Relativity, part 2

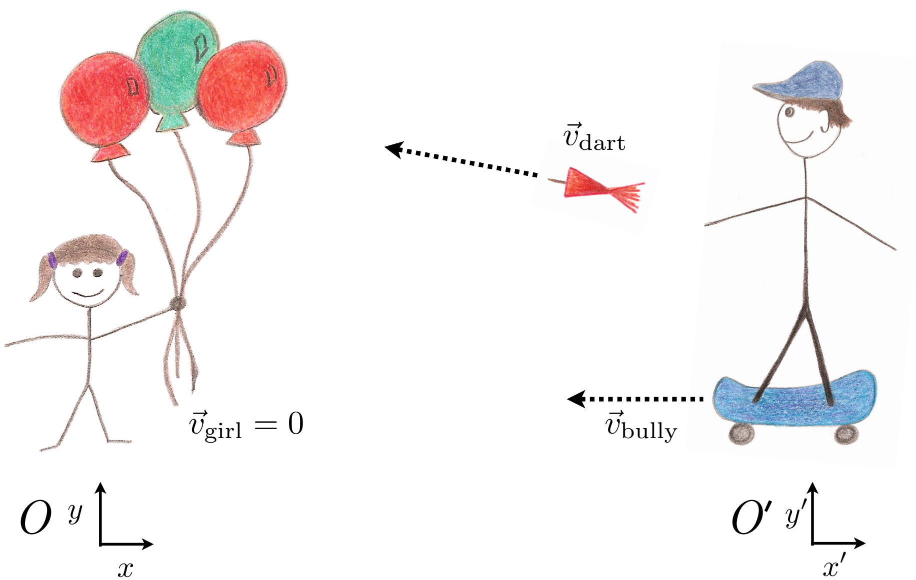
What is allowed?

- relativity: physics is the same for all observers
- so light travels at the same speed for everyone

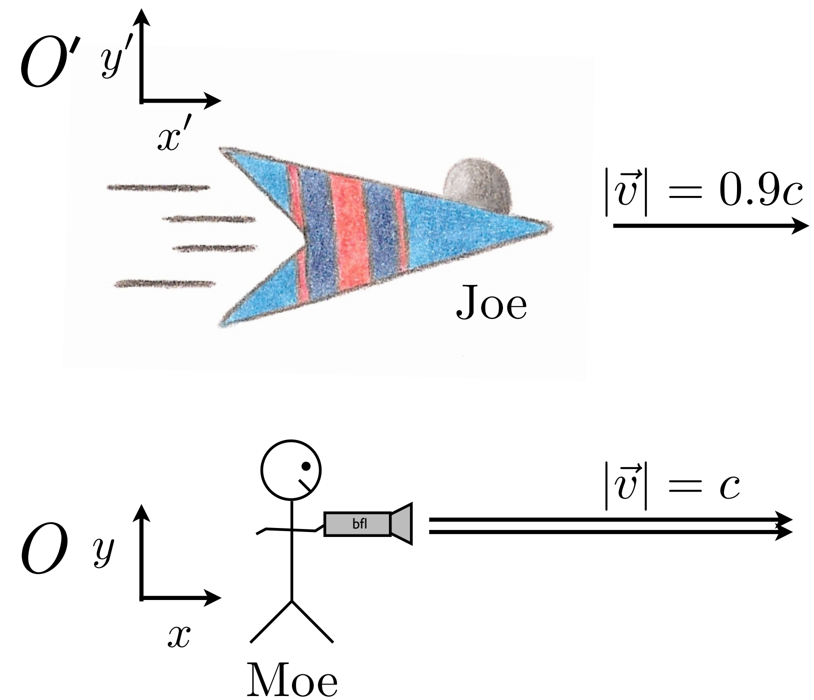
- so what?

Compare ...

how fast does the dart



how fast does light go?



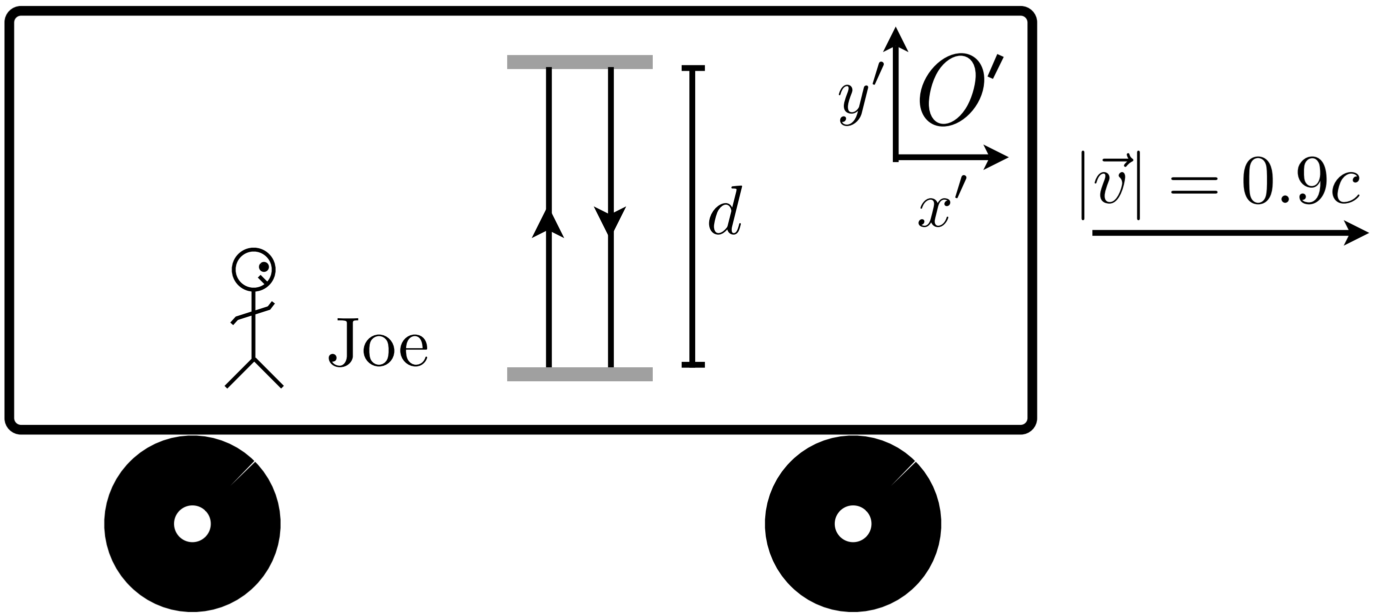
we can't be consistently right in both cases
but if light obeys velocity addition, logical

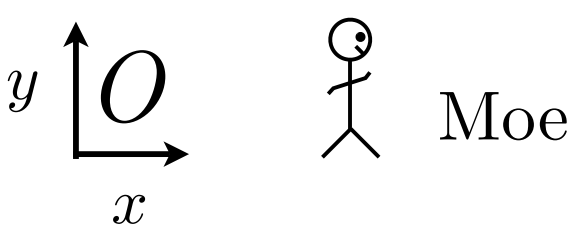
Consequences:

- the passage of time is relative
- finite light speed ... "now" is subjective
- the rate your clock moves *depends*
- speed of light is a *cosmic speed limit*

- weird, but no logical problems!

