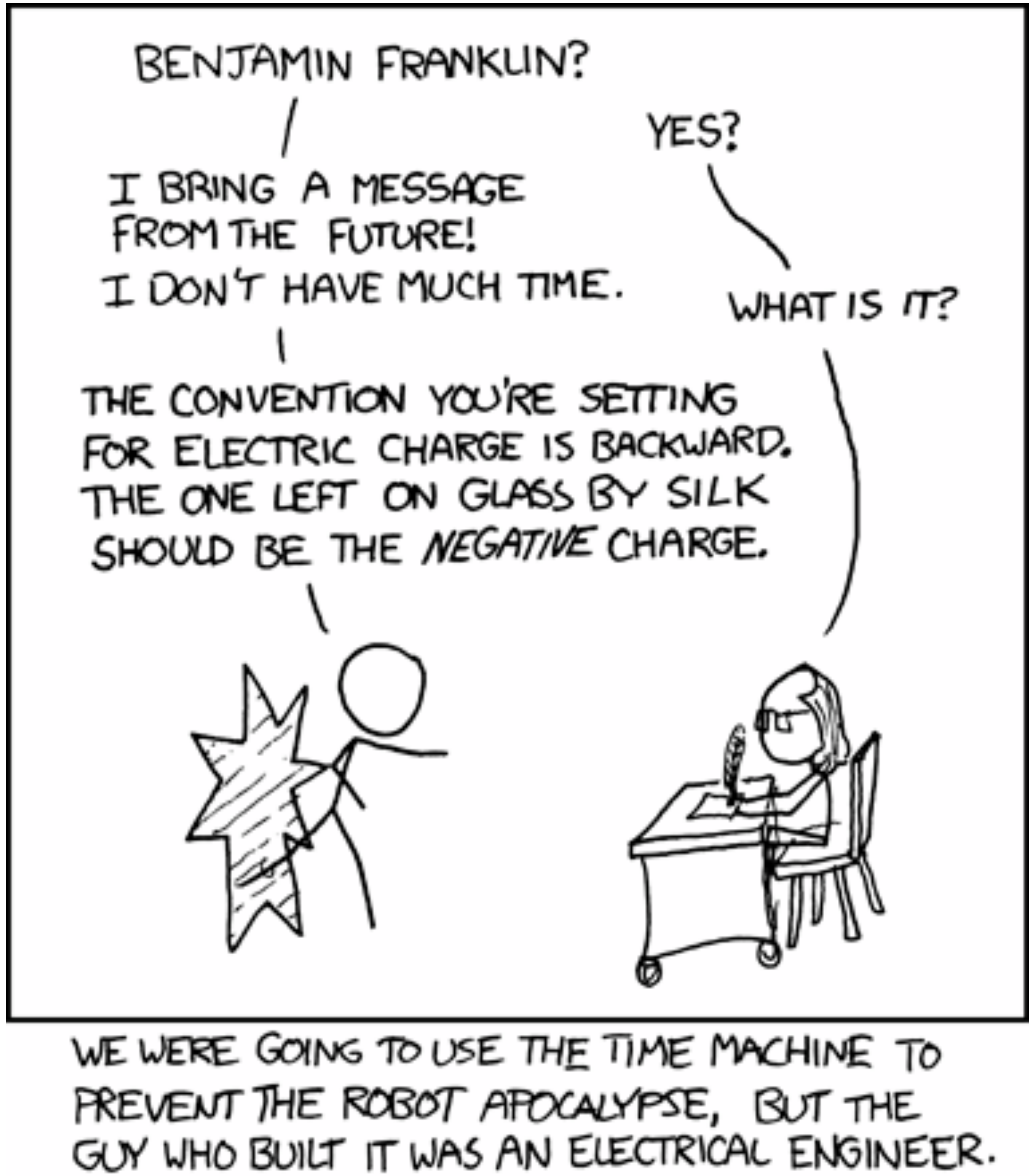


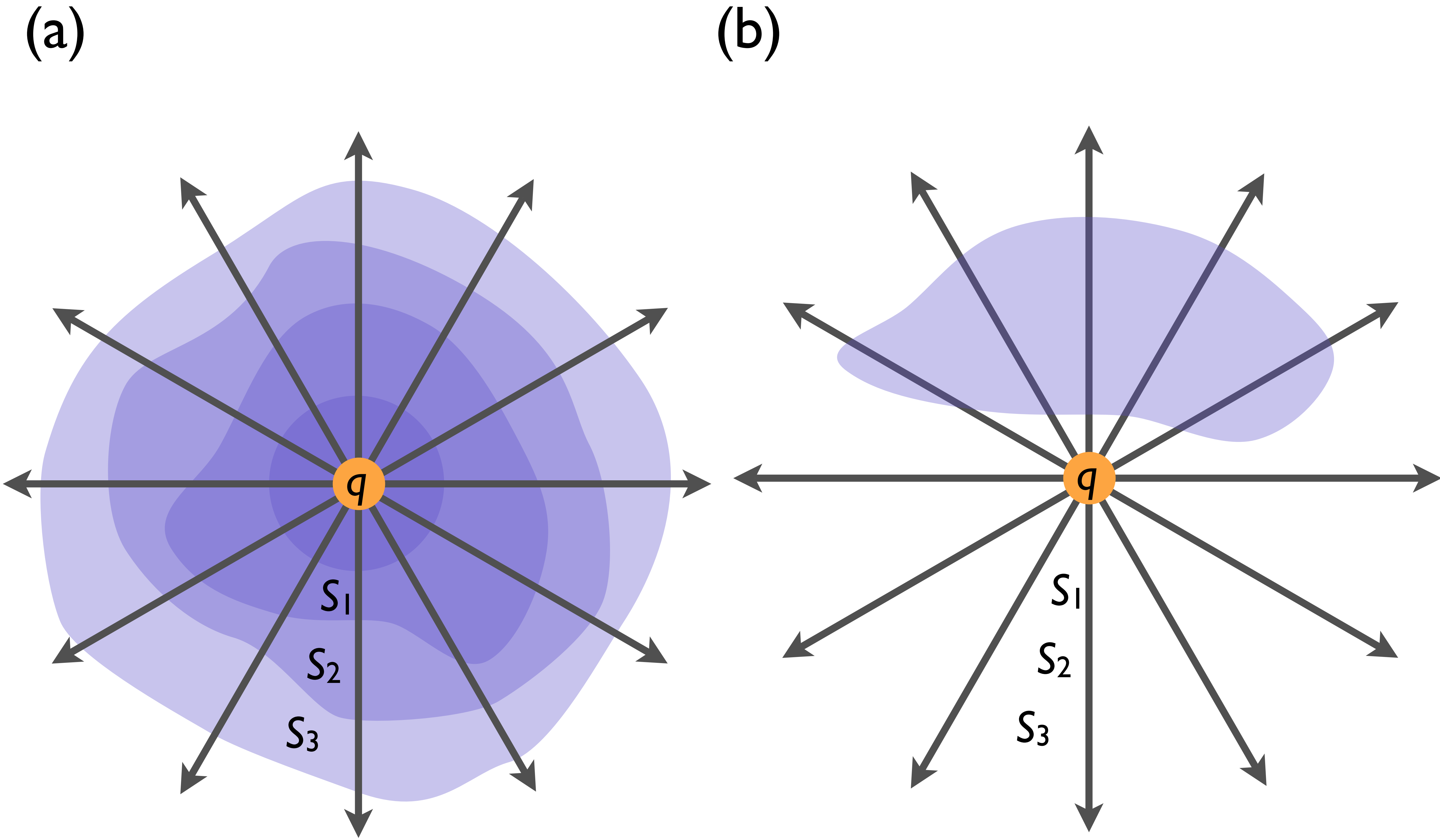
http://xkcd.com/567/

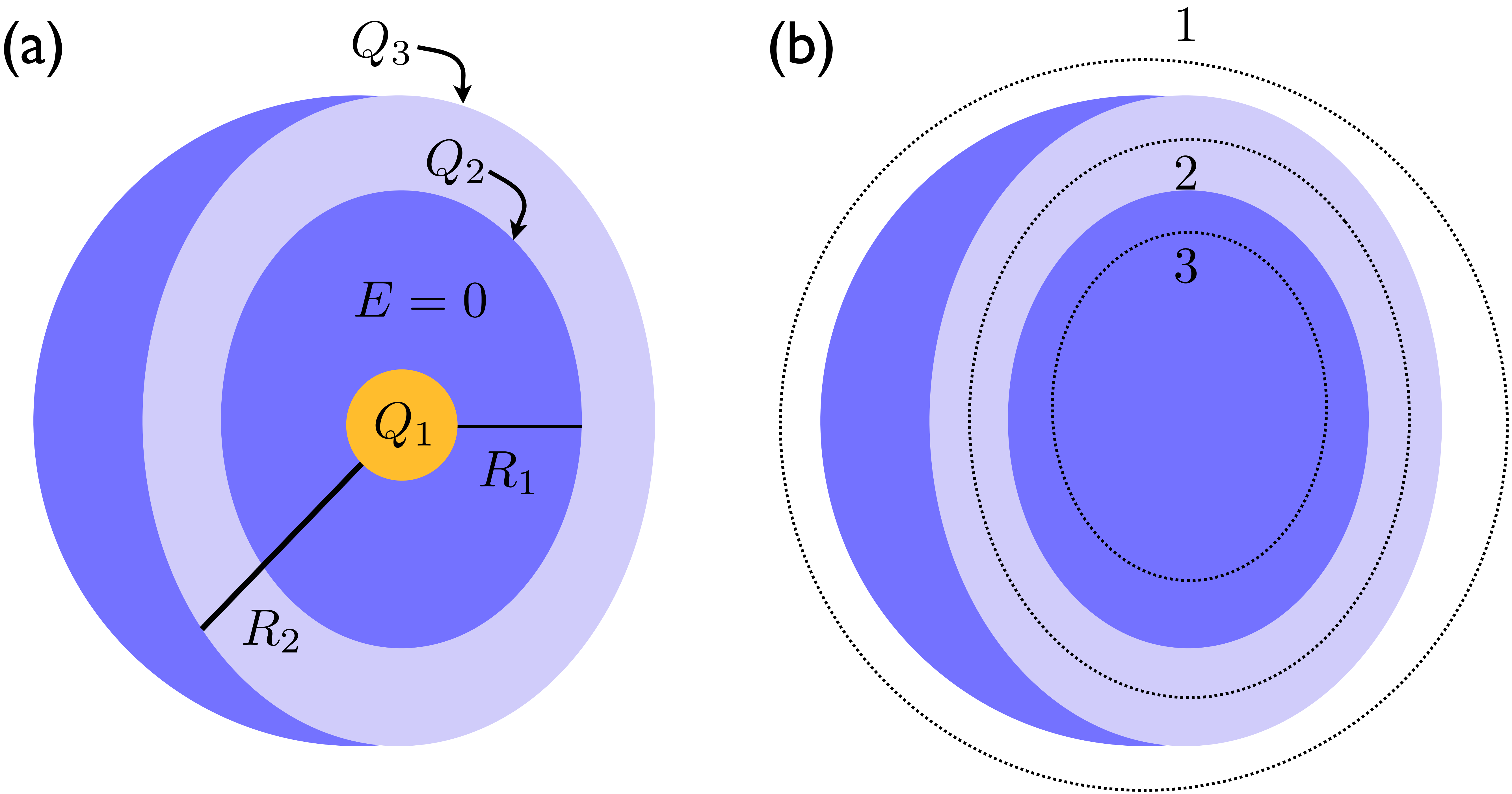
electrical energy & capacitance

- today & tomorrow
- first: wrap up Gauss' law, then potential
- capacitors/dielectrics tomorrow
- rest of the week: circuits/current/resistance
- NEXT MON: exam I

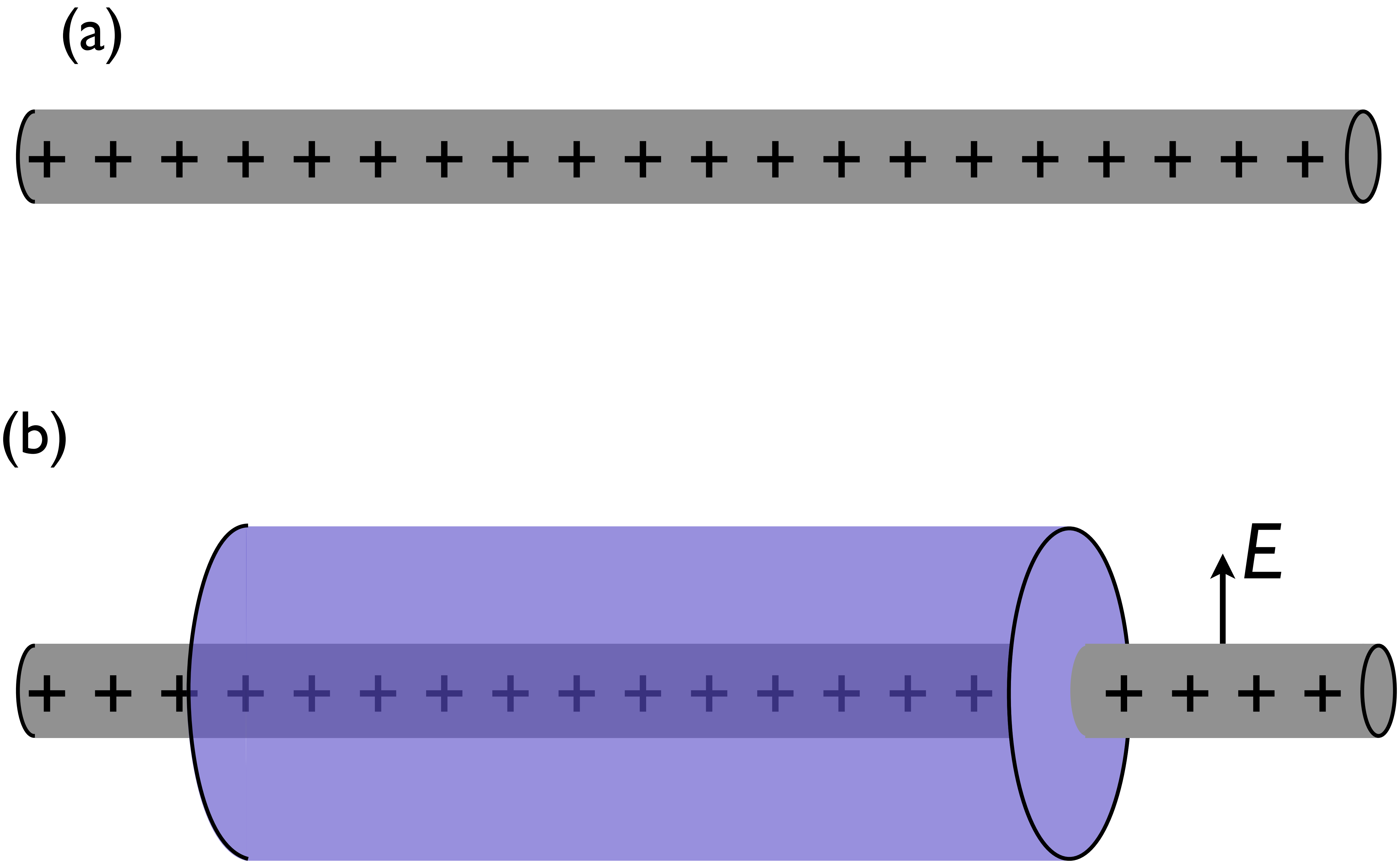
  problem-based, cumulative

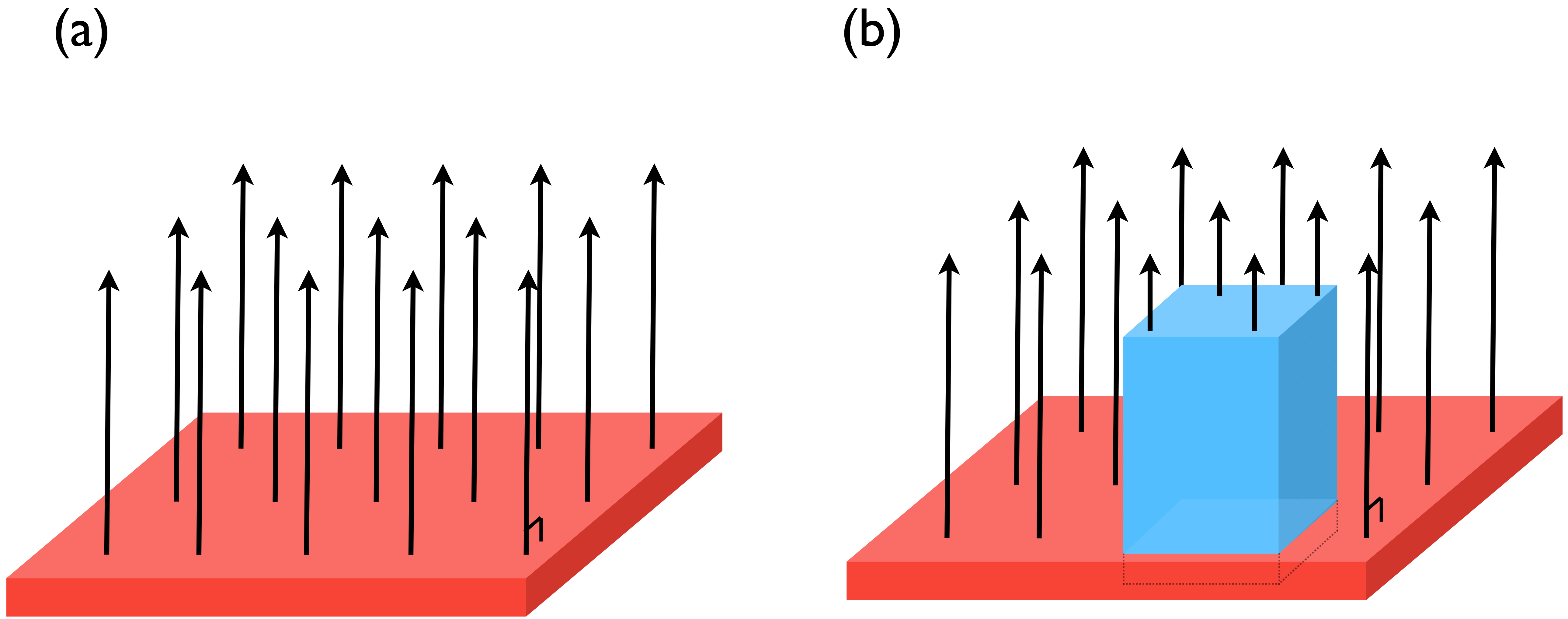
  more details throughout the week

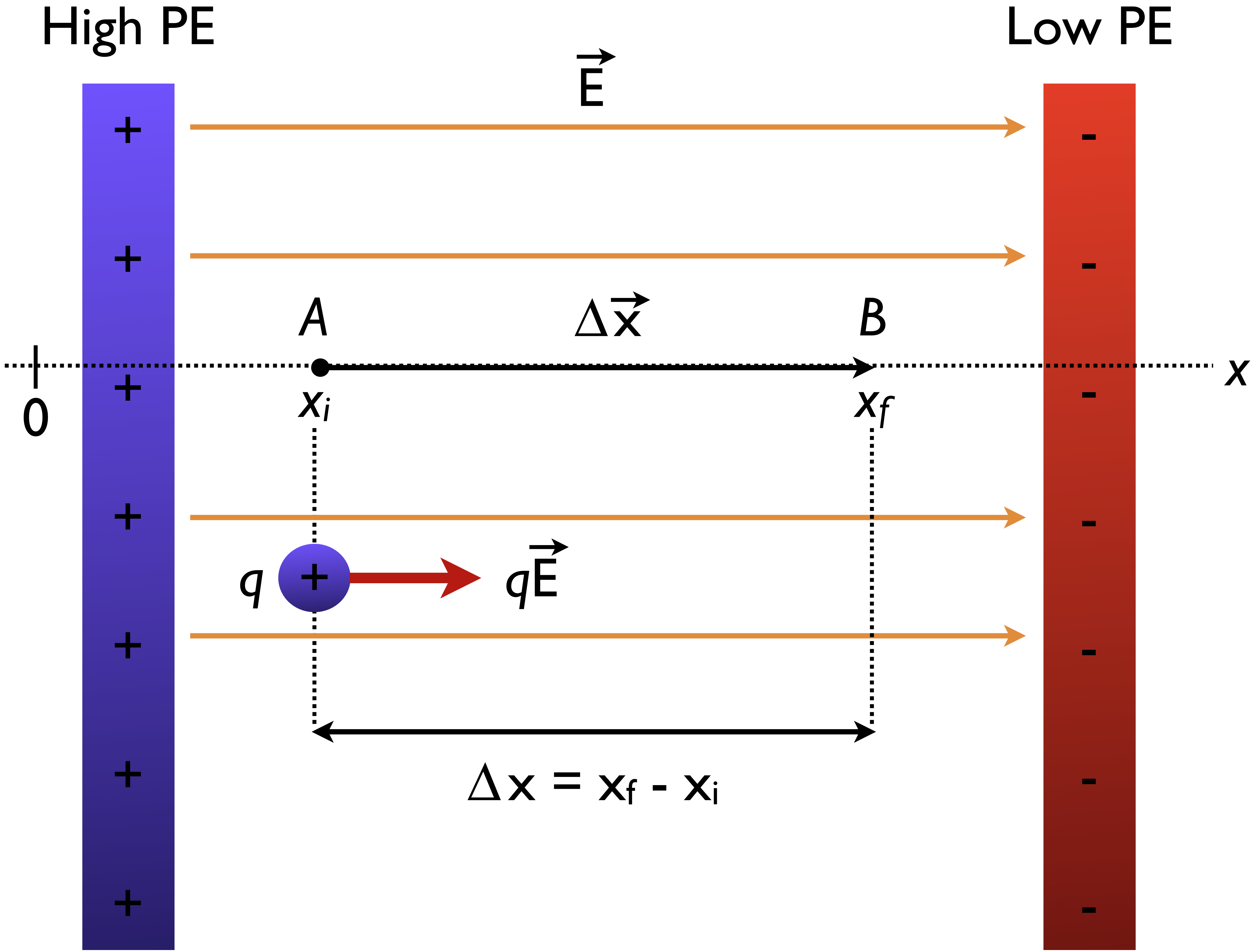

(a)
(b)
q
S1
S2
S3
q
S1
S2
S3

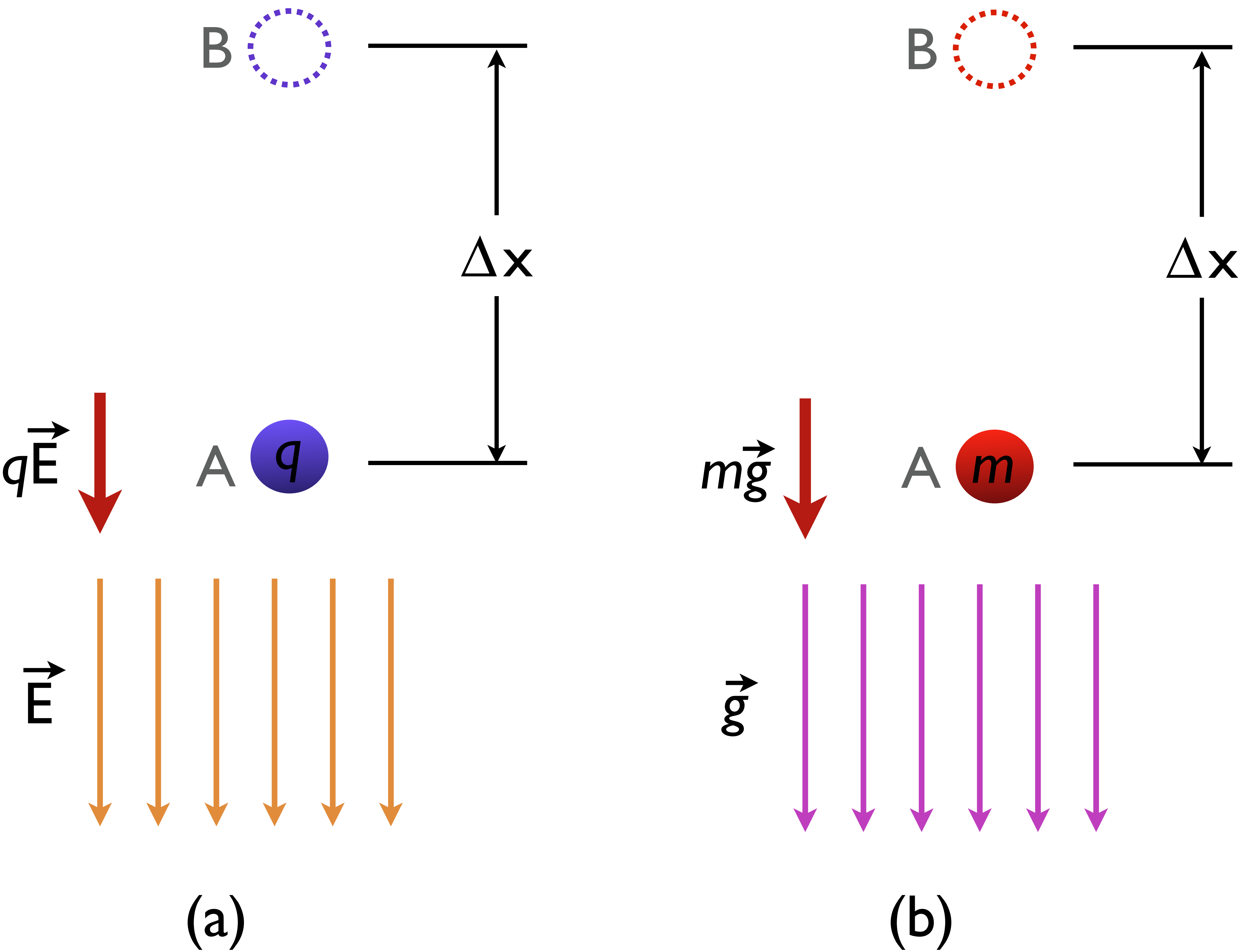


(a)
Q_3
Q_2
$E = 0$
Q_1
R_1
R_2
(b)
1
2
3

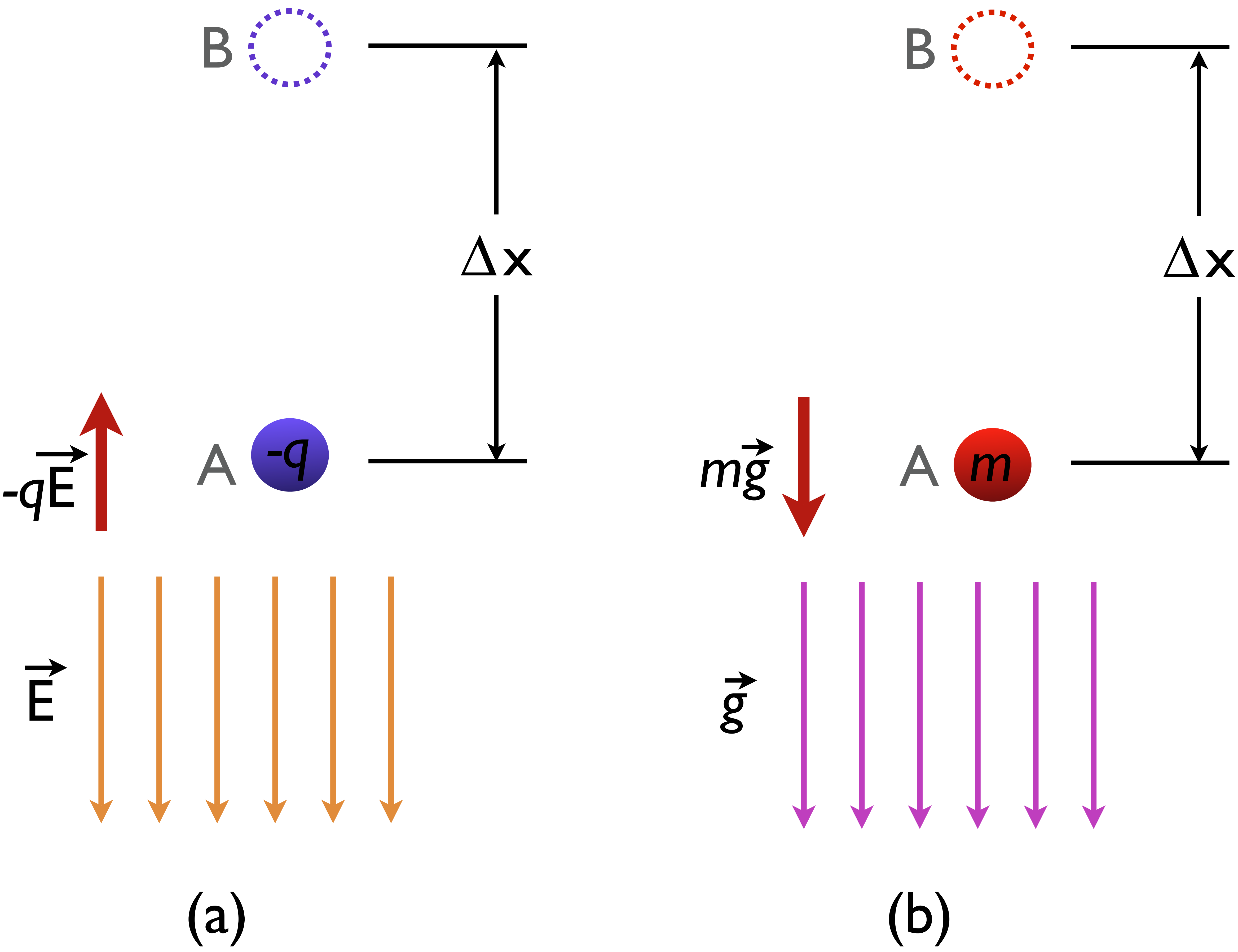

(a)
(b)
E


(a)
(b)

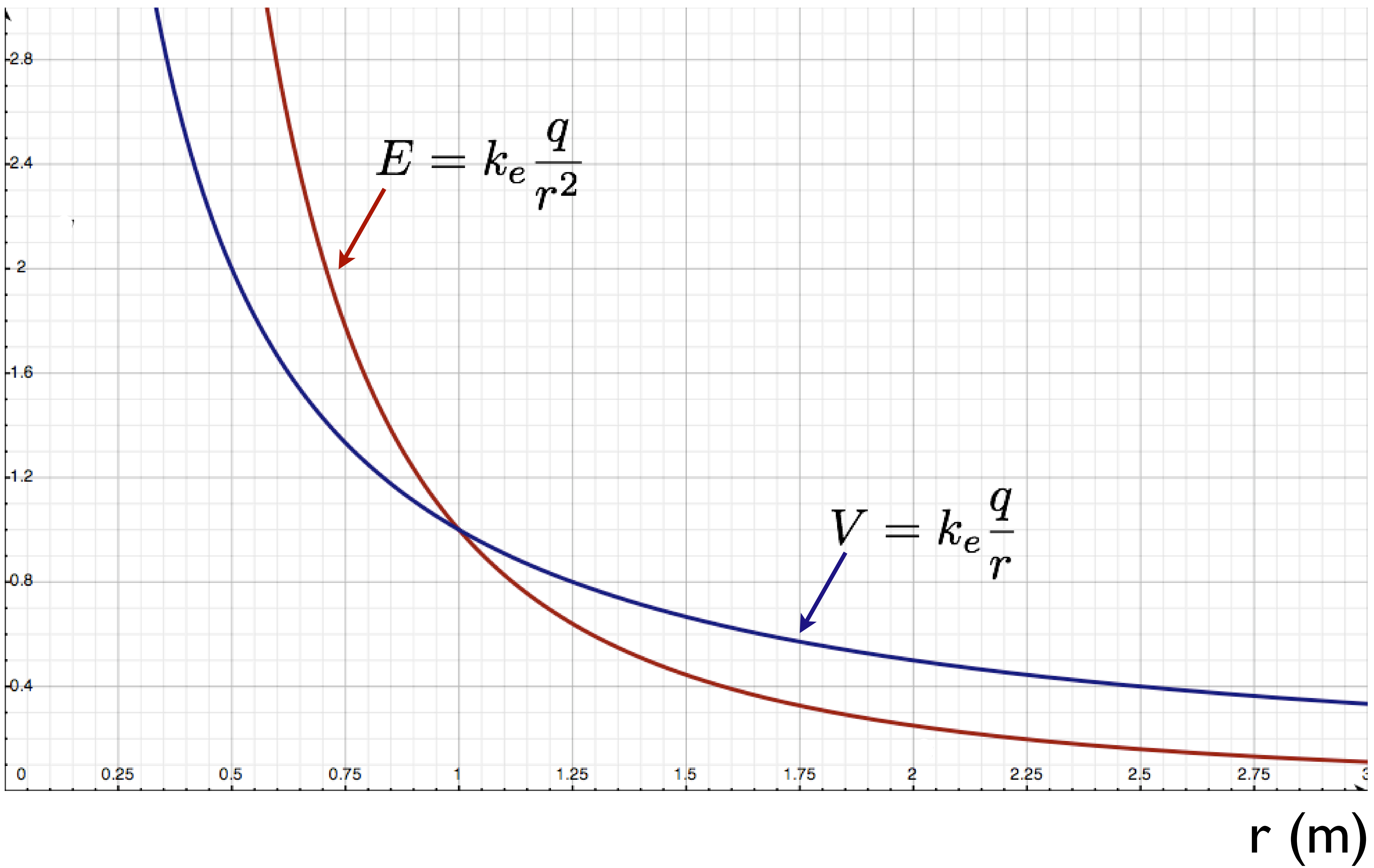

High PE
Low PE
$\vec{E}$
A
$\Delta\vec{x}$
B
x
0
x_i
x_f
q
$q\vec{E}$
$\Delta x = x_f - x_i$



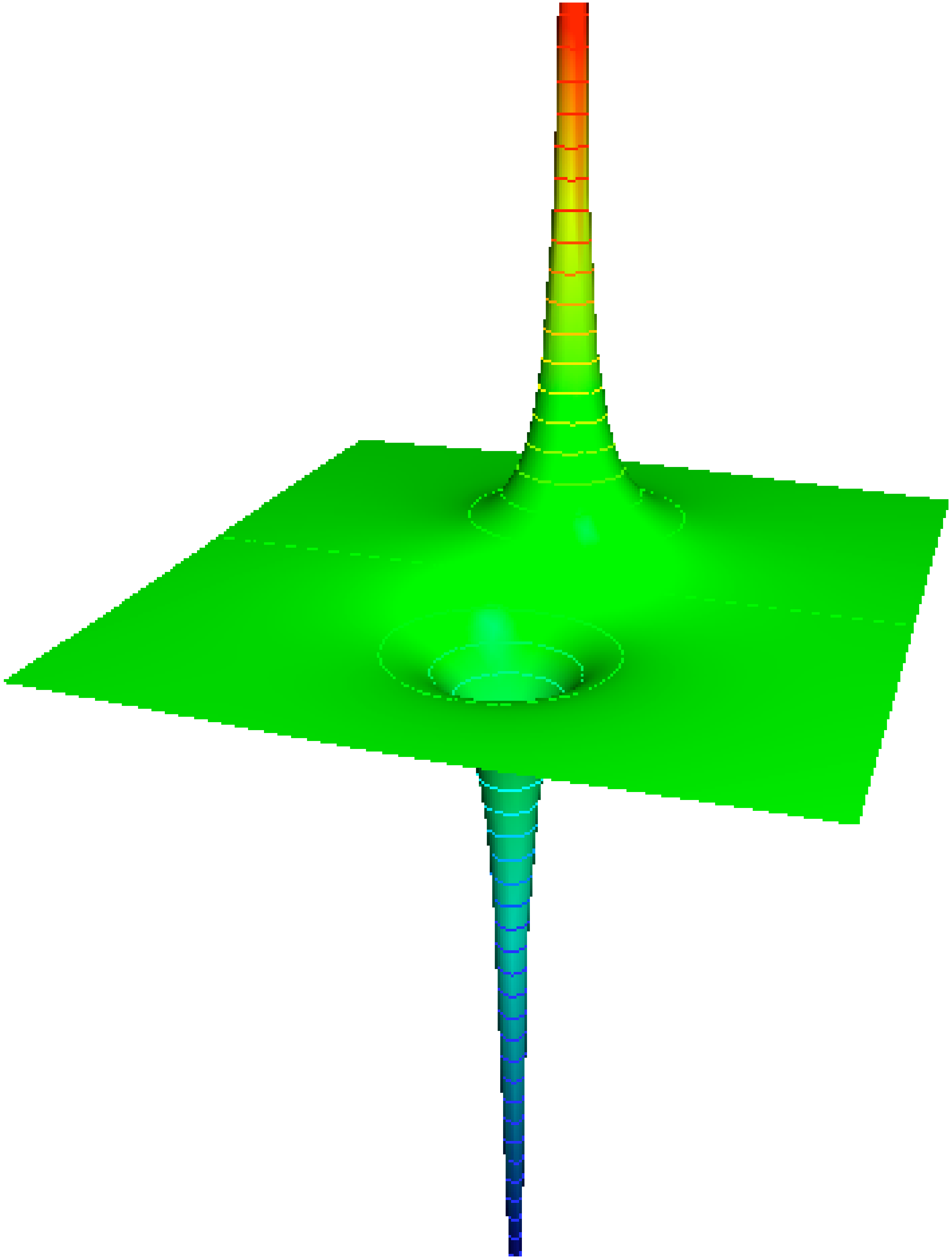
B
Δx
A
q
$q\vec{E}$
$\vec{E}$
(a)
B
Δx
A
m
$m\vec{g}$
$\vec{g}$
(b)

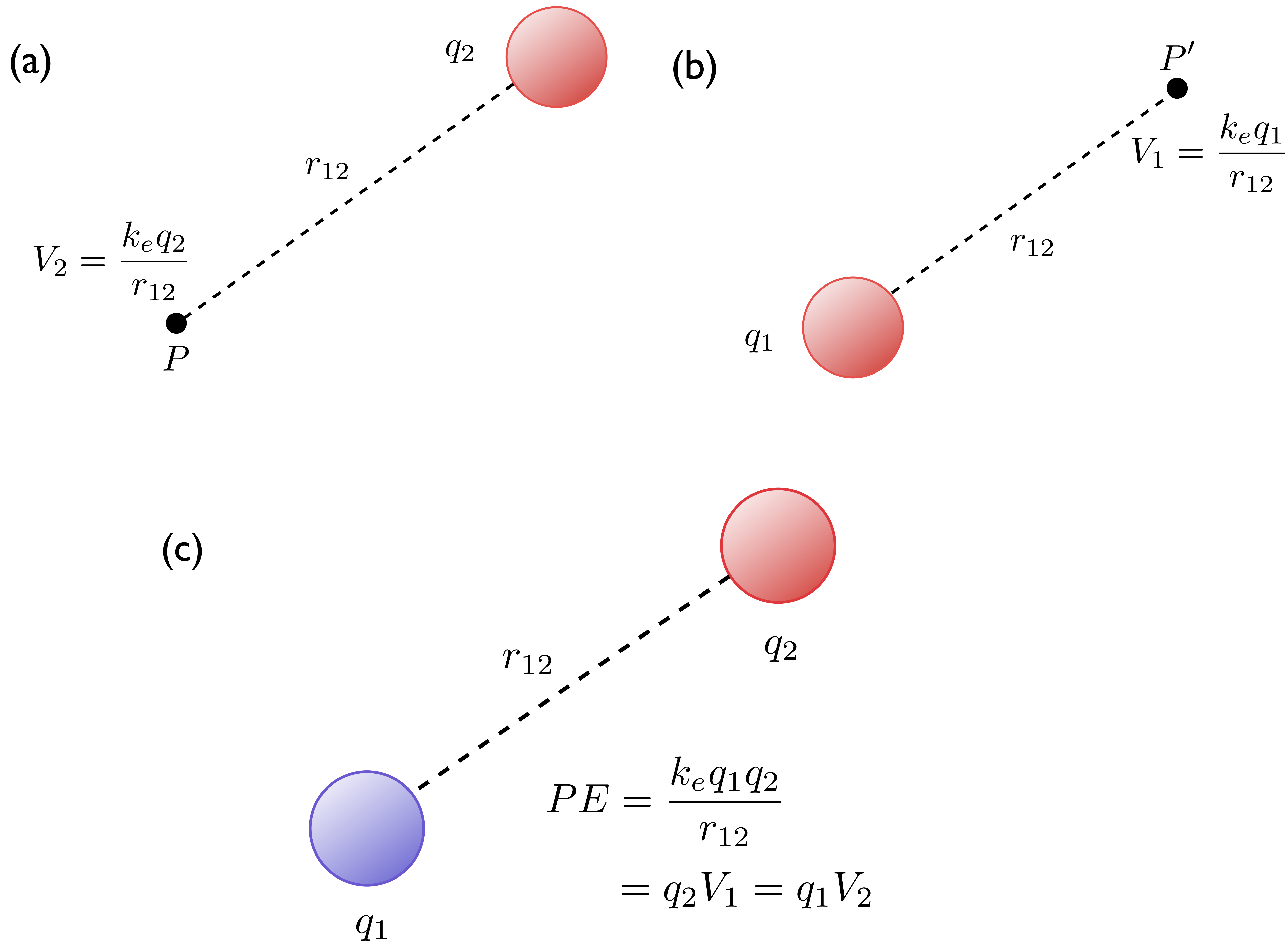


B
Δx
A
-q
-qE
E
(a)
B
Δx
A
m
mg
g
(b)