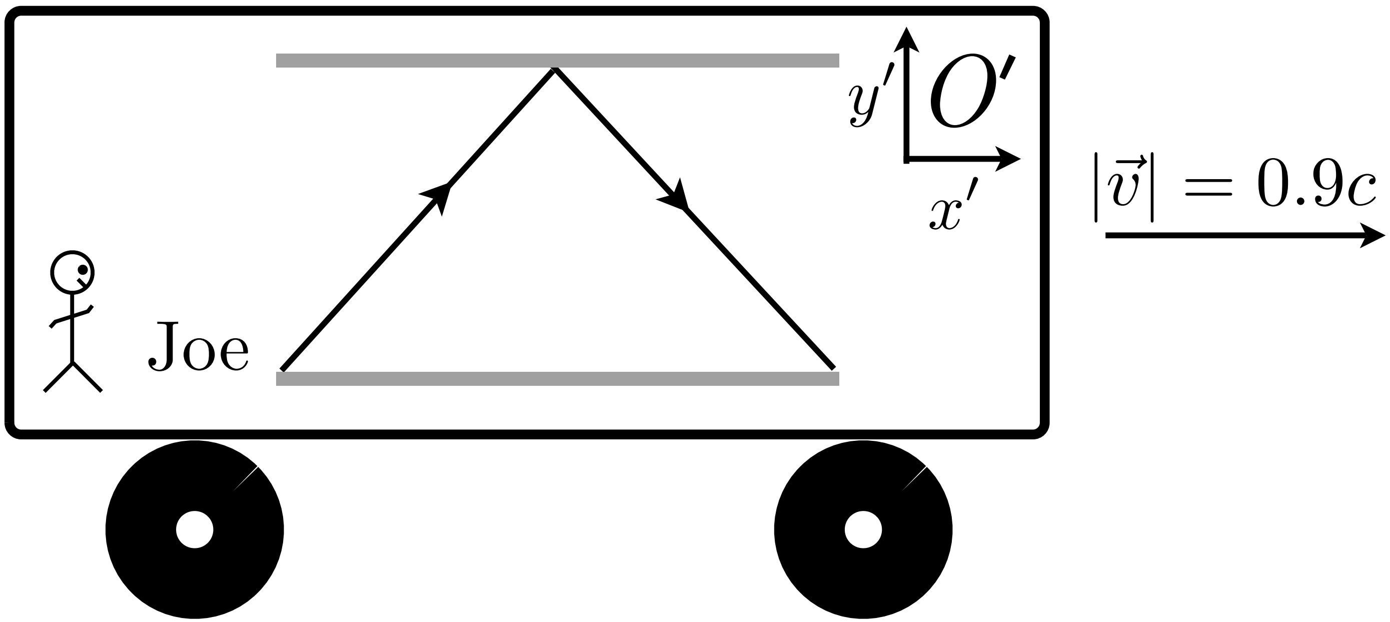
Rate of time passage

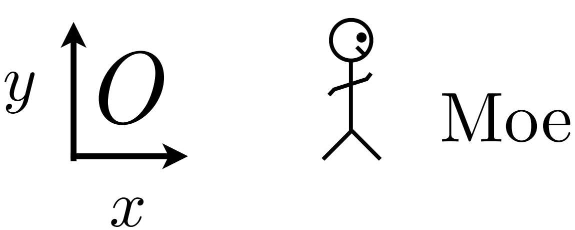




Joe bounces a laser off of some mirrors
he counts the round trips
this measures distance

Rate of time passage





Moe sees the boxcar move;
once the light is created, it does not.
Moe sees a triangle wave

So what?

- Moe sees light travel *farther* than Joe
- If the speed of light is the same ...
  - Moe thinks it takes longer!

- More time passes for Moe!

Time dilation

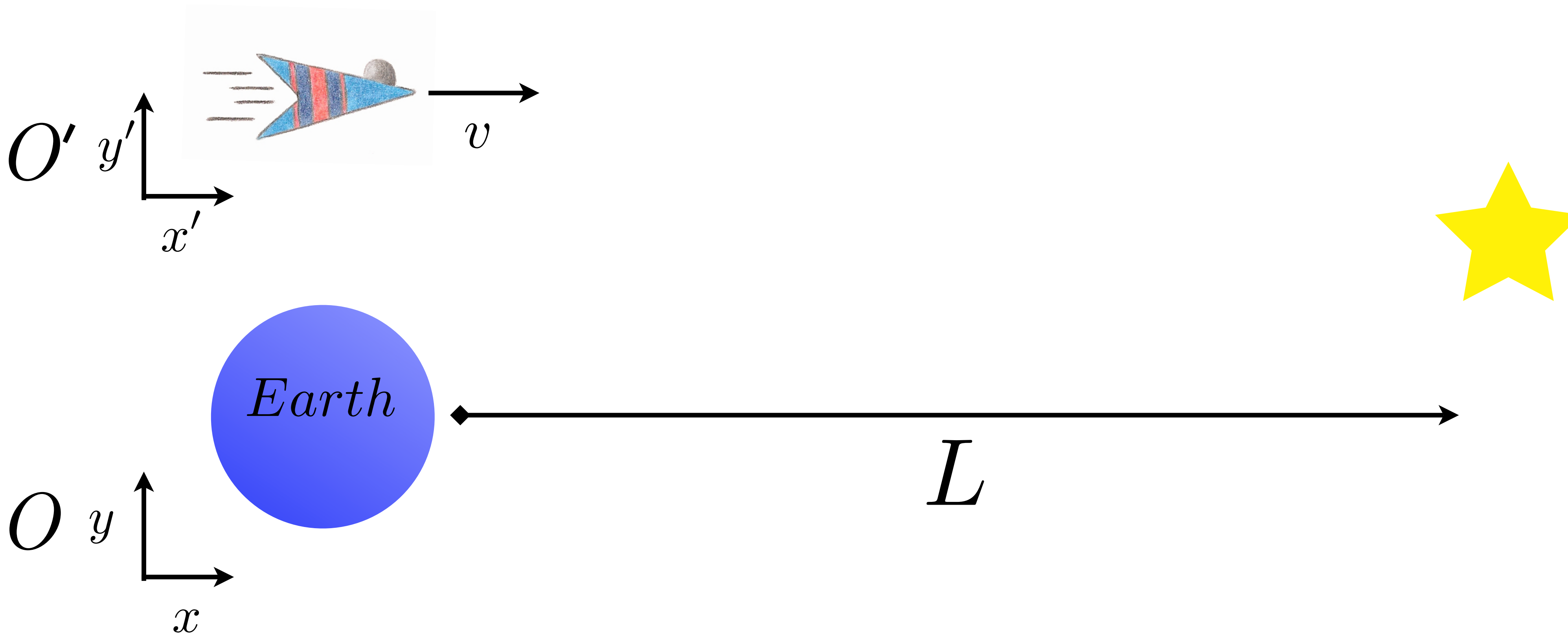
- time slows down moving observers!
- *experimentally observable!*
  - 747 experiment with atomic clocks
  - GPS relies on it
  - particle accelerators / decay