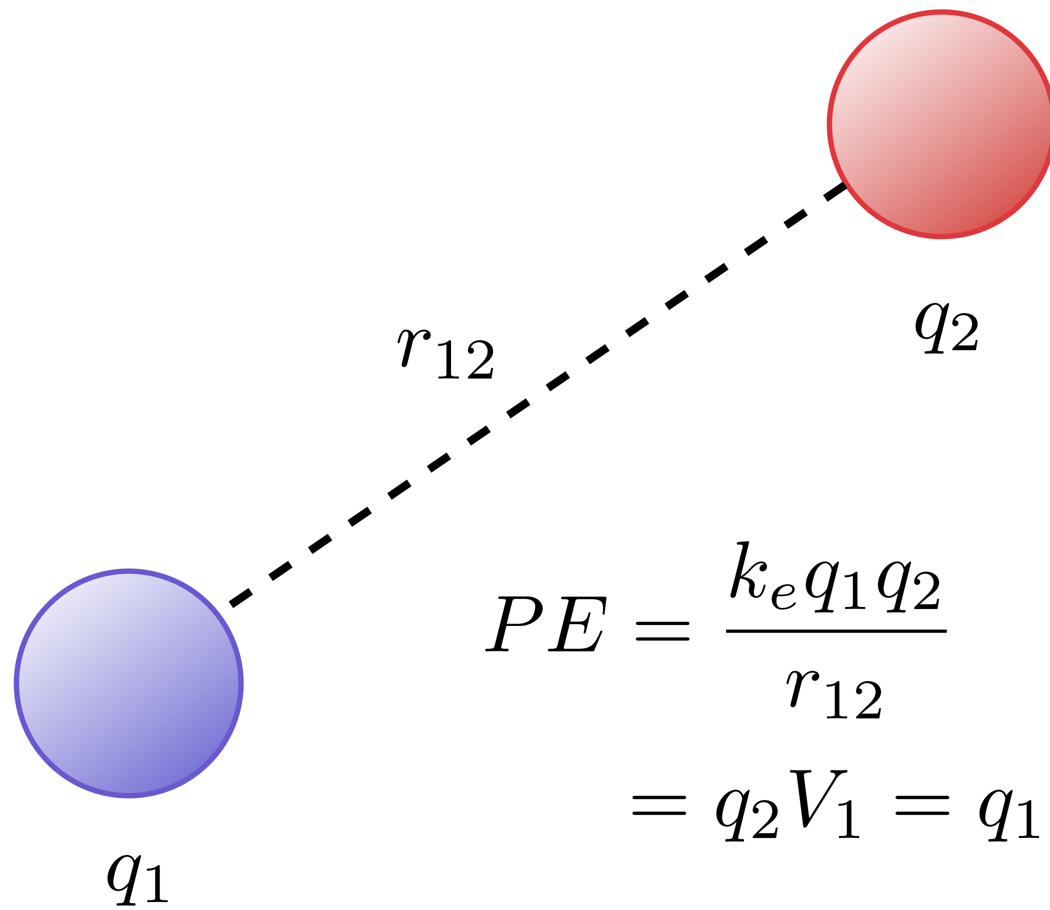
$E = k_e \frac{q}{r^2}$
$V = k_e \frac{q}{r}$
2.8
2.4
2
1.6
1.2
0.8
0.4
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3
r (m)





(a)
q_2
r_{12}
$V_2 = \frac{k_e q_2}{r_{12}}$
P
(b)
P'
$V_1 = \frac{k_e q_1}{r_{12}}$
r_{12}
q_1
(c)
r_{12}
q_2
q_1
$PE = \frac{k_e q_1 q_2}{r_{12}}$
$= q_2 V_1 = q_1 V_2$

(c)



$$PE = \frac{k_e q_1 q_2}{r_{12}}$$
$$= q_2 V_1 = q_1 V_2$$

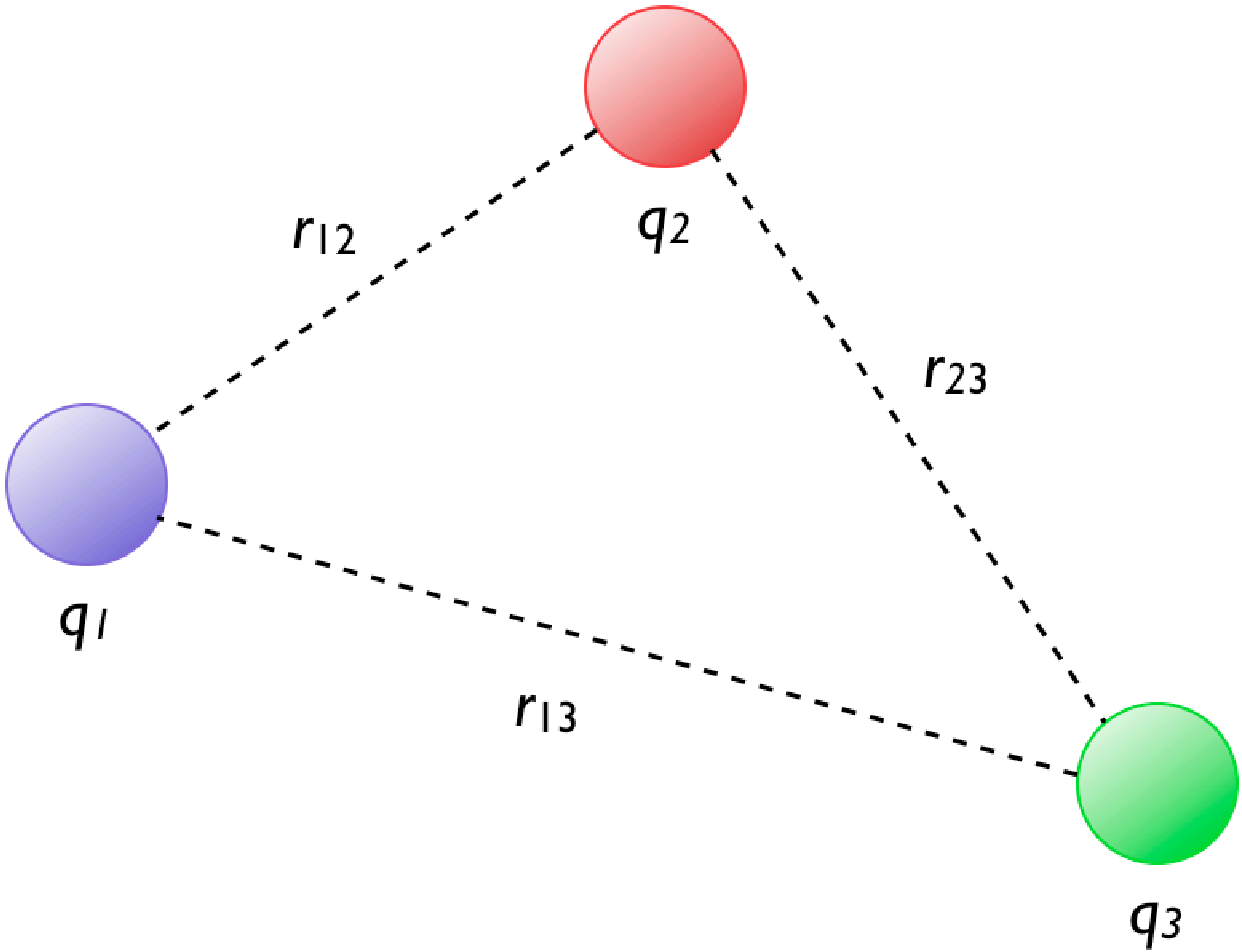
PE = (1 due to 2) = (2 due to 1)

PE = (E to bring 1 close to 2)
(E to bring 2 close to 1)

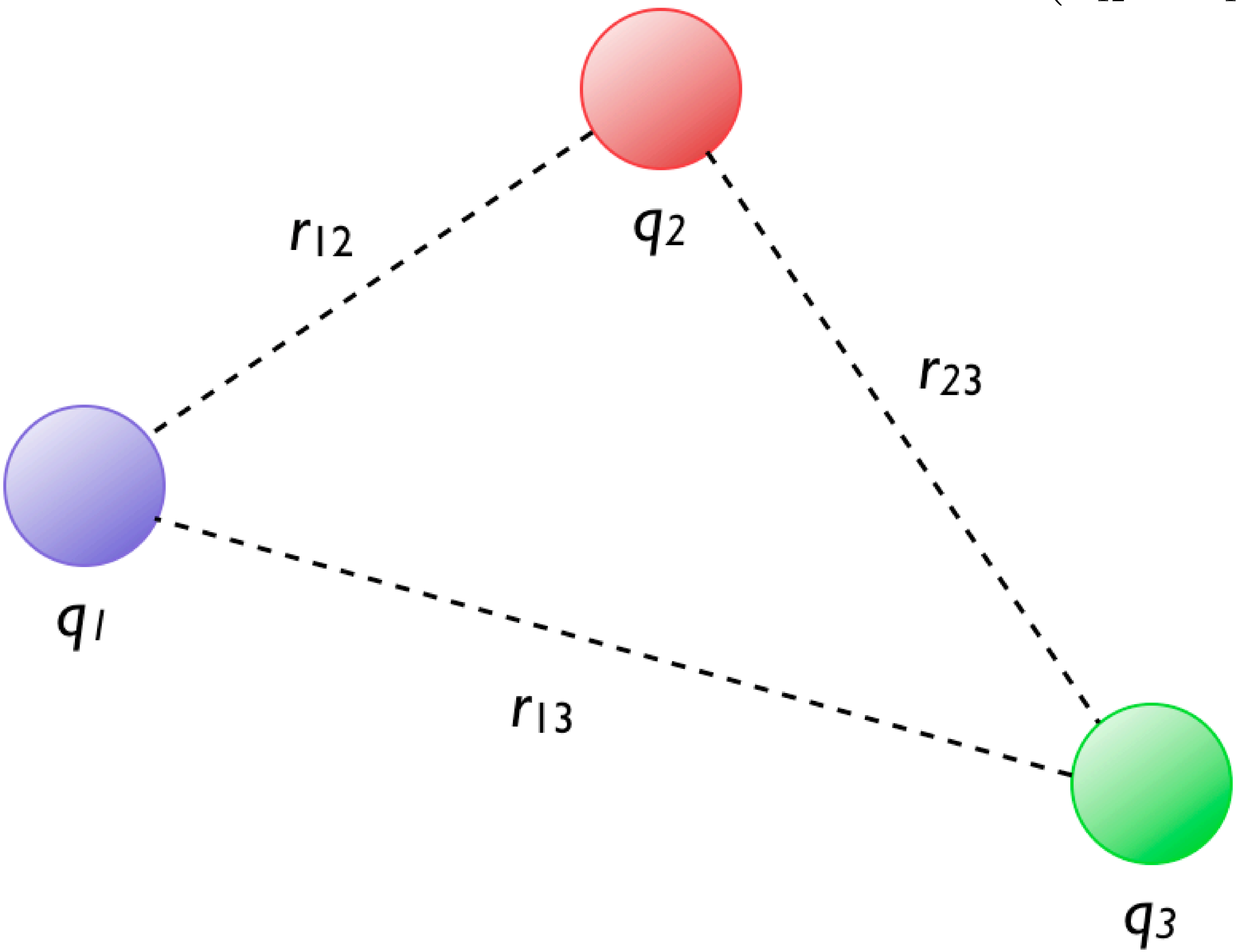
PE = (E required to build this thing)

oh noes, what about three charges?

still just pairs.



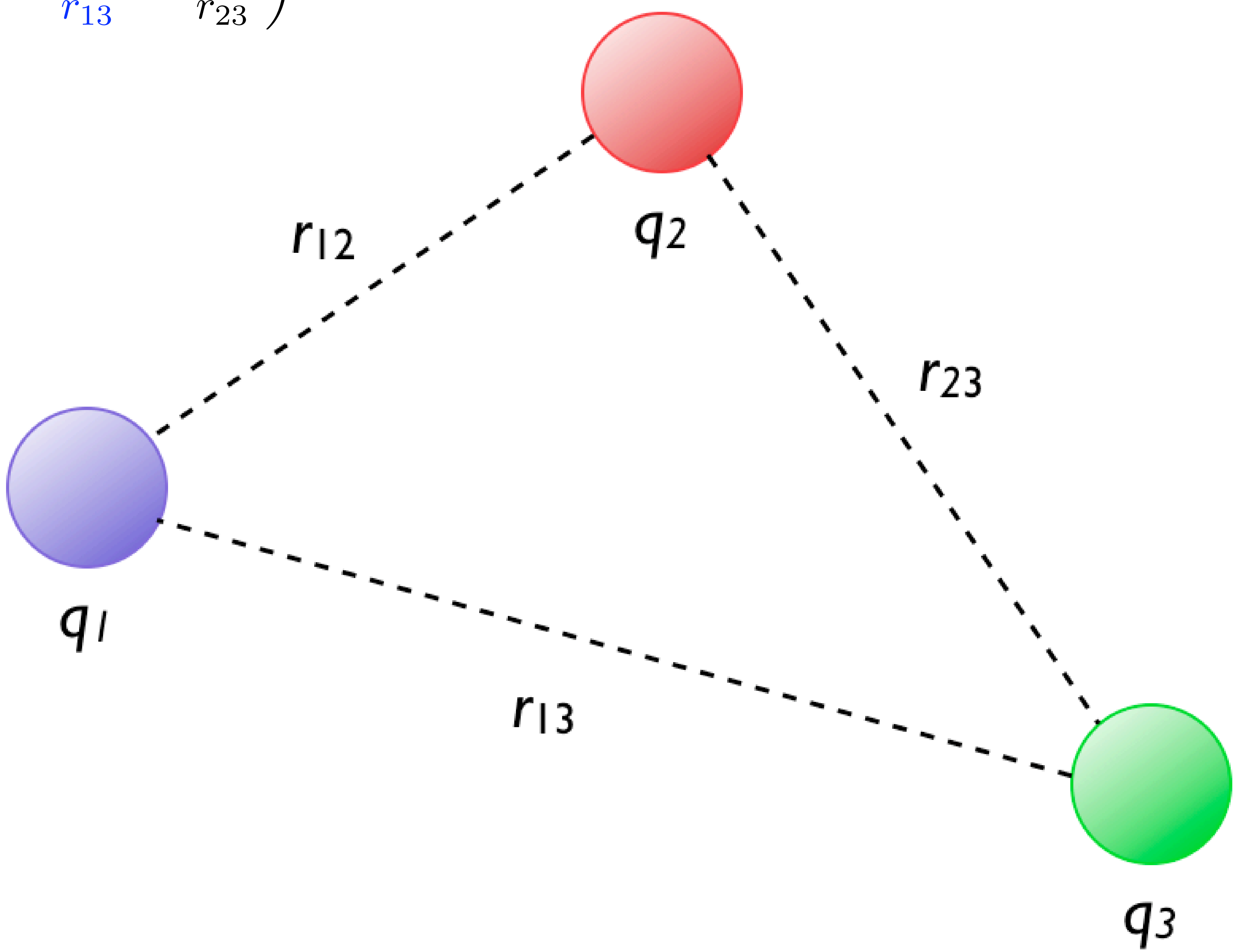
$$PE = PE_{1\&2} + PE_{2\&3} + PE_{1\&3} = PE_{2\&1} + PE_{3\&2} + PE_{3\&1} = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$



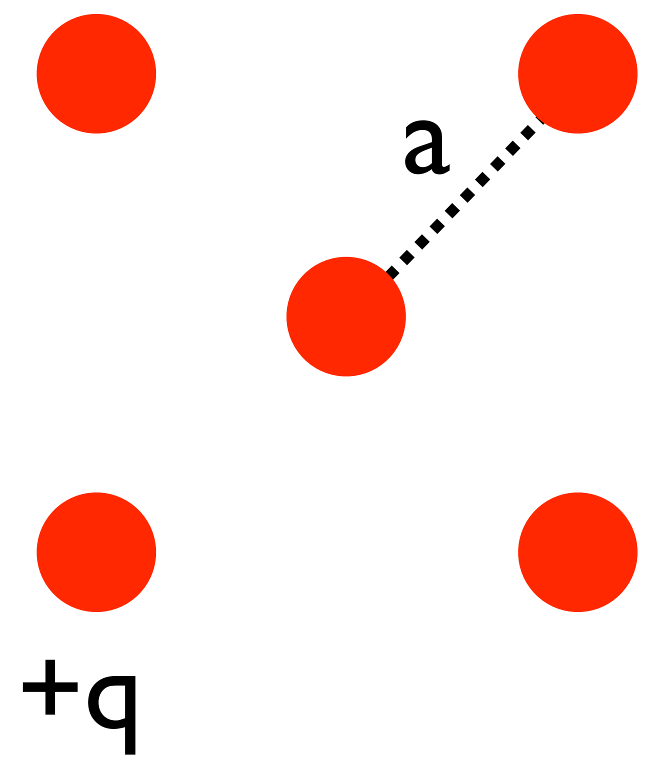
$$PE = \frac{1}{2}\sum_{i=1}^{3}\sum_{\substack{j=1\\j\neq i}}^{3}\frac{k_e q_i q_j}{r_i j}$$

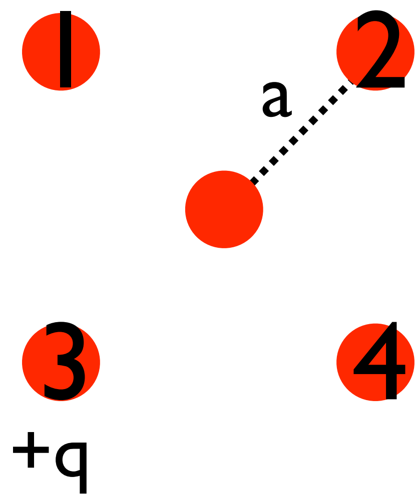
$$= \frac{1}{2}\left(\frac{k_e q_2 q_1}{r_{21}} + \frac{k_e q_3 q_1}{r_{31}} + \frac{k_e q_1 q_2}{r_{12}} + \frac{k_e q_3 q_2}{r_{32}} + \frac{k_e q_1 q_3}{r_{13}} + \frac{k_e q_2 q_3}{r_{23}}\right)$$

$$= k_e\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$



what is the potential energy of the “crystal”