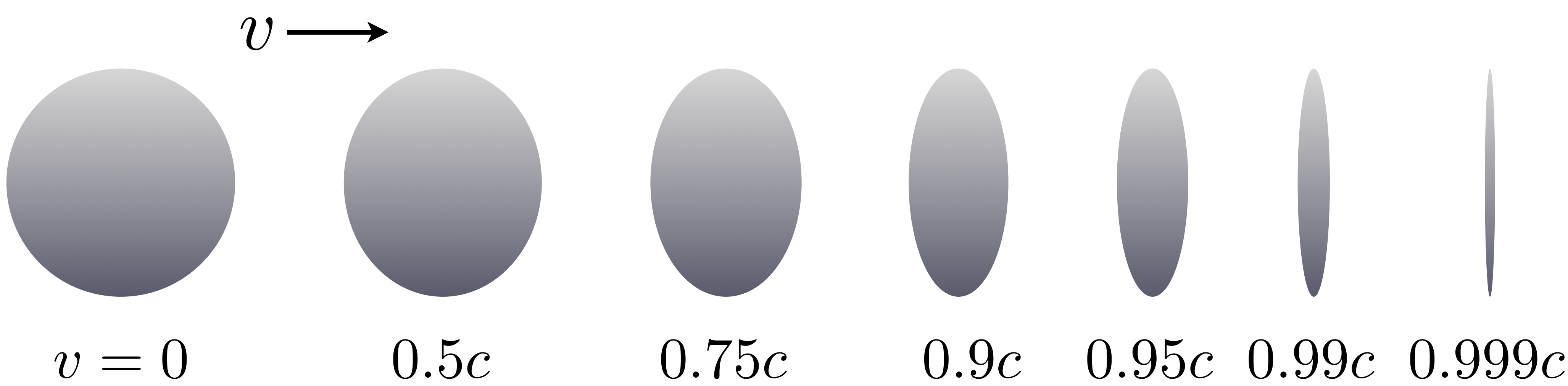
Twin “paradox”

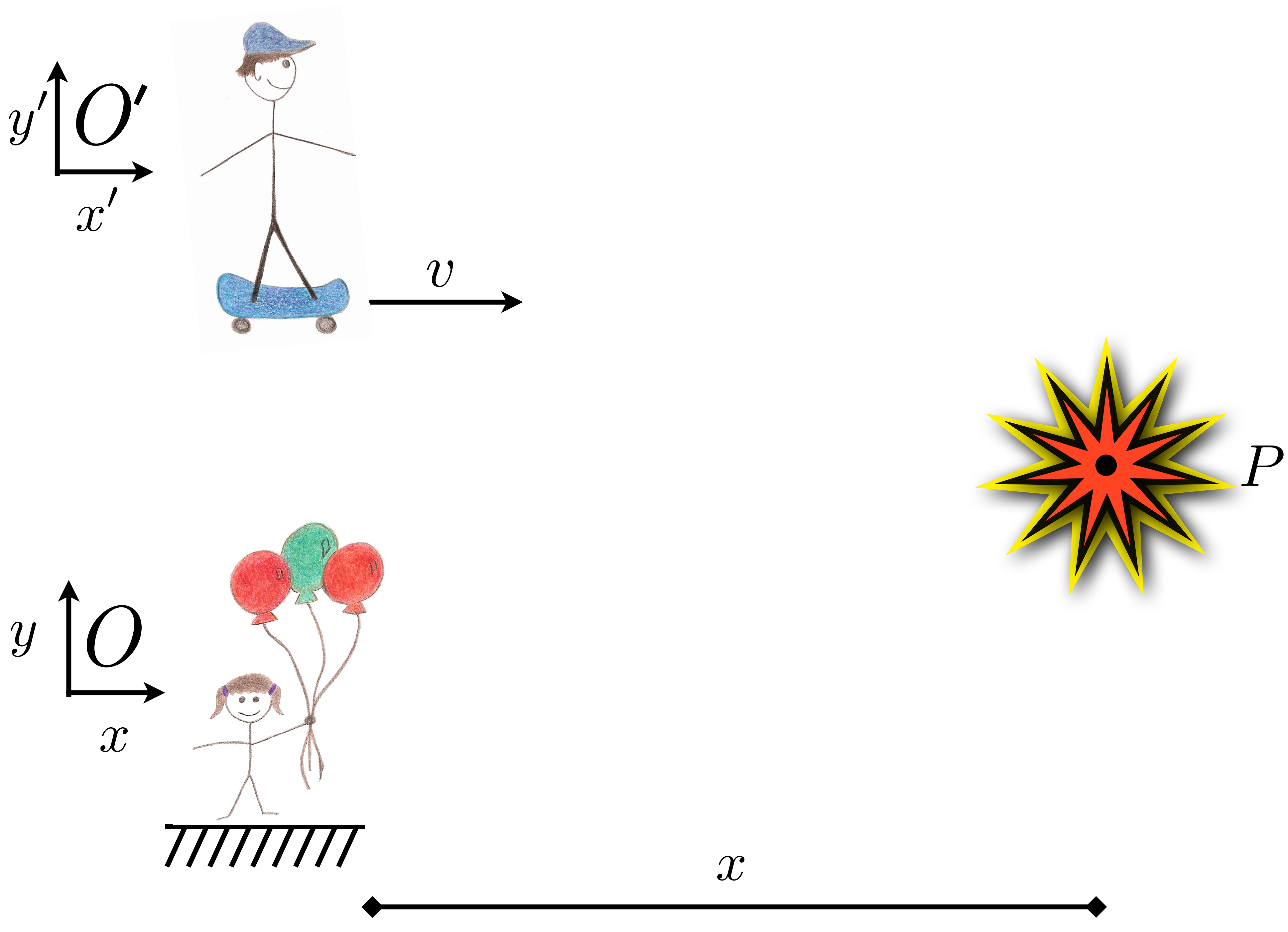
- One twin stays on earth
- One on a rocket at 80% of light speed
- 10 years pass on earth
- *only 6 years pass on the ship*

- Merely surprising; no logical or physical paradox
- Is this a form of time travel?

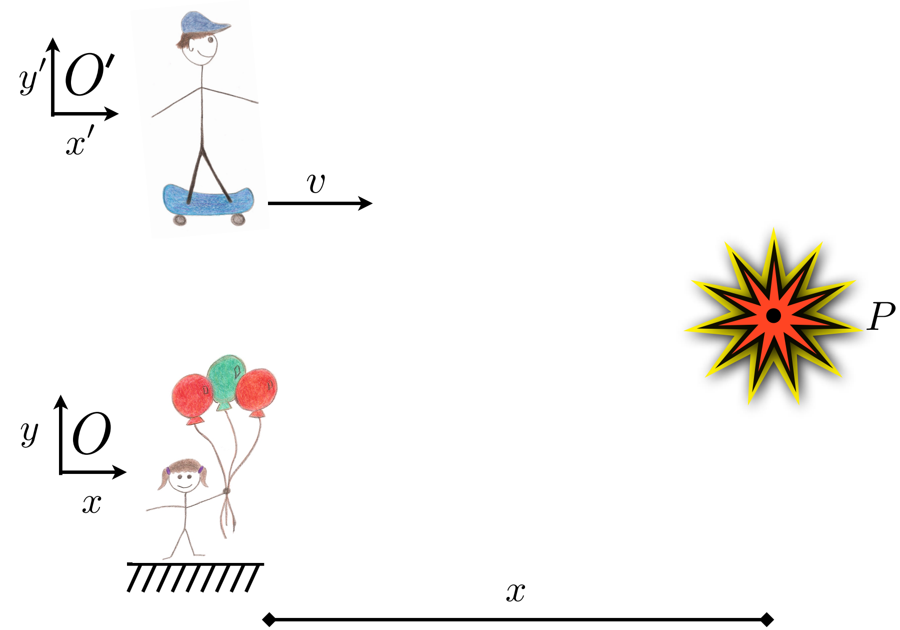
Length contraction




v
$v = 0$
$0.5c$
$0.75c$
$0.9c$
$0.95c$
$0.99c$
$0.999c$



y'
O'
x'
v
P
y
O
x
x



girl:

nova observed after $t = \frac{x}{c}$

boy:

distance = (girl's distance contracted) - (closing rate)

$$x = vt + \frac{x'}{\gamma}$$

girl:

distance = (her to boy) + (boy to nova, un-contracted)

$$x' = \frac{x}{\gamma} - vt'$$

Algebra ensues ...

- have 2 equations in x, x’ and t, t’ ...
- solve for x’ in terms of x, t’ in terms of t

Transformation of distance between reference frames:

$$x' = \gamma(x - vt) \quad (1.3$$

$$x = \gamma(x' + vt') \quad (1.3$$

Here (x,t) is the position and time of an event as measured by an observer in O stationary
it. A second observer in O', moving at velocity v, measures the same event to be at positic
and time (x',t').

Time measurements in different non-accelerating reference frames:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \quad (1.46)$$

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right) \quad (1.47)$$

Here (x,t) is the position and time of an event as measured by an observer in O stationary to it. A second observer in O', moving at velocity v, measures the same event to be at position and time (x',t').

Summary

- simultaneity is relative ... so "now" is ill-defined!
- rate of time passage is relative
  - moving observers: less time passes
- lengths along direction of motion are contracted
  - but not in *own* rest frame
- can relate times & positions for observers

Elapsed times between events in non-accelerating reference frames:

$$\Delta t' = t_1' - t_2' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right) \quad (1.4$$

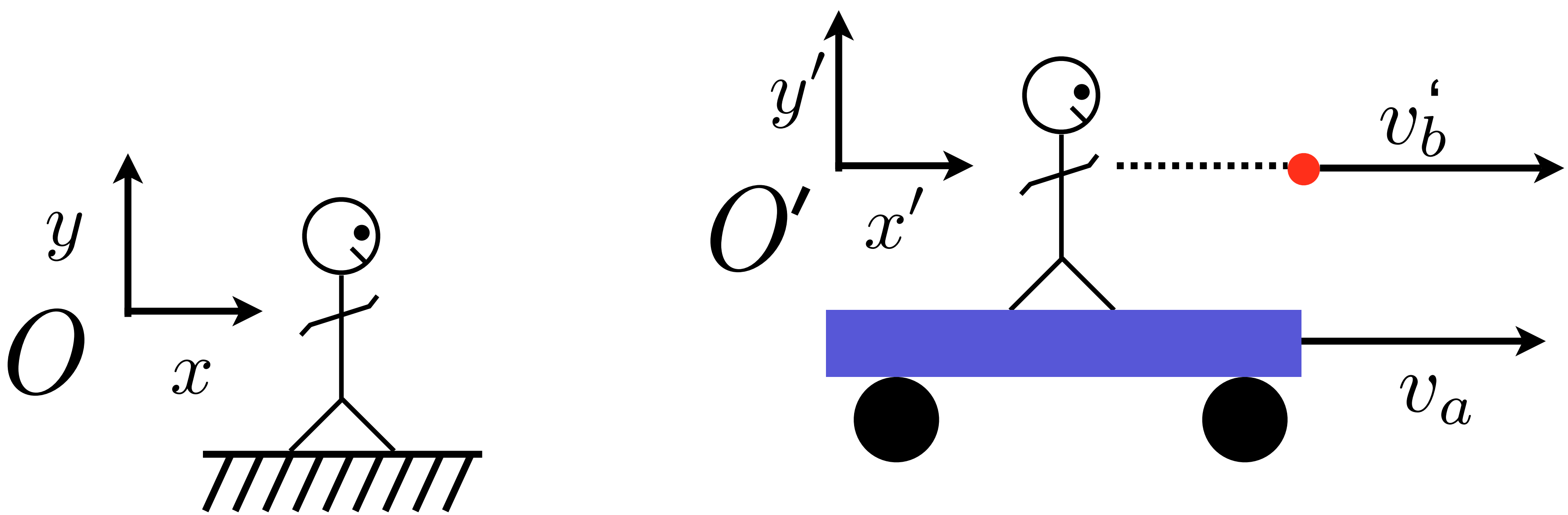
- for events to be simultaneous ...
  - both time intervals must be zero

- this can only happen if
  - *events are not spatially separated*
  - *no relative motion*

- this means defining "now" is ill-defined ...
  - not great for nowism

One more problem: flashlight on a rocketship?