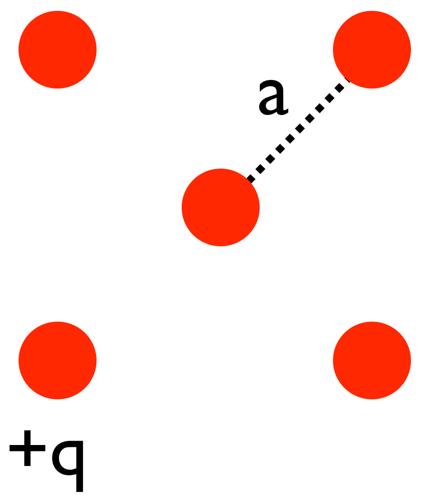
we just have to sum the energy of all unique pairs of charges.

so how many are there?

$$\text{ways of choosing pairs from five charges} = \binom{5}{2} = {}^5C_2 = \frac{5!}{2!\,(5-2)!} = \frac{5\cdot4\cdot3\cdot2\cdot1}{2\cdot1\cdot3\cdot2\cdot1} = 10$$

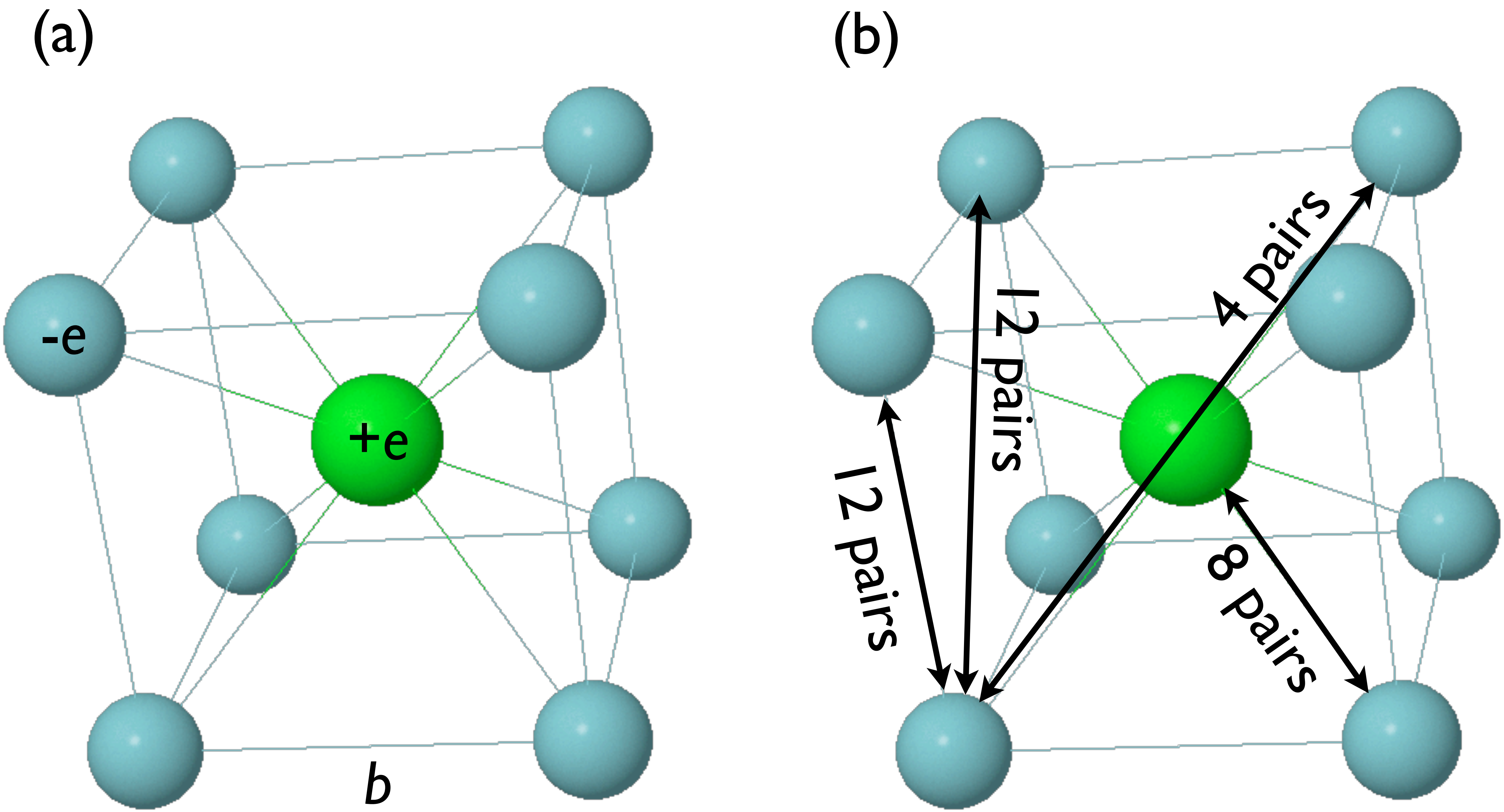
$$\begin{array}{llll}
(1,2) & (1,3) & (1,4) & (1,5) \\
(2,3) & (2,3) & (2,5) & \\
(3,4) & (3,5) & & \\
(4,5) & & &
\end{array}$$

#, pairing type	**separation**				**pairs**
4, center-corner	a	$(1,5)$	$(2,5)$	$(3,5)$	$(4,5)$
4, adjacent corners	$a\sqrt{2}$	$(1,4)$	$(3,4)$	$(2,3)$	$(1,2)$
2, far corner	$2a$			$(1,3)$	$(2,4)$

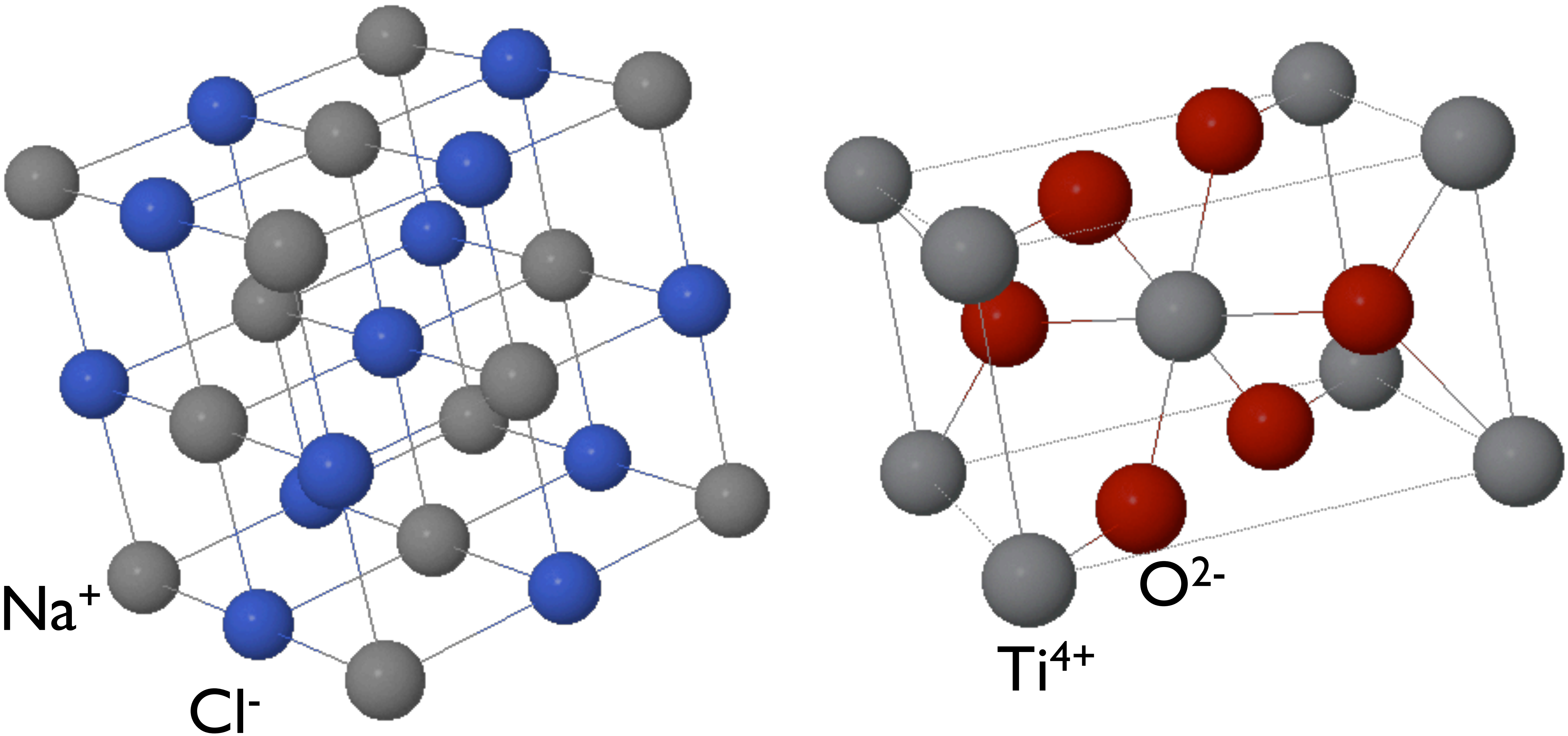


$$
\begin{aligned}
PE_{\text{square}} &= 4\,(\text{energy of center-corner pair}) + 2\,(\text{energy of far corner pair}) + 4\,(\text{energy of adjacent corner pair}) \\
&= 4\left[\frac{k_e q^2}{a}\right] + 2\left[\frac{k_e q^2}{2a}\right] + 4\left[\frac{k_e q^2}{a\sqrt{2}}\right] \\
&= \frac{k_e q^2}{a}\left[4 + 1 + \frac{4}{\sqrt{2}}\right] \\
&= \frac{k_e q^2}{a}\left[5 + 2\sqrt{2}\right] \approx 7.83\frac{kq^2}{a}
\end{aligned}
$$

it works for more complicated stuff



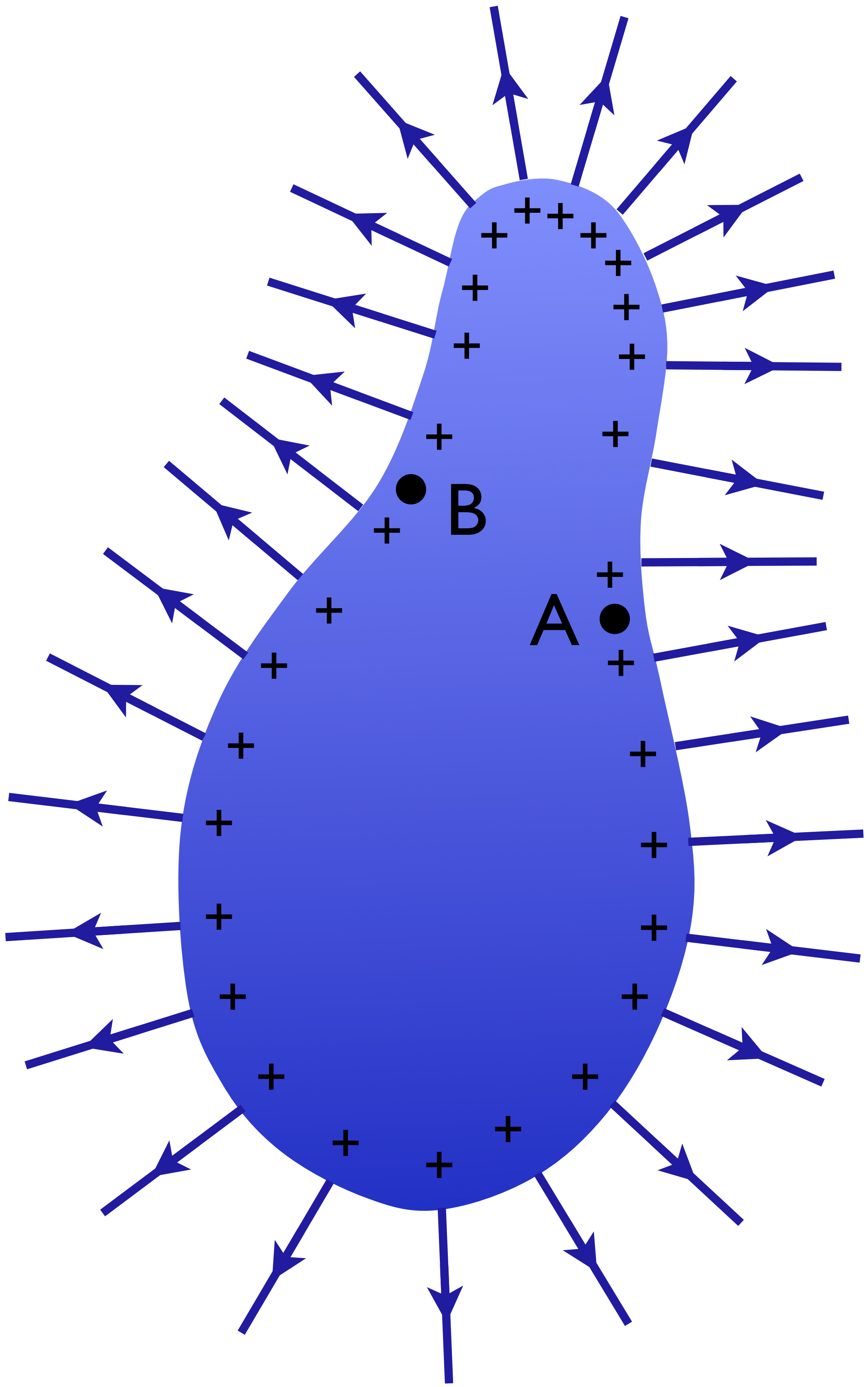
(a) Rocksalt (b) Rutile



M = -1.75

M = -4.82

Potential & Conductors

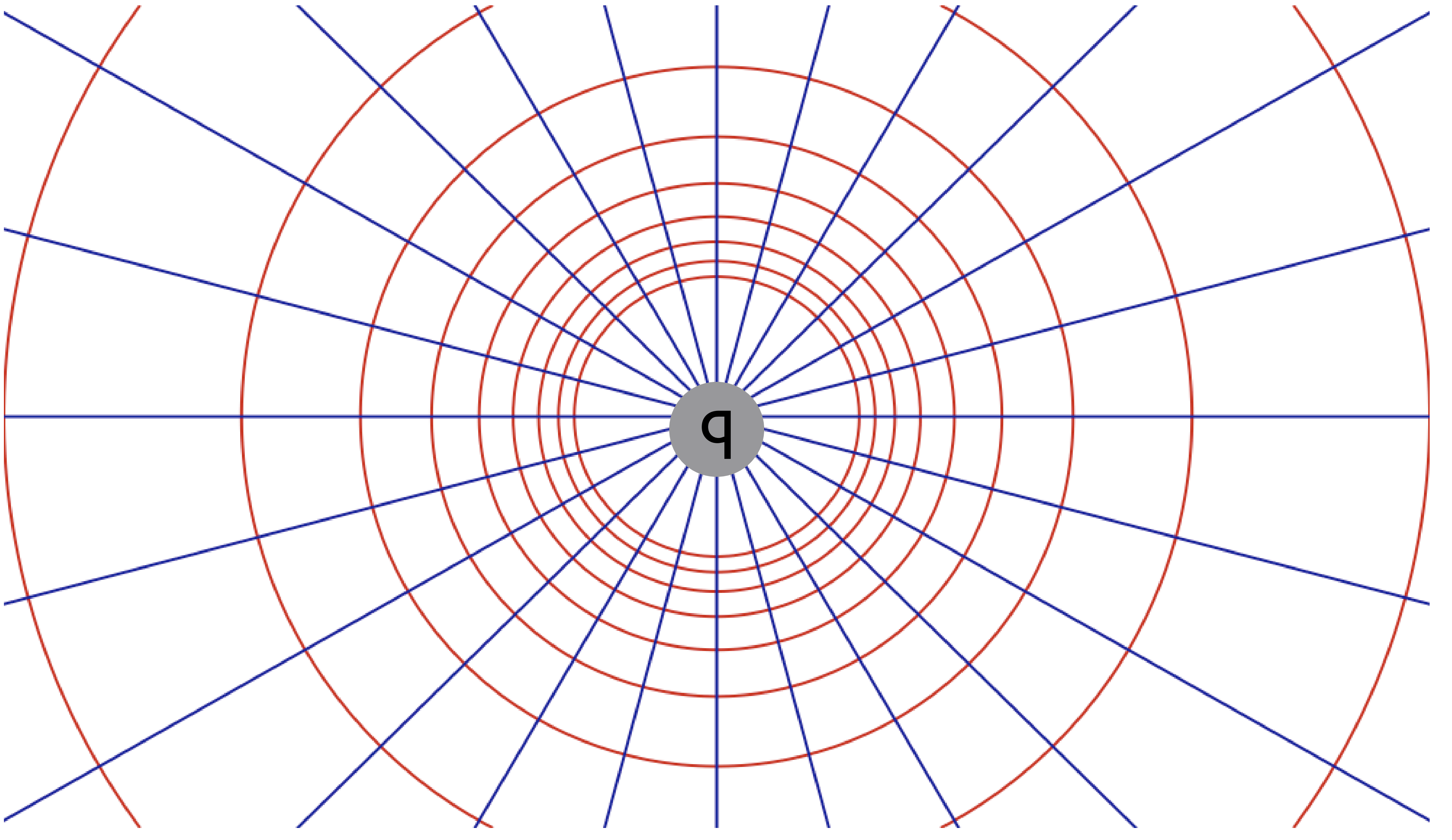


travel along surface:
E perpendicular to path everywhere

no work done!
can move on surface for free

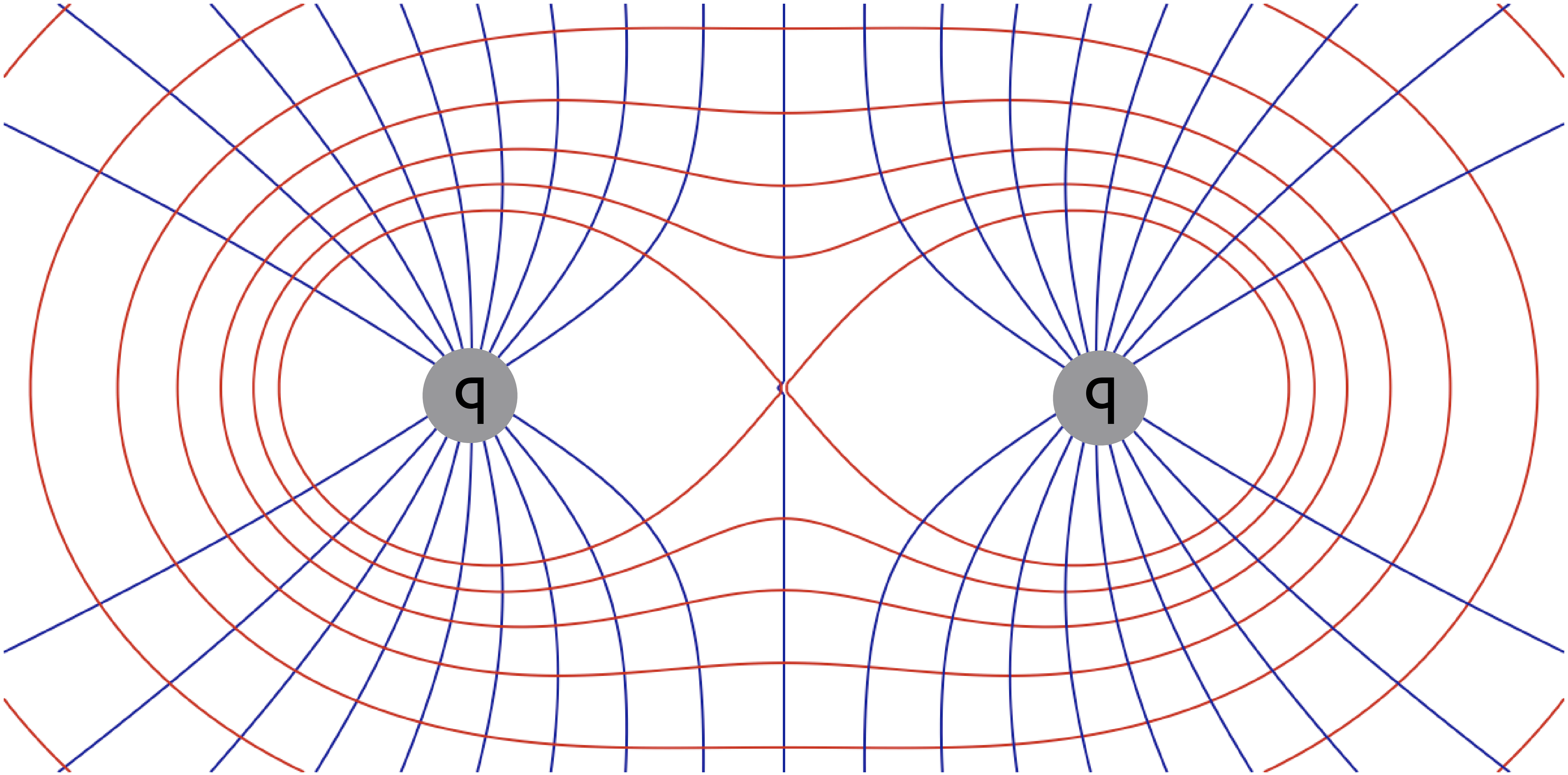
electric force is conservative ...