Adding velocities



Say car is 0.75c, ball is 0.5c off of car ...
adding as normal, ball at 1.25c relative to ground?

clearly not OK ... account contraction/dilation

Adding speeds correctly

Relativistic velocity addition:
We have an observer in a frame O, and a second observer in another frame O′ who are moving relative to each other at a velocity v. Both observers measure the velocity of another object in their own frames (v_{obj} and v'_{obj}). We can relate the velocities measured in the different frames as follows:

$$v_{\text{obj}} = \frac{v + v'_{\text{obj}}}{1 + \frac{v v'_{\text{obj}}}{c^2}} \qquad v'_{\text{obj}} = \frac{v_{\text{obj}} - v}{1 - \frac{v v_{\text{obj}}}{c^2}} \tag{1.53}$$

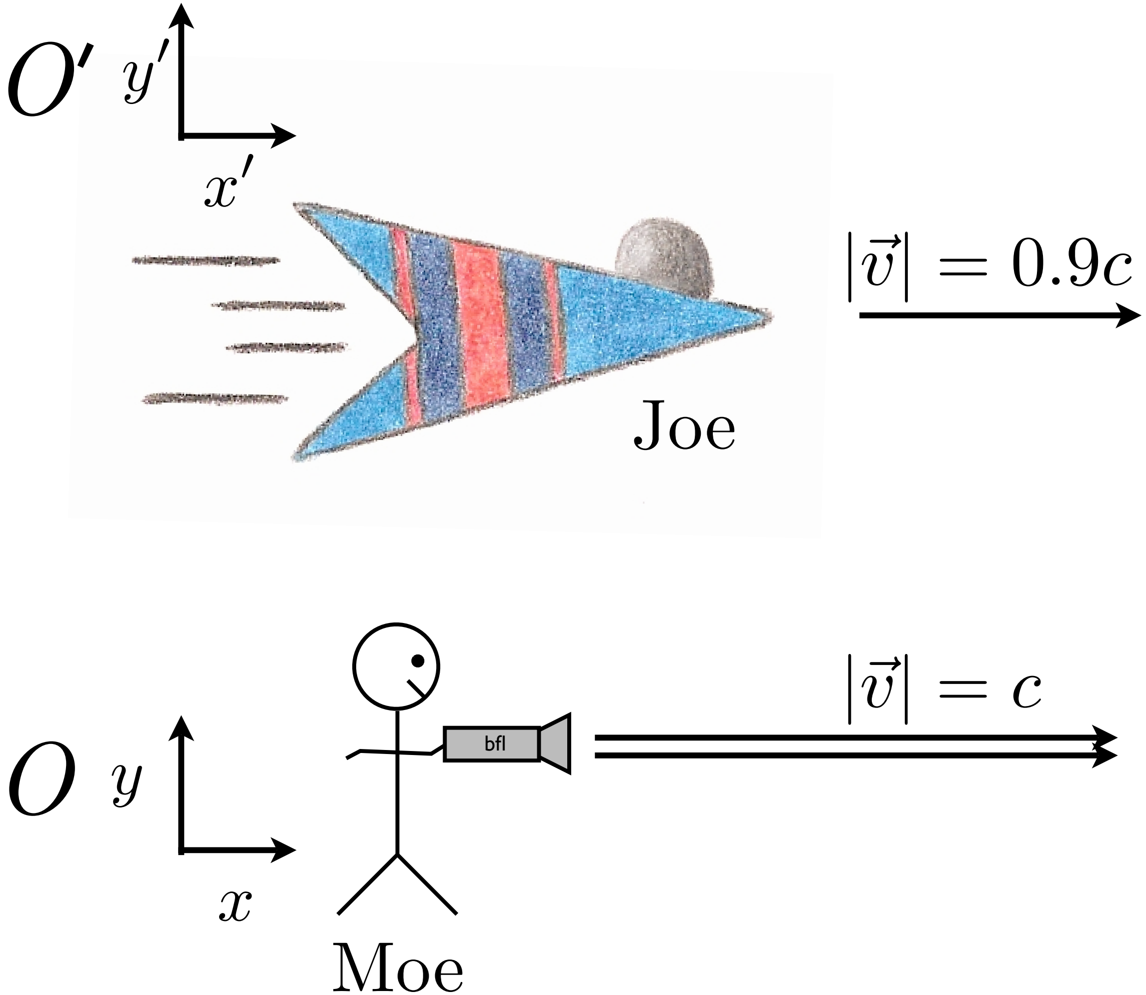
Again, v_{obj} is the object's velocity as measured from the O reference frame, and v_{obj} is its velocity as measured from the O′ reference frame.

$v'_{\text{obj}} = 0.5c$
$v = 0.75c$

now we get $v_{\text{obj}} = 0.91c$

never ends up with $v > c$!

(add or subtract? do this as normal, correct formula follows)



how about this?

$$v'_{\text{light}} = \frac{v_{\text{light}} - v_{\text{rocket}}}{1 - \frac{v_{\text{rocket}} v_{\text{light}}}{c^2}} = \frac{c - 0.99c}{1 - \frac{(0.99c)(c)}{c^2}} = \frac{0.01c}{1 - 0.99} = c$$

what if Joe has the light?

$$v_{\text{light}} = \frac{v_{\text{rocket}} + v'_{\text{light}}}{1 + \frac{v_{\text{rocket}} v'_{\text{light}}}{c^2}} = \frac{0.99c + c}{1 + \frac{(0.99c)(c)}{c^2}} = \frac{1.99c}{1 + 0.99} = c$$

(add or subtract? do this as normal, correct formula follows)

1. An astronaut traveling at $v = 0.80c$ taps her foot 3.0 times per second. What is the frequency of taps determined by an observer on earth? (*Hint: be careful about the difference between time and frequency!*)

- ○ 5.0 taps/sec
- ○ 6.7 taps/sec
- ○ 1.8 taps/sec
- ○ 3.0 taps/sec

2. A spaceship moves away from earth at high speed. How do experimenters on earth measure a clock in the spaceship to be running? How do those in the spaceship measure a clock on earth to be running?