potential energy depends *only* on position, not path



lines of equal potential are perpendicular to field lines

equal *field* = constant force; *move* this way, change E
equal *potential* = equal energy; can *stay* here, but need KE

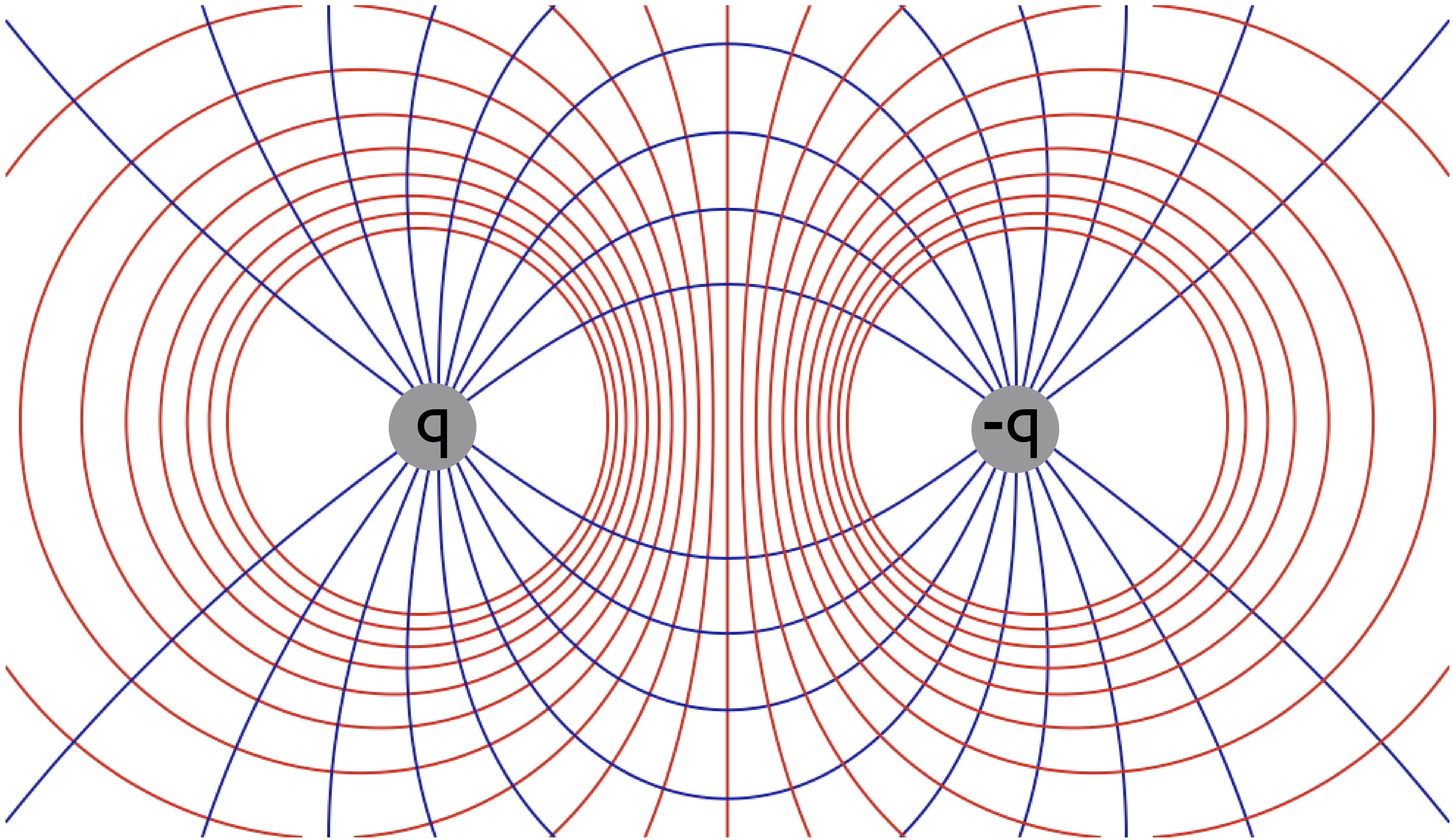


contours of const E = constant force

can move along these, but change energy

contours of const V = constant energy

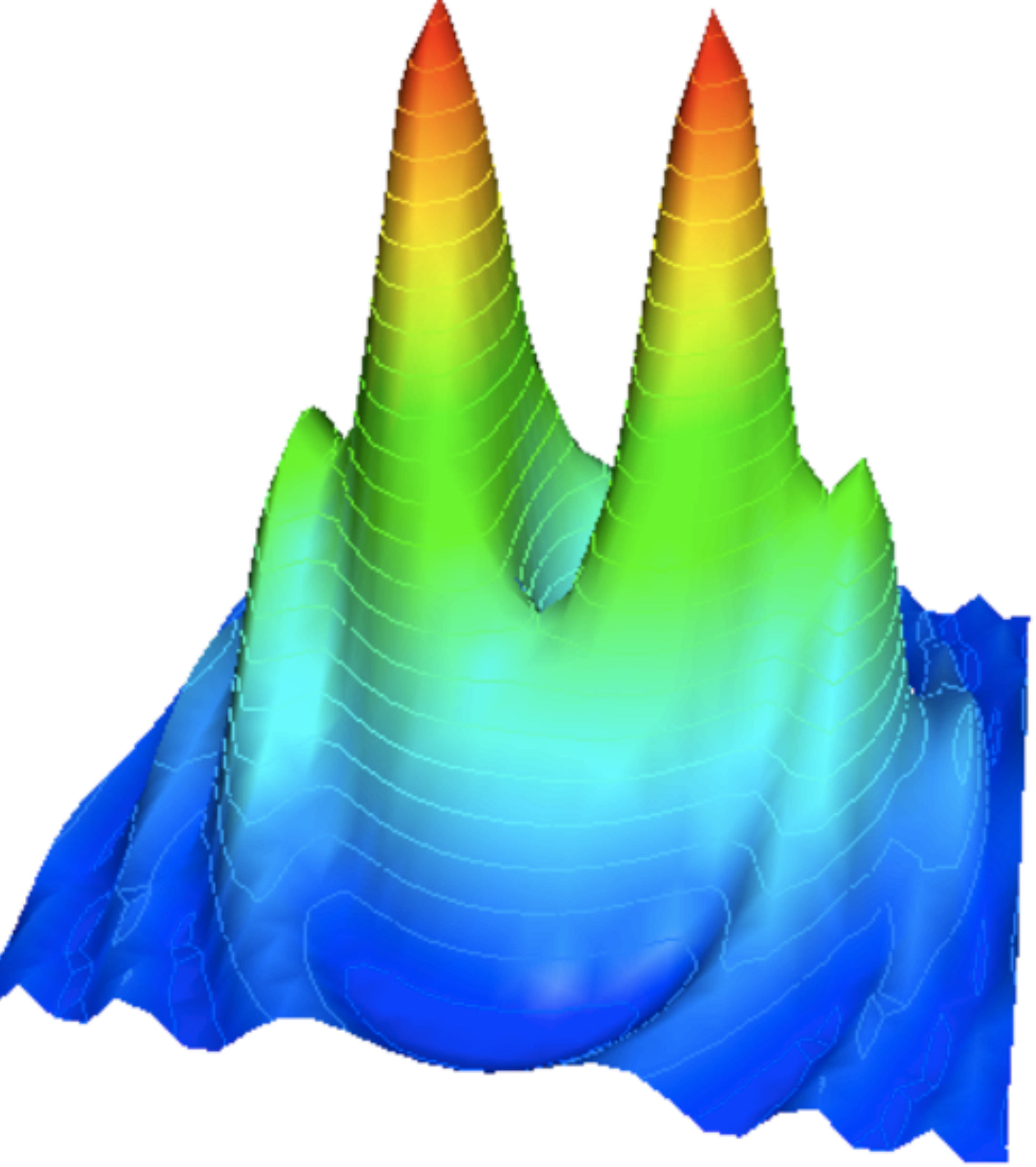
can move along these without changing E



V contours - places you can exist with a certain energy

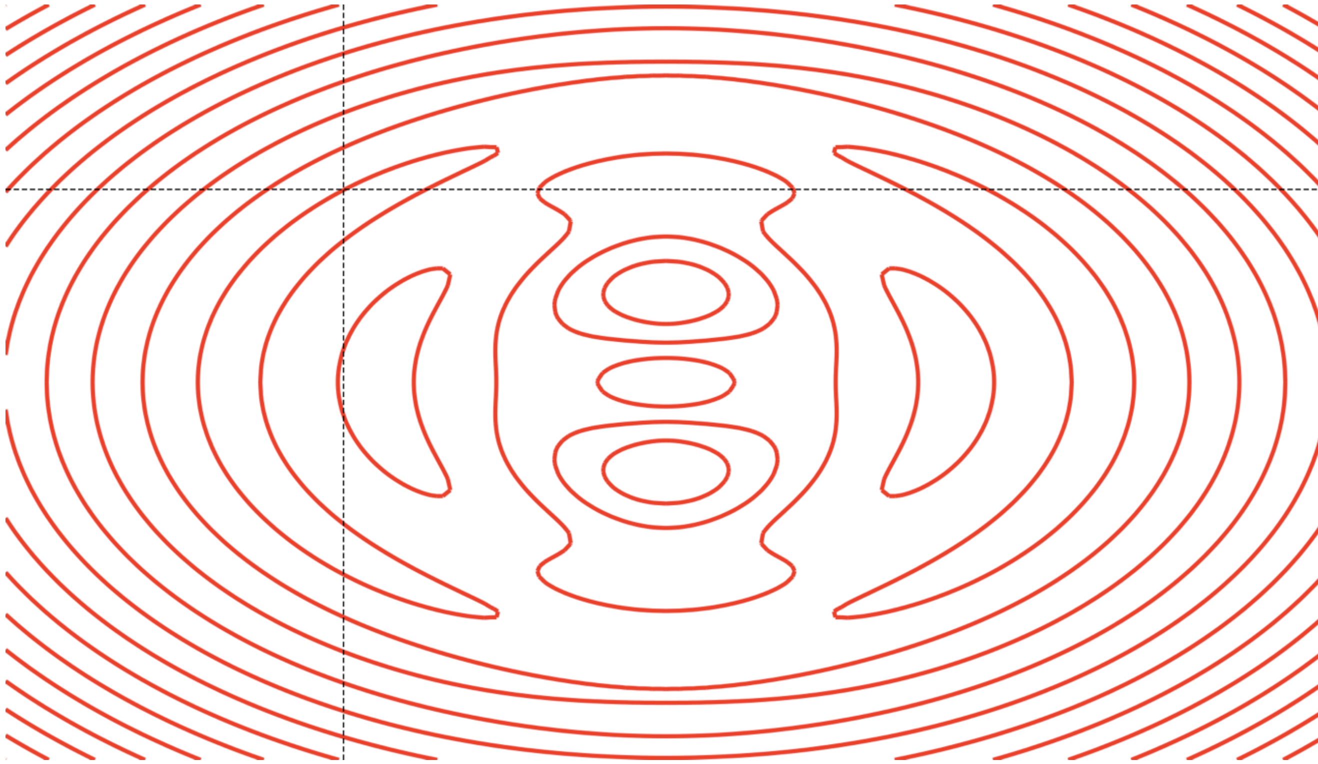
equipotential lines?
contours of constant V

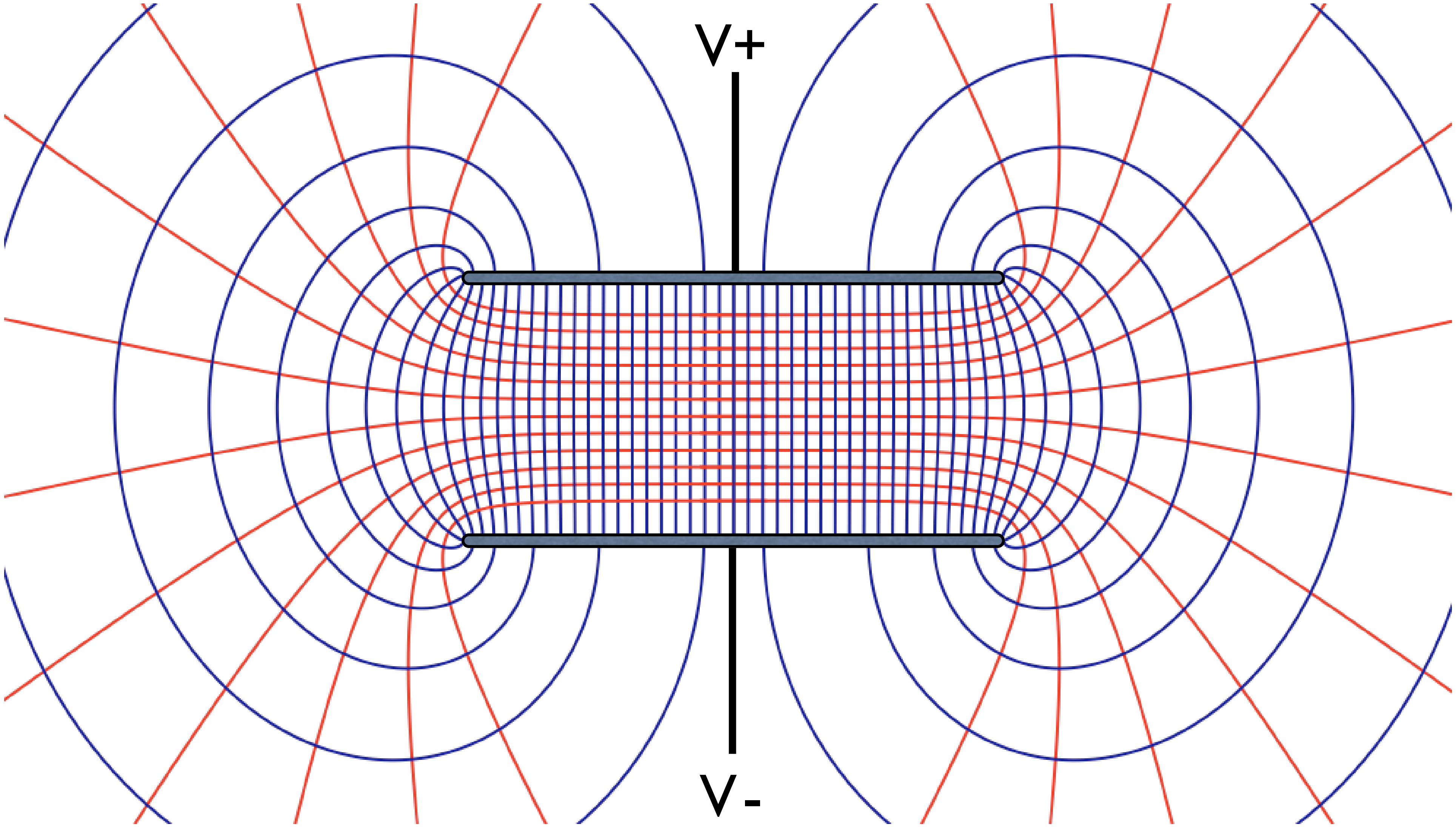
no work to
move along them
(like gravity)