- ○ slow; fast
- ○ slow; slow
- ○ fast; slow
- ○ fast; fast

3. If you are moving in a spaceship at high speed relative to the earth, would you notice a difference in your pulse rate? In the pulse rate of the people back on earth?

- ○ no; yes
- ○ no; no
- ○ yes; no
- ○ yes; yes

4. The period of a pendulum is measured to be 3.00 in its own reference frame. What is the period as measured by an observer moving at a speed of $0.950c$ with respect to the pendulum?

- ○ 6.00 sec
- ○ 13.4 sec
- ○ 0.938 sec
- ○ 9.61 sec

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A view of spacetime

- 2 observers in different frames (O, O')
- observer in O' traveling at v relative to O
- their origins coincide at t=t'=0
- light pulse emitted from origin at this moment
- where is light pulse at a later time?

Distance light pulse covers?

according to O:

$$r = \sqrt{x^2 + y^2 + z^2} = c\Delta t$$

according to O':

$$r' = \sqrt{x'^2 + y'^2 + z'^2} = c\Delta t'$$

no surprises: we know how to relate distances and times
but look more closely ...

They can agree on ...

For the light pulse, both can agree on:

$$s^2 = r^2 - c^2 \Delta t^2 = r'^2 - c^2 \Delta t'^2 = 0$$

s is the *spacetime interval*
like the distance formula, but with time as a coordinate
  time coordinate is imaginary (mathematically)
  metric 'signature' is +++-

all observers can agree on this - *invariant*
  even though they can't with dist, time separately

3 classes of intervals

$$s^2 = r^2 - c^2 \Delta t^2$$

- r = spatial separation of events
- t = time between events

- $s^2 < 0$ ... separation too big for light to cover
- $s^2 > 0$ ... separation small enough for light
- $s^2 = 0$ ... an interval traveled by light

$$s^2 = r^2 - c^2 \Delta t^2 < 0$$

- in time *t*, light goes farther than dist btw events
- i.e., events close enough photon could be at both
- causal connection is *possible*

- OTOH: events *cannot* be simult. in any frame
  - for that, need time interval zero => $s^2 > 0$
- clear time ordering of events for given observer

$$s^2 = r^2 - c^2 \Delta t^2 < 0$$

- if we talk about the motion of objects?

- on these paths, $r < ct$, so speed is less than c
- these are ‘time-like’ paths particles can follow
- paths along with causal connections possible
- light covers larger intervals

$$s^2 = r^2 - c^2\Delta t^2 > 0$$

- now r > ct ... events too far apart for light!
- “space-like” intervals; causality impossible
- can’t speak of past/future ordering
- *can* find a frame in which they are simult.

- so far apart even light can’t be at both events

types of intervals

- $s^2 > 0$ ... space-like, impossible paths
  - no absolute ordering, simultaneity relative