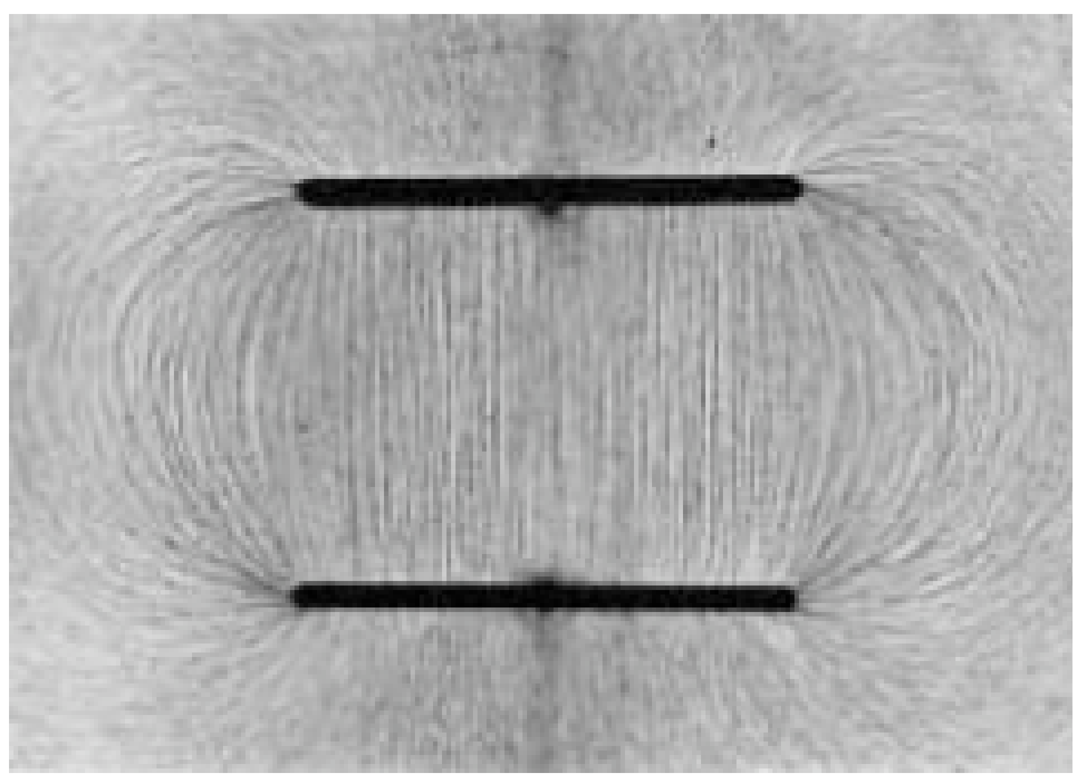


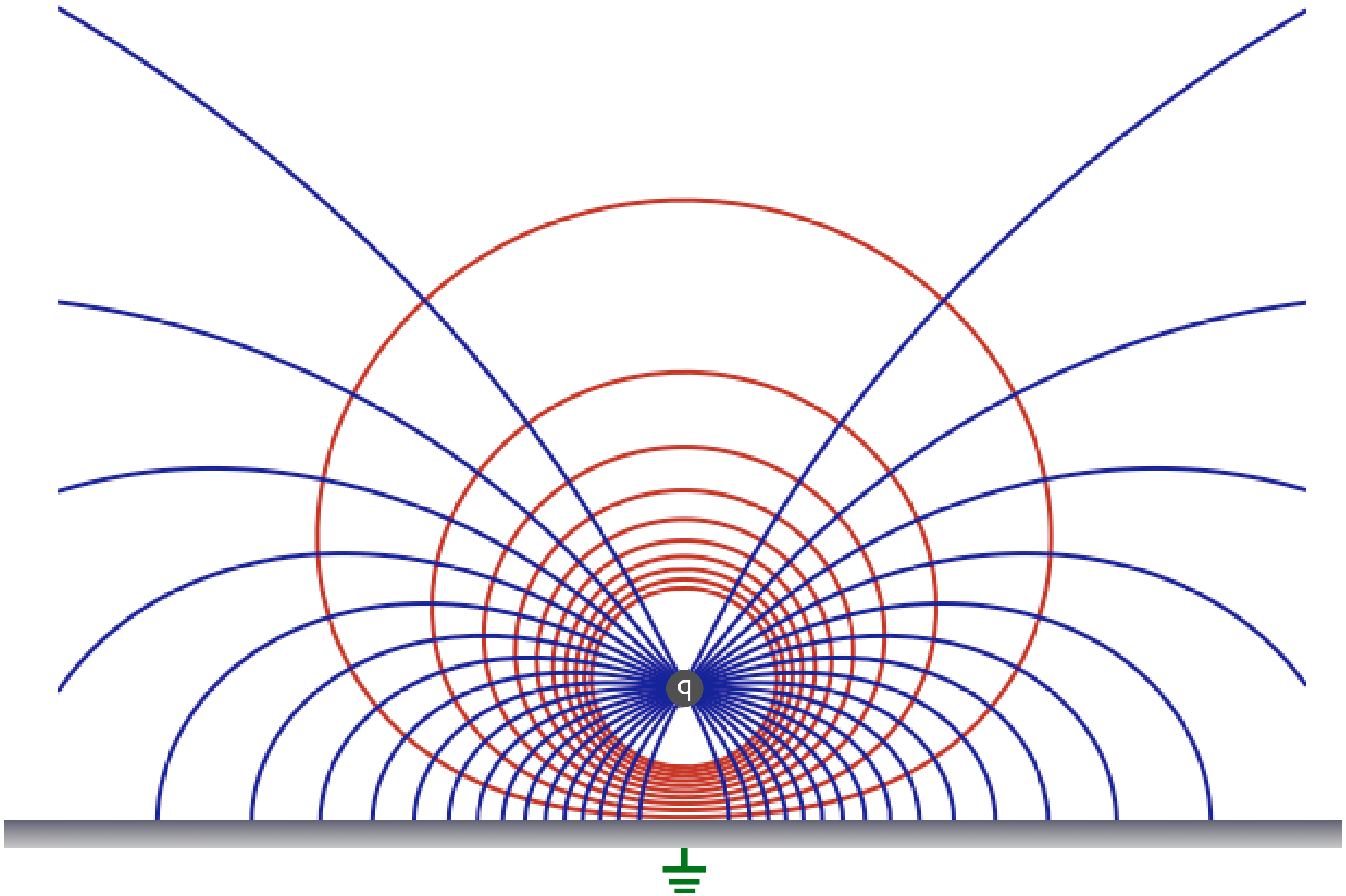
x, y = spatial coordinates
z = electric potential
3d

x, y = spatial coordinates
potential constant on lines
2d

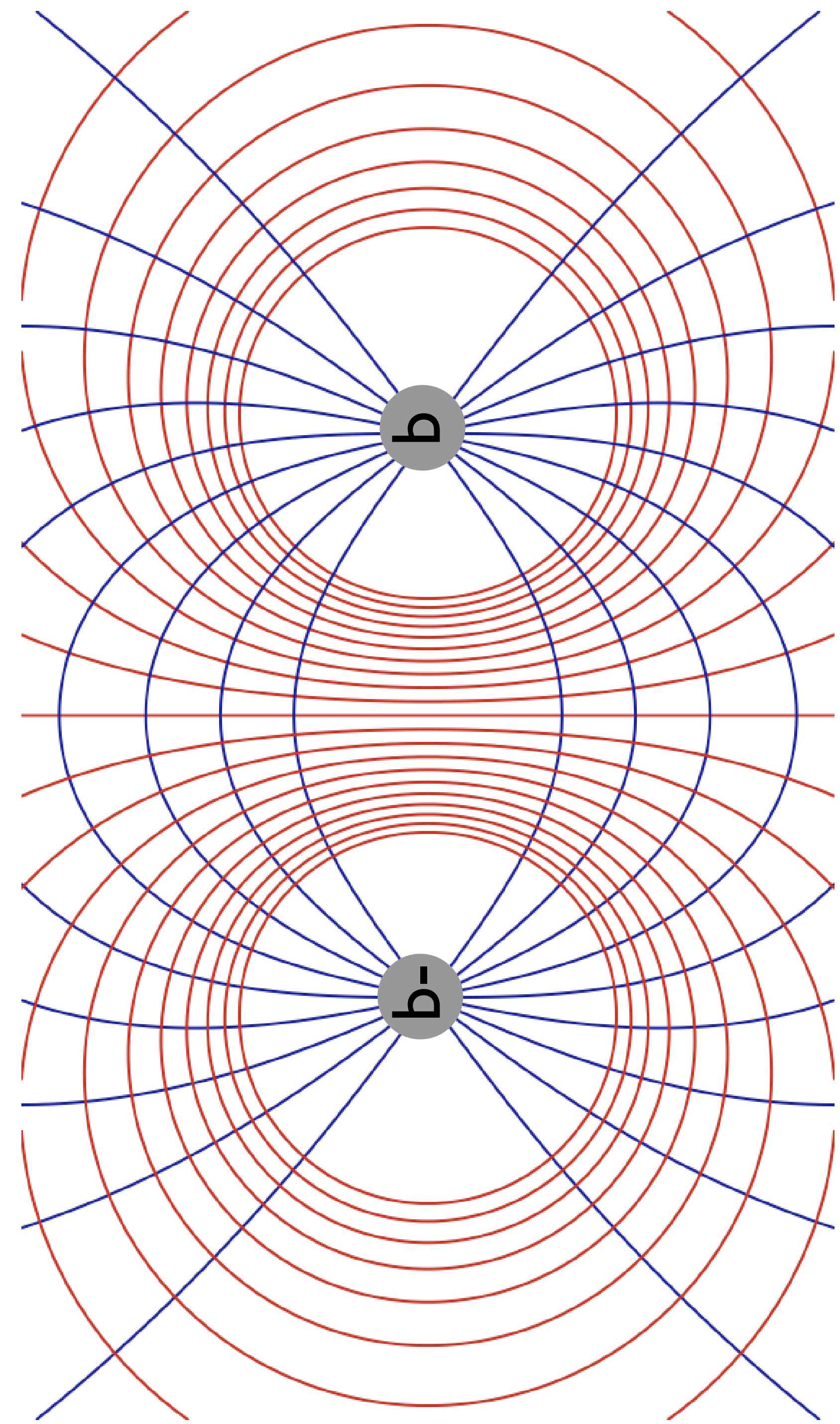



V+
V-

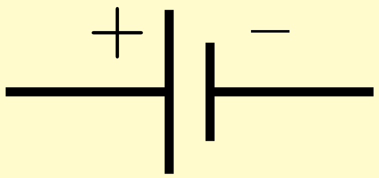
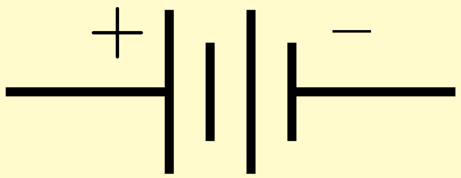


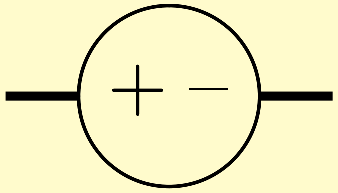


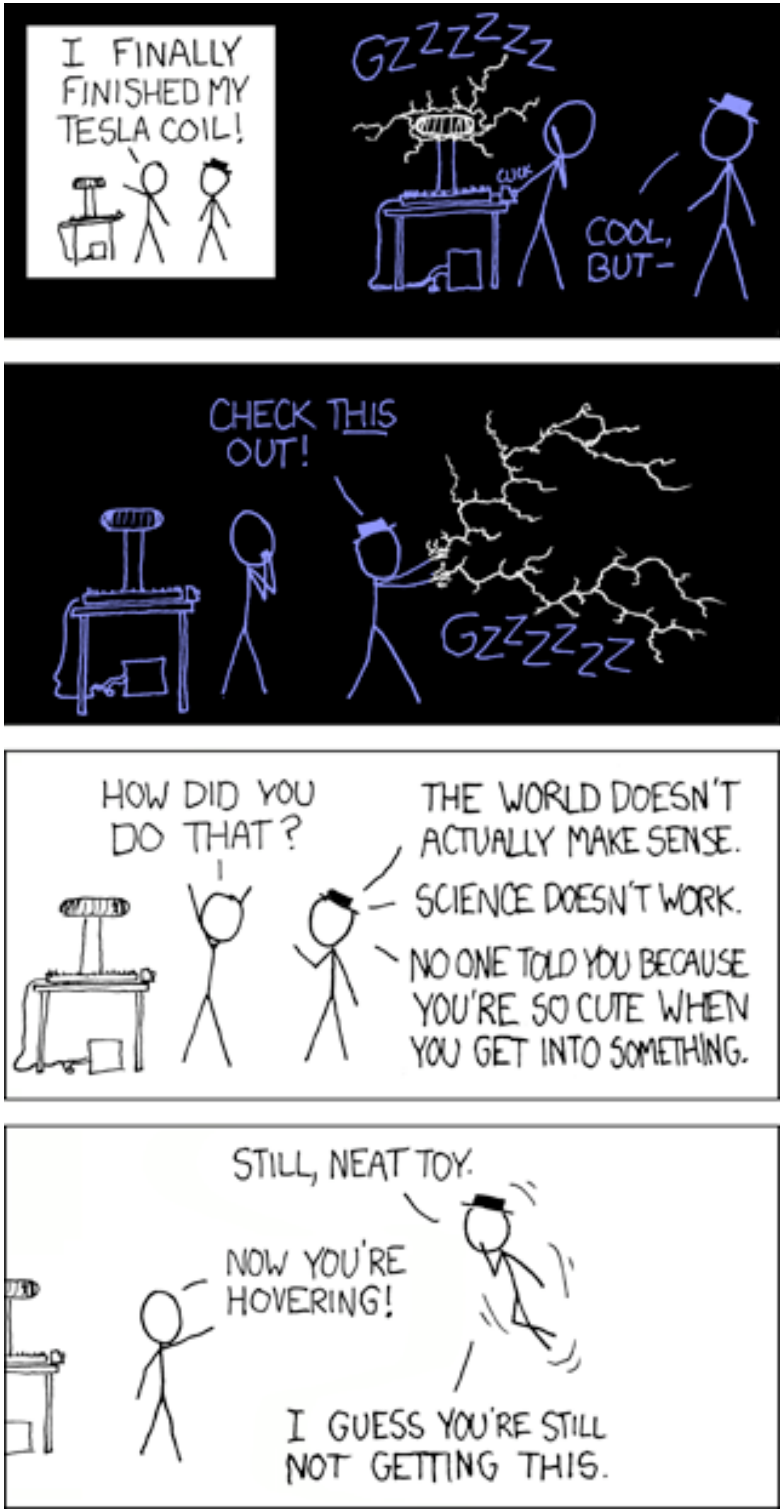
conductor = mirror for field & potential lines

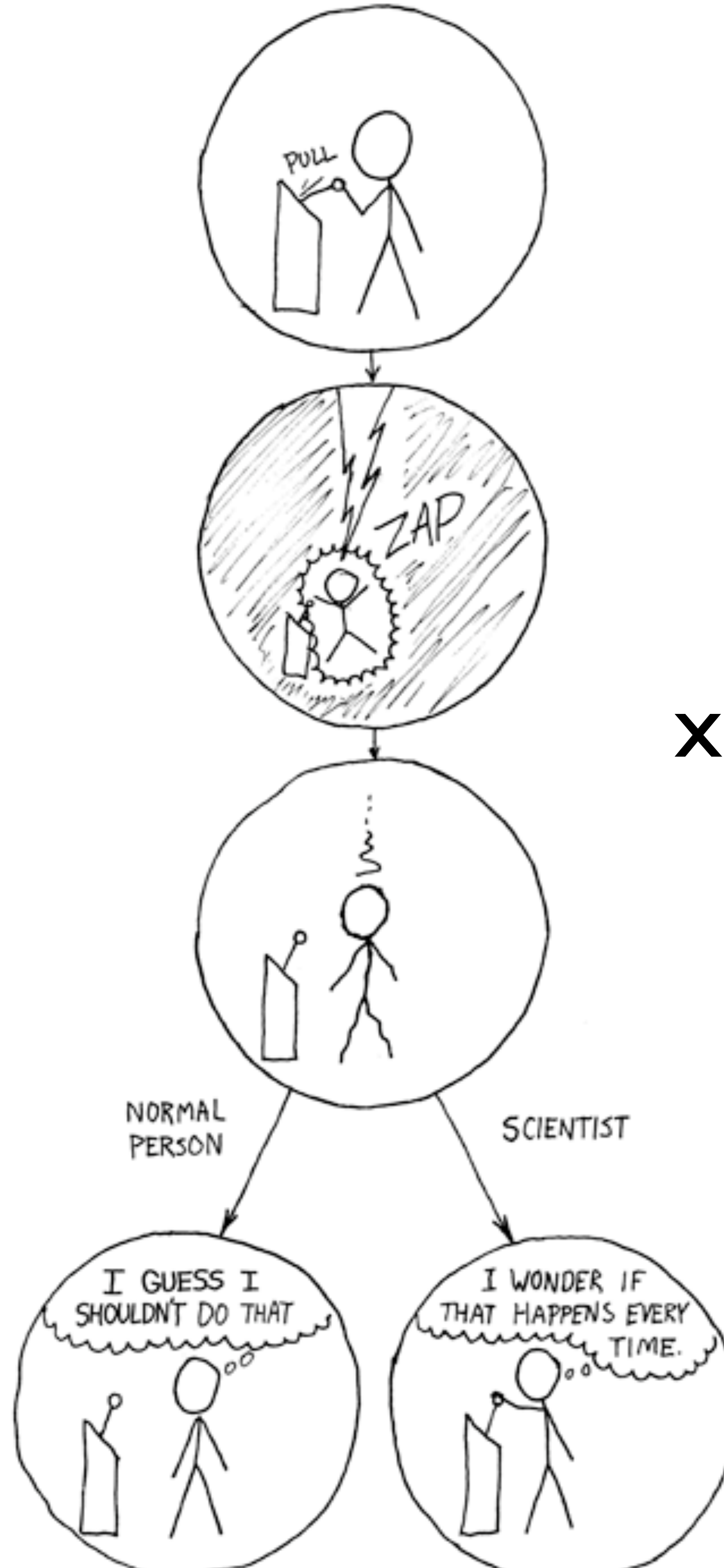


Circuit diagram symbol for voltage sources:

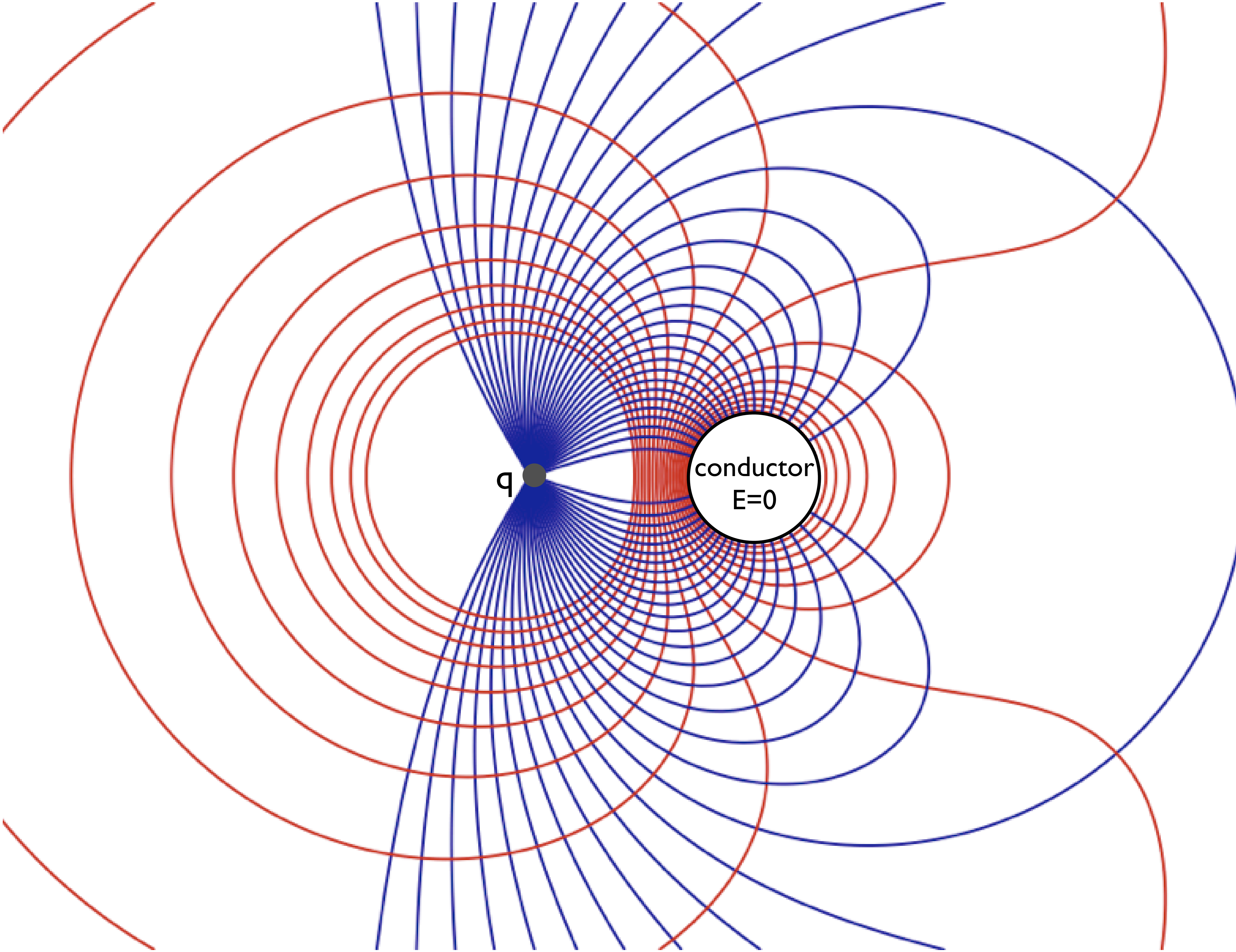
Batteries:  