- $s^2 < 0$ ... time-like, particle paths
  - time ordering is absolute

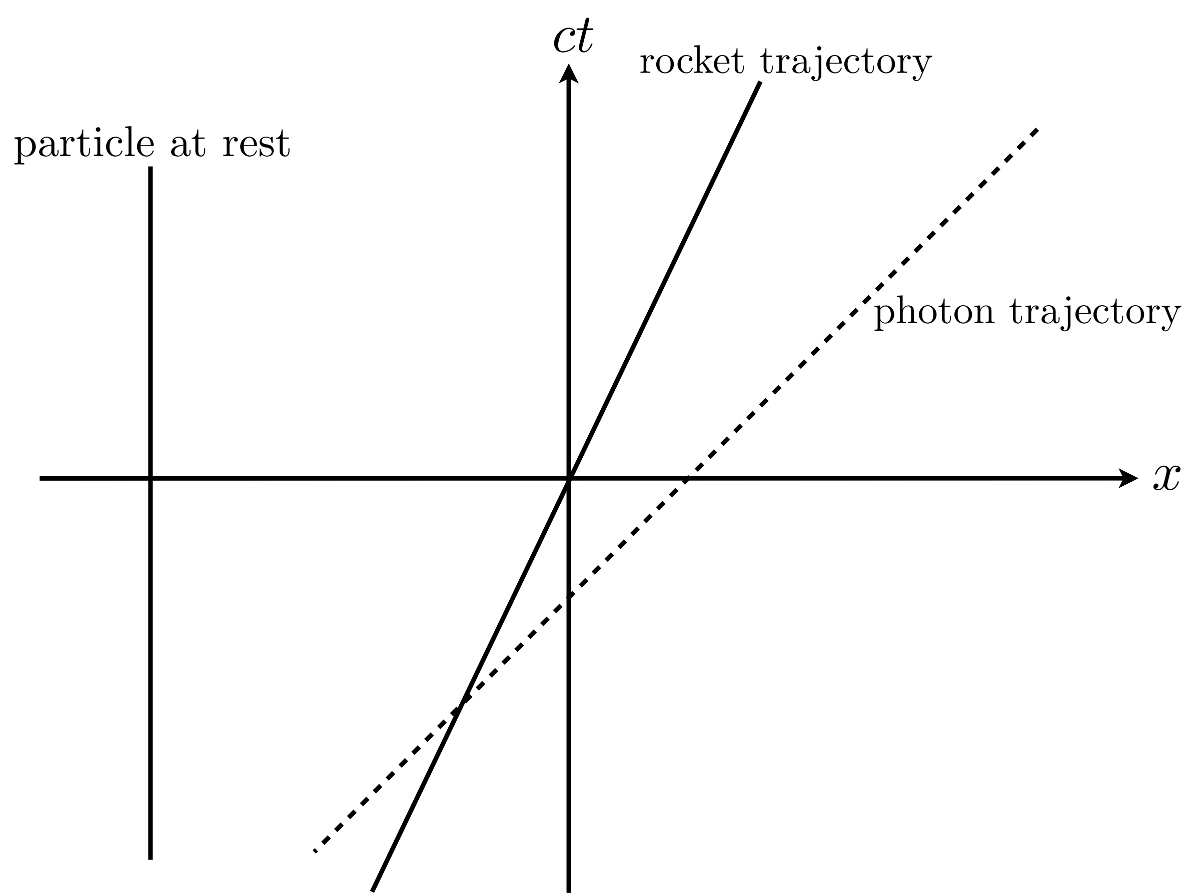
- $s^2 = 0$ ... light paths

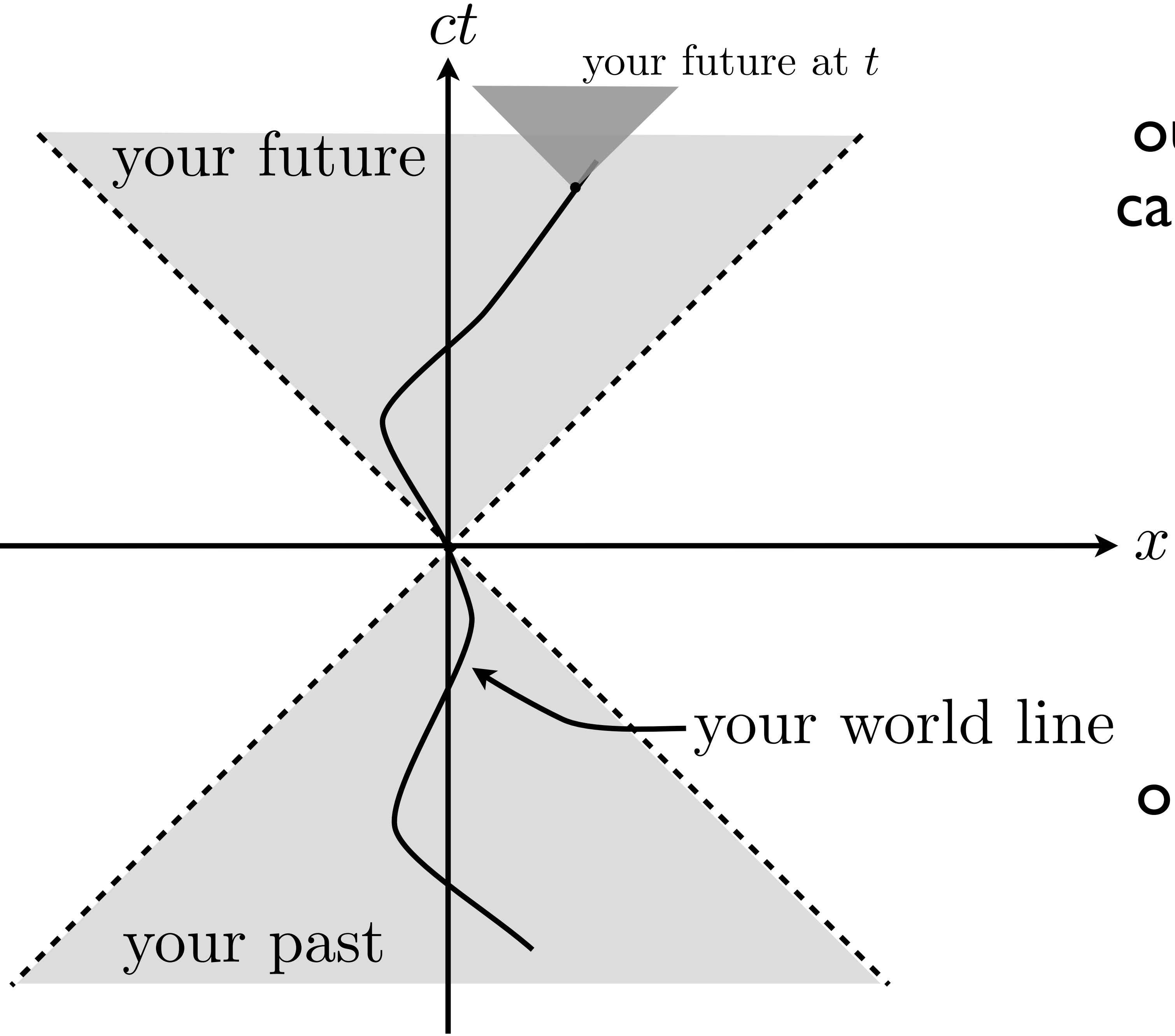
spacetime diagrams

- "Minkowski diagrams"
- way of visualizing intervals
- typically 1 spatial dimension + time

object paths
=
"worldlines"

path through
space & time

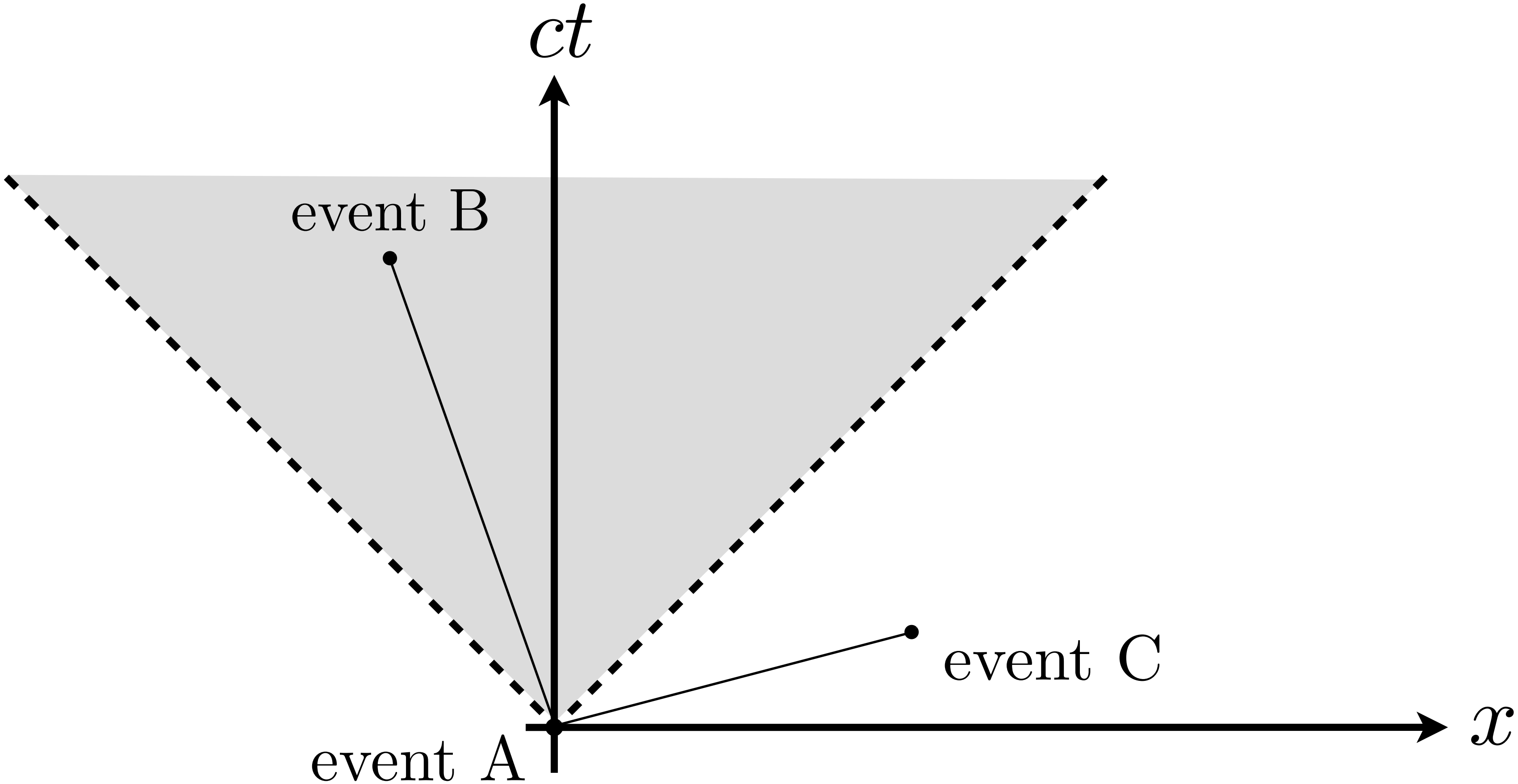




outside cone: no causal connection

only see outside events *later*

inside cone:
can be part of your *present or past*



A & C: x > ct ... space-like ... no causal connection
A & B: x < ct ... time-like ... can be causal connection

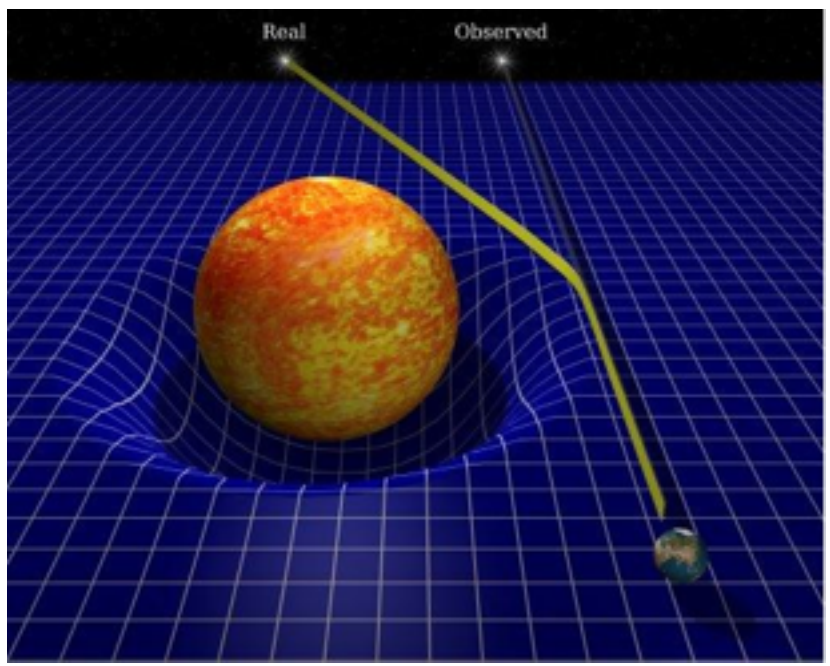
look at it like a triangle:
  time leg is shorter = space-like = acausal
  distance leg is shorter = time-like = possibly causal

Summary

- rate of time passage is relative
- lengths along direction of motion are contracted
- can relate times & positions for observers
- simultaneity is relative ... so “now” is ill-defined!
- can place constraints on causality
- much more on energy & momentum ...

General relativity

- gravity is masses "bending" spacetime
- earth's worldline bends around the sun
- what if world lines bent so much they looped?



http://physics.highpoint.edu/~mdewitt/phy1050

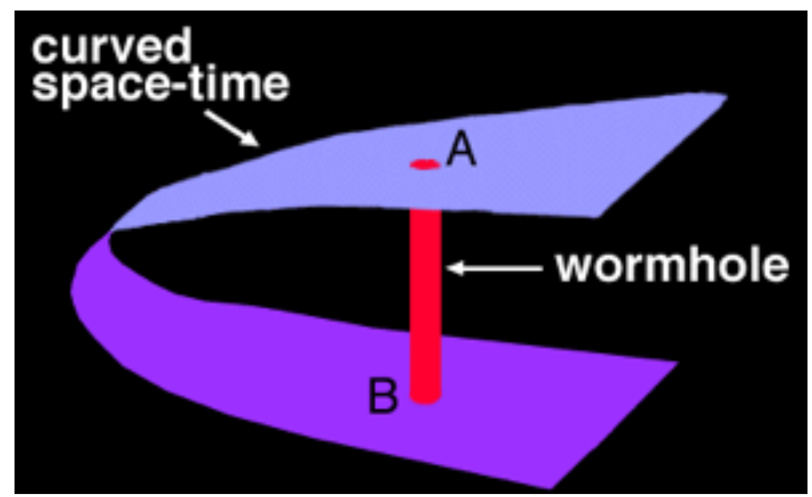
Closed Timelike Curves

- CTC = world line that loops back on itself
- would make a closed loop in space and time!
- i.e., Groundhog Day

- *mathematically* allowed by general relativity
- just a loop, not arbitrary time travel

Wormholes

- a ‘shortcut’ through curved space
- like a tunnel to China ...
- can play games with moving ends, etc ...
- but still can’t travel to time before you entered!



http://www.eclipse.net/~cmmiller/BH/

So where do we stand?

- time travel to the future is a real thing
  - but just slow your own time passage
- time travel to the past *may* be a real thing
  - but only to point after starting 'journey'
- still, nothing explicitly *forbids* time travel!
  - take causality/paradoxes seriously though ...

Other issues

- no known way to make CTCs
- wormholes require exotic matter ...
- locality/autonomy - how to avoid chaos?

  (block time?)
- that one can't go back to moment *before* initiating time travel helps!
- what about energy?

  (even information costs energy)