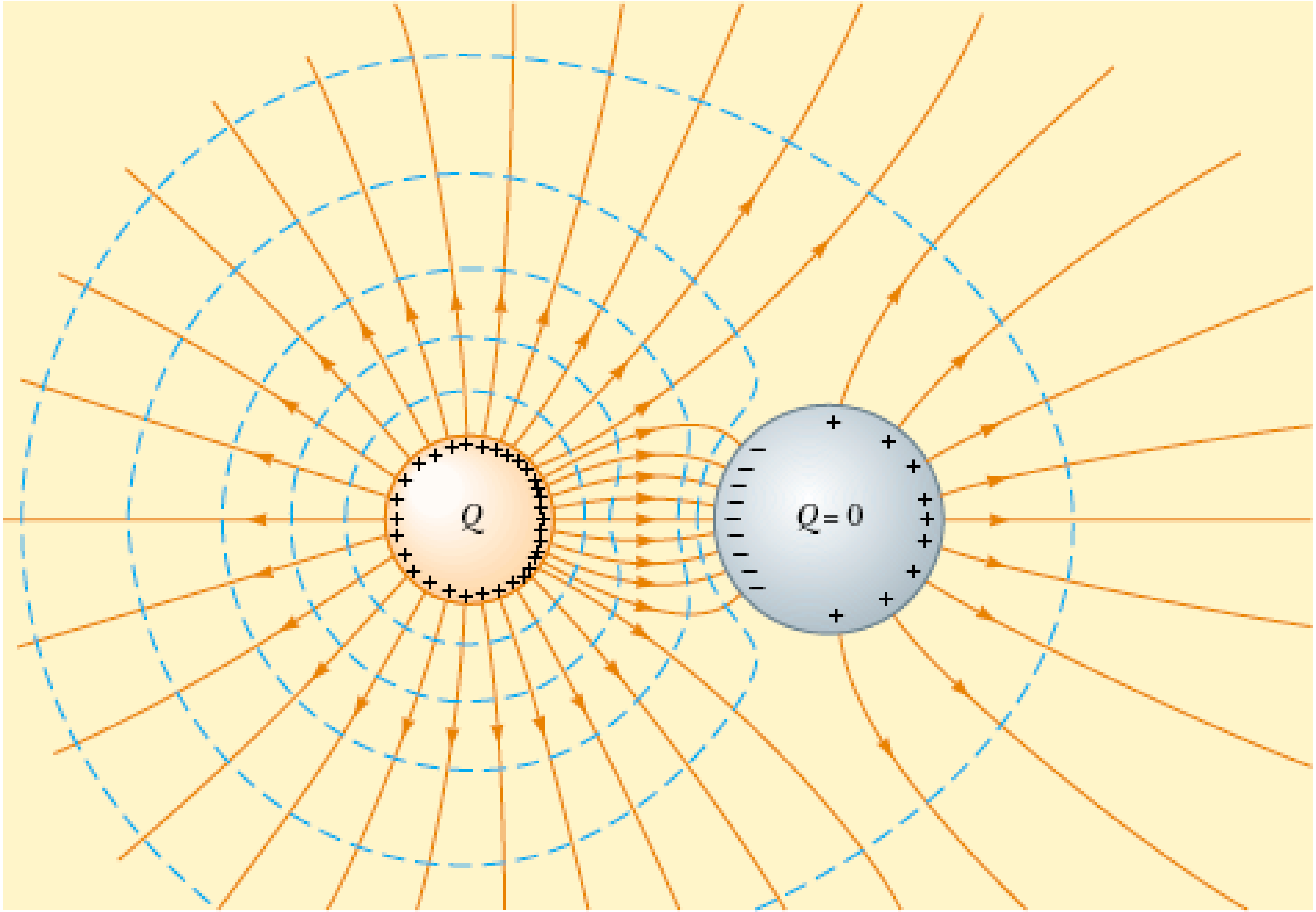
General constant voltage source: 


I FINALLY FINISHED MY TESLA COIL!
GZZZZZZ
CLICK
COOL, BUT-
CHECK THIS OUT!
GZZZZZZ
HOW DID YOU DO THAT?
THE WORLD DOESN'T ACTUALLY MAKE SENSE.
SCIENCE DOESN'T WORK.
NO ONE TOLD YOU BECAUSE YOU'RE SO CUTE WHEN YOU GET INTO SOMETHING.
STILL, NEAT TOY.
NOW YOU'RE HOVERING!
I GUESS YOU'RE STILL NOT GETTING THIS.

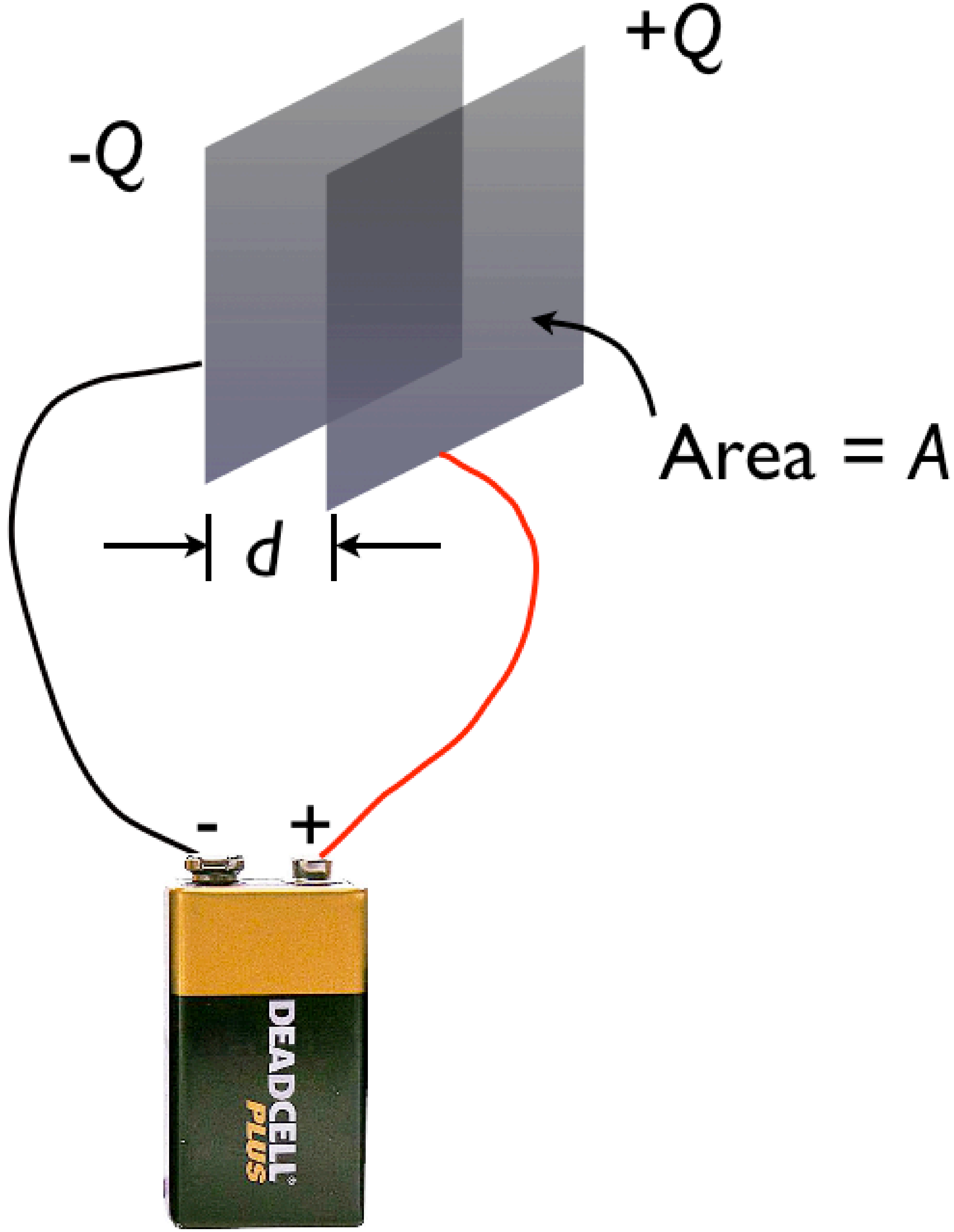

PULL
ZAP
NORMAL PERSON
SCIENTIST
I GUESS I SHOULDN'T DO THAT
I WONDER IF THAT HAPPENS EVERY TIME.

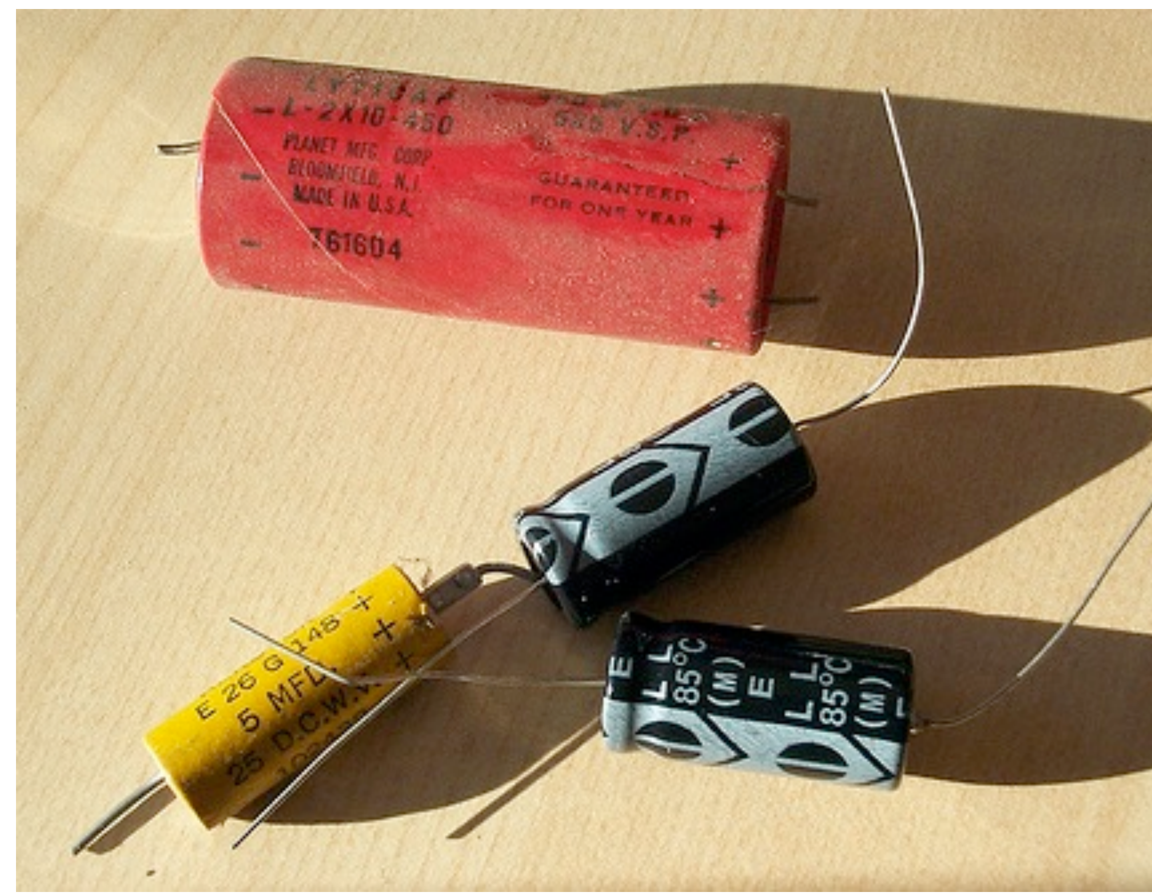
xkcd.com

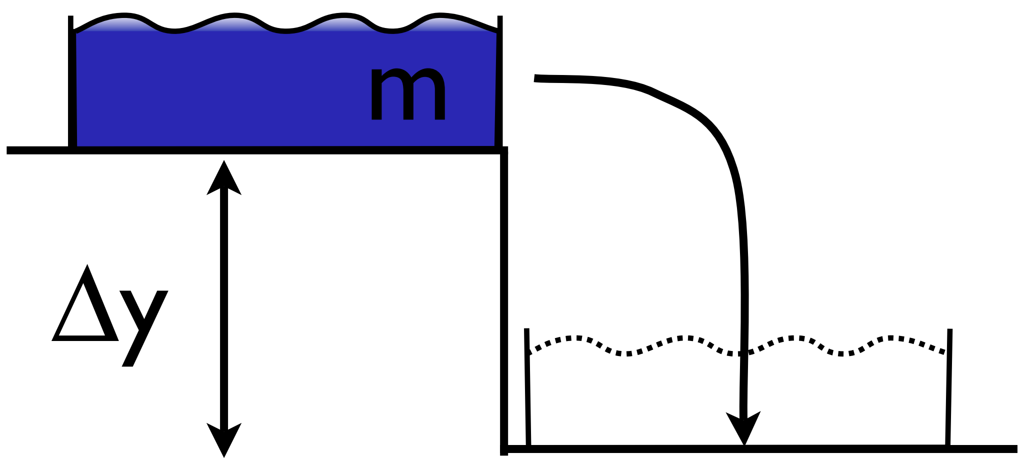

q
conductor
E=0



Q
$Q = 0$

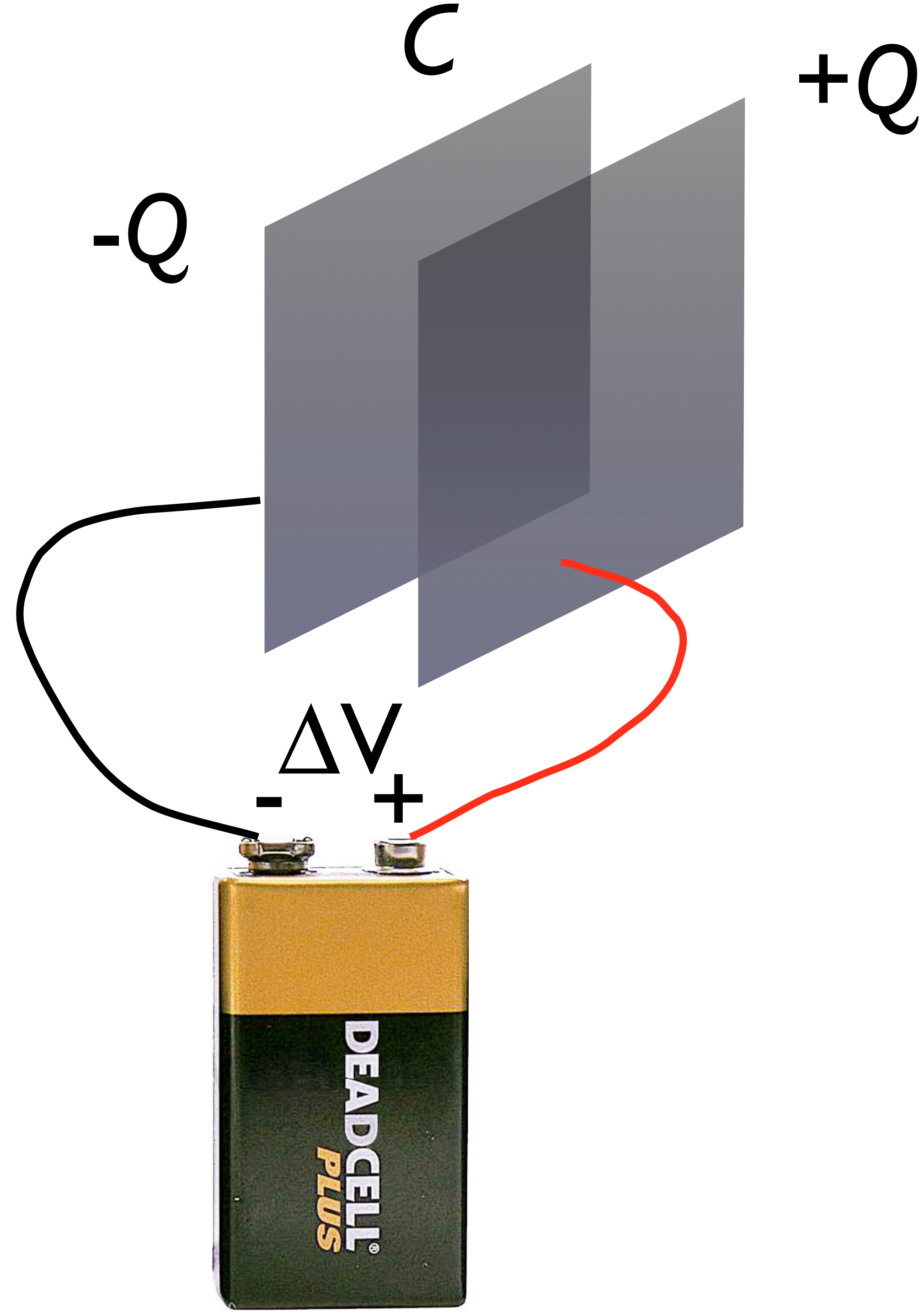

+Q
-Q
Area = A
d
-
+
DEADCELL
PLUS


GUARANTEED
FOR ONE YEAR
85°C



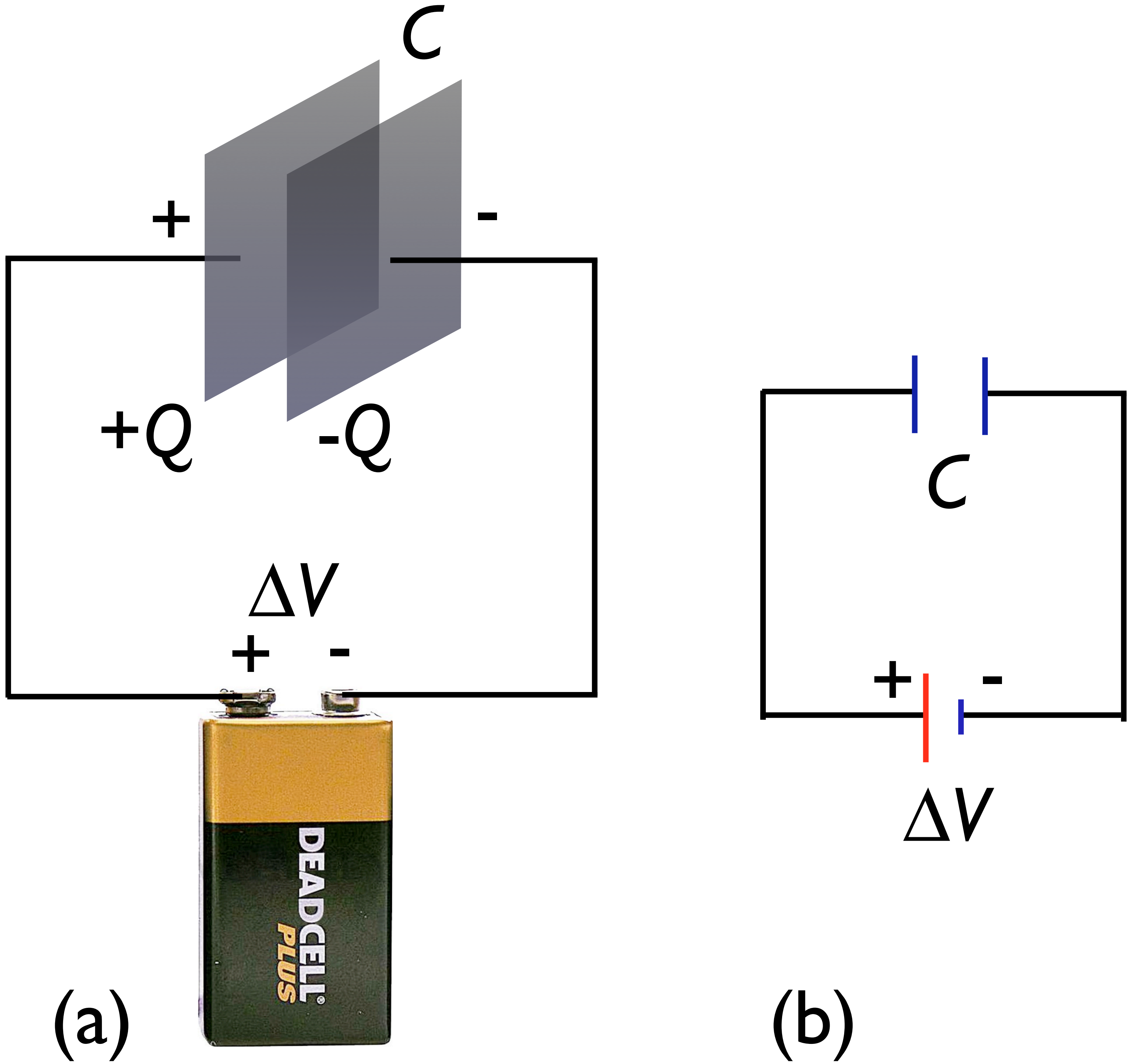
$$W = mg\Delta y$$

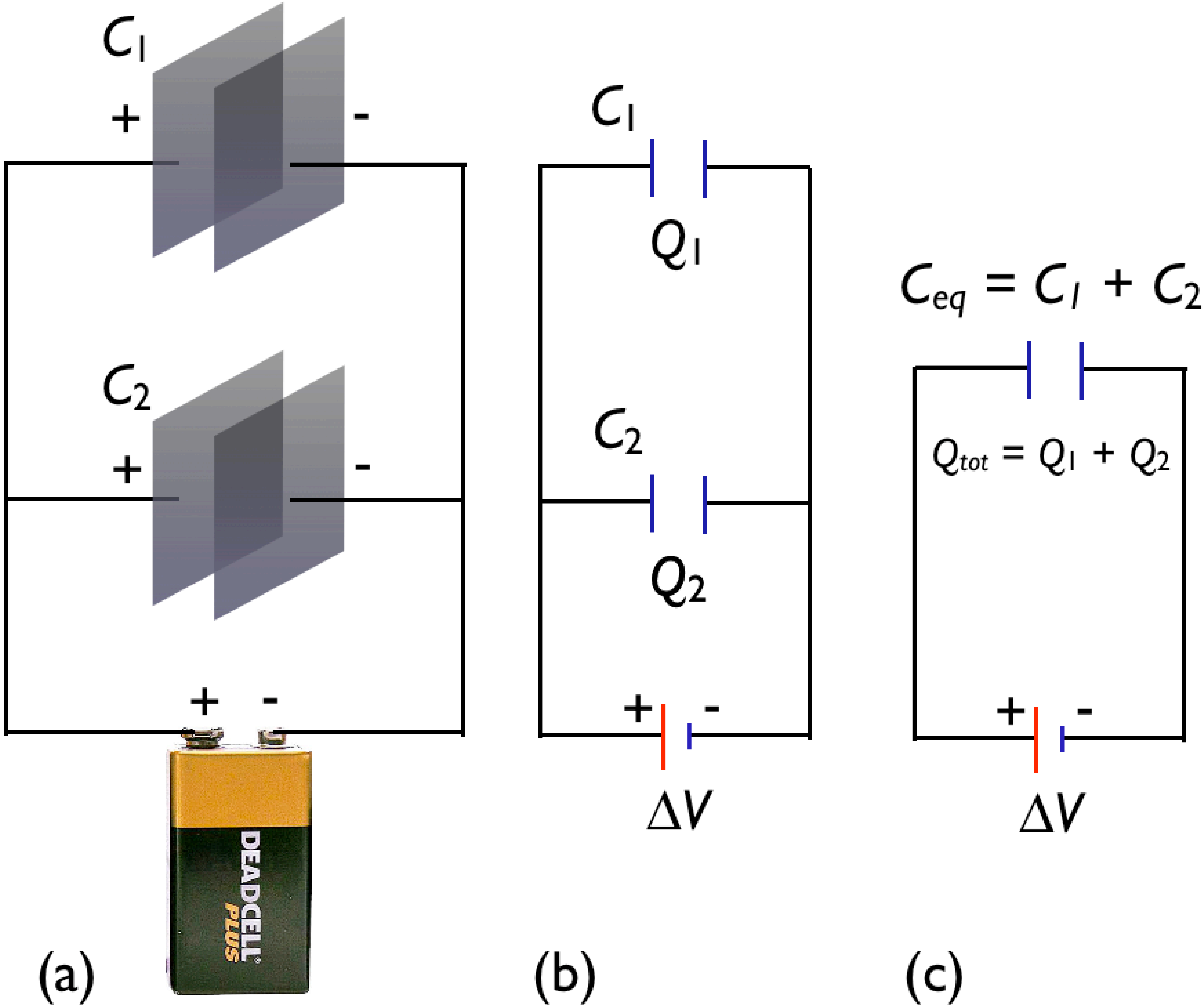
(a)



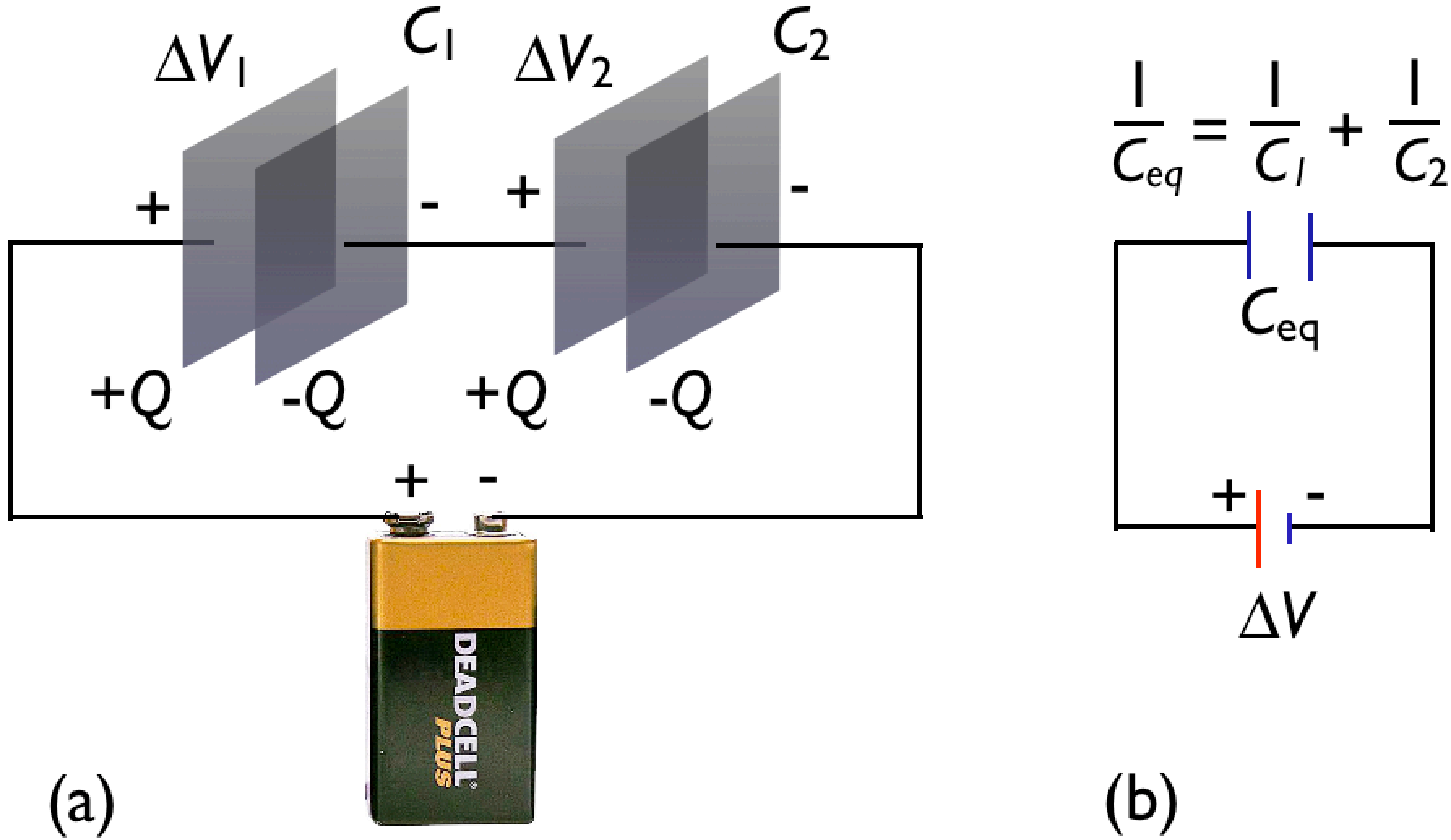
$$W = \tfrac{1}{2} Q\Delta V = \tfrac{1}{2} Q^2/C$$

(b)

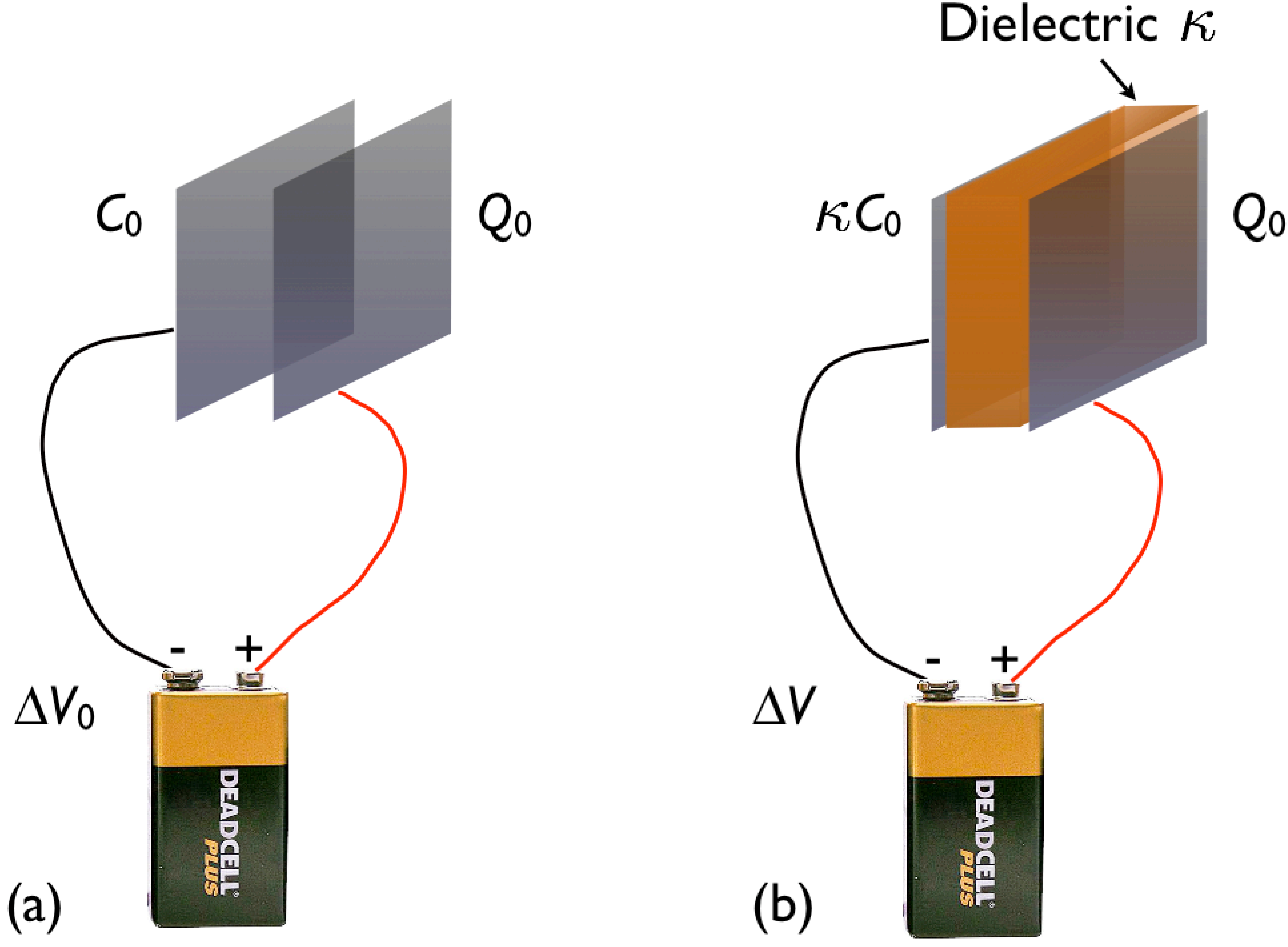

C
+
-
+Q
-Q
ΔV
+
-
DEADCELL
PLUS
(a)
C
+
-
ΔV
(b)



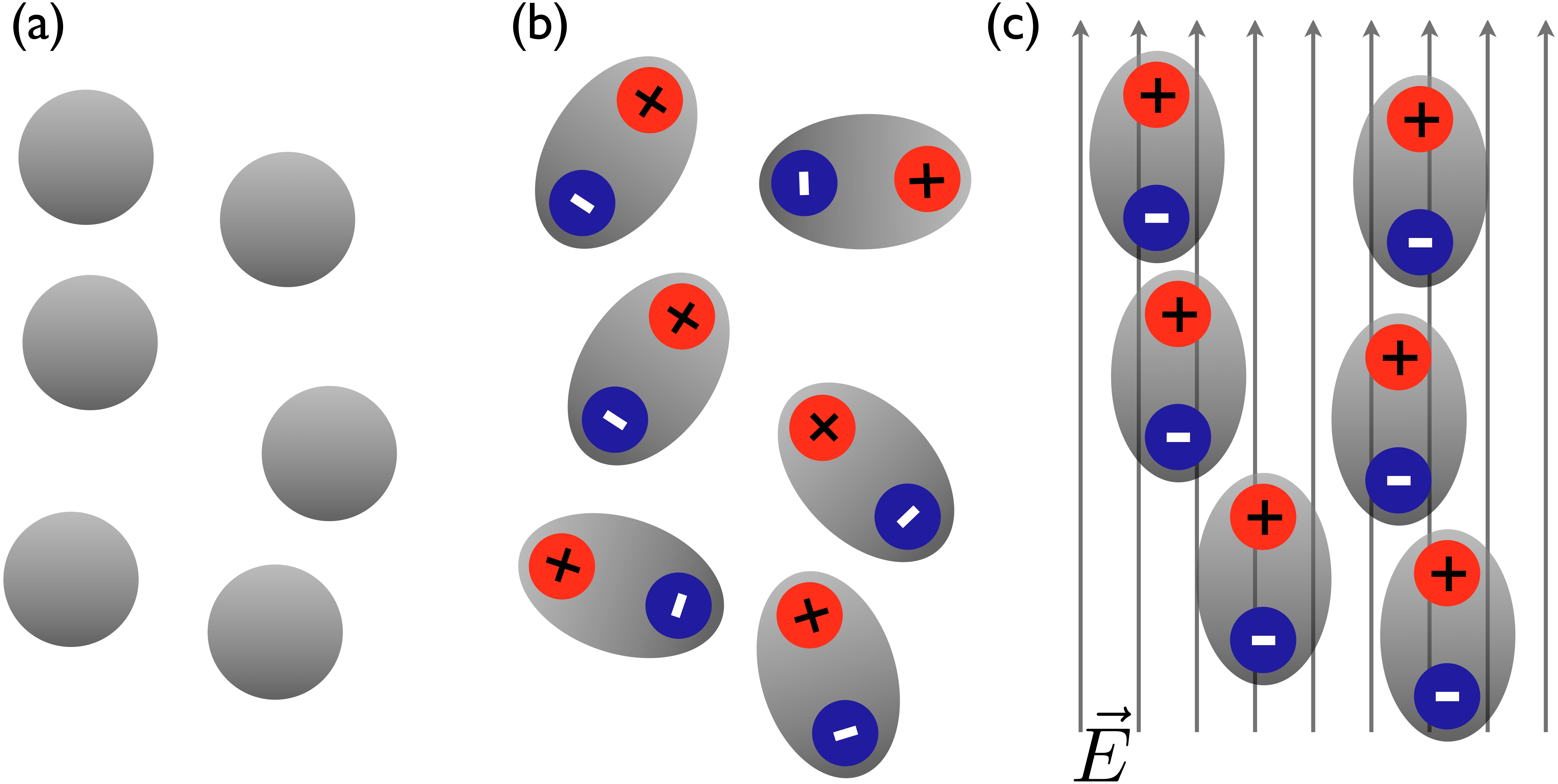
C1
+
-
C2
+
-
+ -
DEADCELL PLUS
(a)
C1
Q1
C2
Q2
+ -
ΔV
(b)
Ceq = C1 + C2
Qtot = Q1 + Q2
+ -
ΔV
(c)

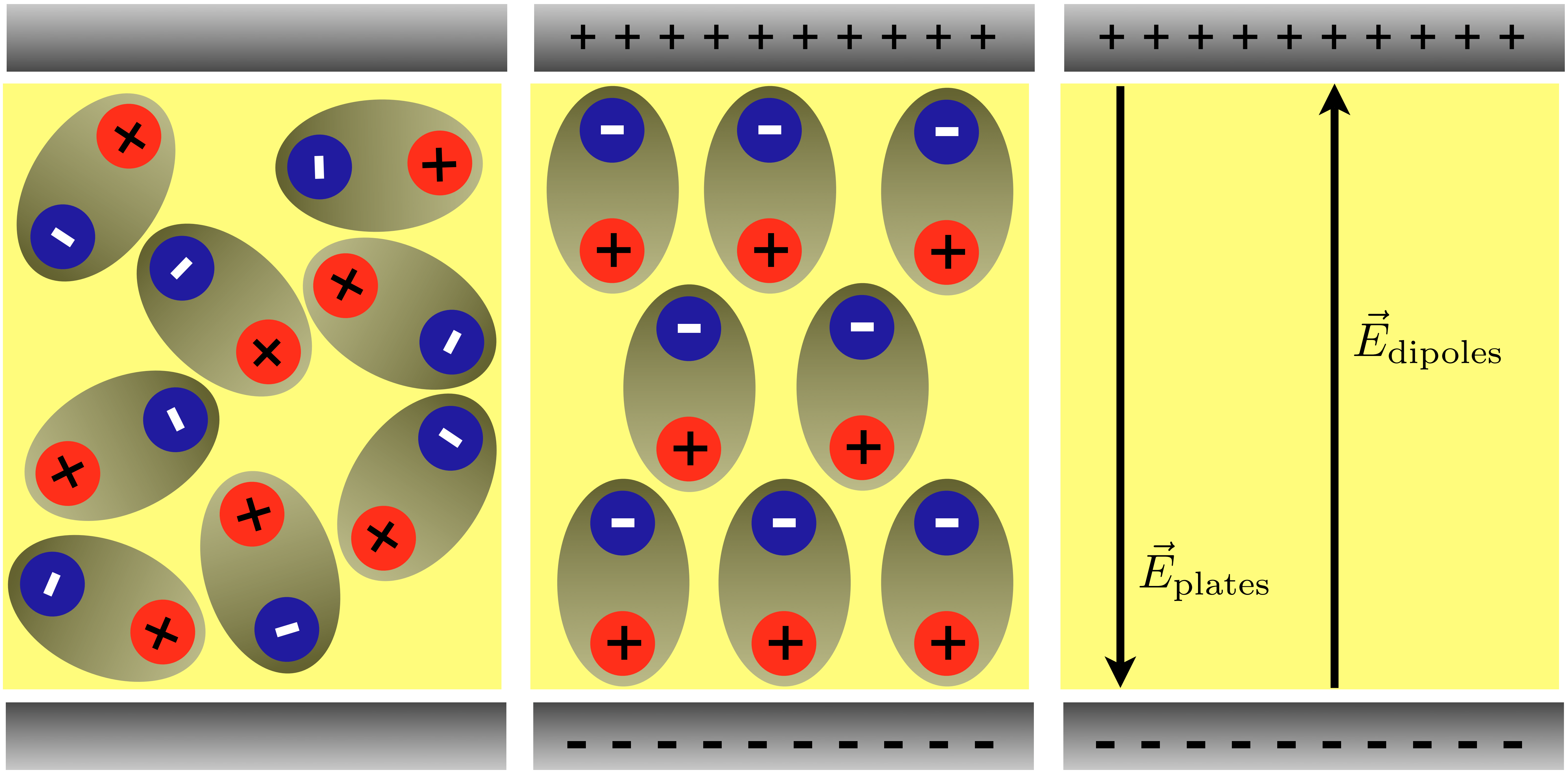


ΔV_1
C_1
ΔV_2
C_2
+
-
+
-
$+Q$
$-Q$
$+Q$
$-Q$
+
-
DEADCELL
PLUS
(a)
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$
C_{eq}
+
-
ΔV
(b)



C_0
Q_0
-
+
ΔV_0
DEADCELL
PLUS
(a)
Dielectric κ
κC_0
Q_0
-
+
ΔV
DEADCELL
PLUS
(b)


(a)
(b)
(c)
+
-
$\vec{E}$


$\vec{E}_{\text{dipoles}}$
$\vec{E}_{\text{plates}}$