Name

PH105 Exam 1 solution

1. B. Given $x(t)$, the velocity at any time is the slope of the $x(t)$ curve at that time. The $x(t)$ curve starts with high positive slope, and as time increases, its slope decreases but remains positive. That means the velocity should start large and positive, and smoothly decrease, as depicted in choice B.

2. E. Given $v(t)$, we integrate to find position. We are told $x(t=0) = 1.00\,\mathrm{m}$, which determines the integration constant.

$$v(t) = 3.00\,\mathrm{m/s} + (4.00\,\mathrm{m/s^3})t^2 \tag{1}$$
$$x(t) = \int v(t)\,dt = \int 3.00\,\mathrm{m/s} + (4.00\,\mathrm{m/s^3})t^2\,dt = (3.00\,\mathrm{m/s})t + \left(\frac{4.00}{3}\,\mathrm{m/s^3}\right)t^3 + C \tag{2}$$
$$x(0) = C = 1.00\,\mathrm{m} \tag{3}$$
$$x(t) = 1.00\,\mathrm{m} + (3.00\,\mathrm{m/s})t + (1.33\,\mathrm{m/s^3})t^3 \tag{4}$$

3. E. Acceleration is the slope of the $v(t)$ graph, dv/dt, so constant acceleration means constant slope. This means $v(t)$ is a straight line graph, and the velocity changes by equal amounts in equal times.

4. C. Instantaneous velocity is $v = dx/dt$:

$$x(t) = bt^2 - ct \tag{5}$$
$$v(t) = \frac{dx}{dt} = 2bt - c \tag{6}$$
$$v(2.2\,\mathrm{s}) = 2(2.0\,\mathrm{m/s^2})(2.2\,\mathrm{s}) - 6.7\,\mathrm{m/s} = 2.1\,\mathrm{m/s} \tag{7}$$

5. B. The total time to travel 541 mi at a constant speed of 65 mi/h is $t = (541\,\mathrm{mi})/(65\,\mathrm{mi/h}) \approx 8.3\,\mathrm{hr}$. Given that you have 9.6 hr, you can stop for 1.3 hr and still make the trip.

6. A. The truck and car travel the same amount of time up until time T, but the truck has been going faster than the car at all times. Therefore, the truck travels farther than the car.

7. D. Acceleration is the slope of the velocity vs. time curve. Constant acceleration means constant slope, meaning the velocity versus time curve has constant slope - a straight line making some angle with the time axis. A horizontal line implies zero acceleration. A vertical straight line implies a multi-valued velocity at a single time, which is unphysical. A parabolic curve would require acceleration changing linearly with time, such that $v = \int a\,dt$ is a quadratic curve.

8. A The time to reach the bottom is independent of mass because the acceleration is independent of mass.

9. B. Acceleration is found from position by $a = d^2x/dt^2$.

$$x = at^3 - bt^2 + ct \tag{8}$$

$$v = \frac{dx}{dt} = 3at^2 - 2bt + c \tag{9}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 6at - 2b \tag{10}$$

$$a(0.7\,\mathrm{s}) \approx 6(4.1\,\mathrm{m/s^3})(0.7\,\mathrm{s}) - 2(2.2\,\mathrm{m/s^2}) \approx 13\,\mathrm{ms^2} \tag{11}$$

10. C. The acceleration of the child is $a = g\sin\theta$ down the ramp. With constant acceleration, displacement is $\Delta x = \frac{1}{2}at^2$. Combining, the known acceleration and time gives the displacement.

$$\Delta x = \frac{1}{2}at^2 = \frac{1}{2}\left(g\sin\theta\right)t^2 \approx 293\,\mathrm{m} \tag{12}$$

11. D. Let us start our clock when the police car begins moving, with the cop's position as $x = 0$. The car travels for one second at $30\,\mathrm{m/s}$, giving it an initial position of $30\,\mathrm{m}$ when the police car starts. Its position as a function of time is then $x_{\mathrm{car}} = 30\,\mathrm{m} + (30\,\mathrm{m/s})t$. The cop starts from rest but has acceleration a, so its position is $x_{\mathrm{cop}} = \frac{1}{2}at^2$. Since we know the car travels $\Delta x = 300\,\mathrm{m}$, we can use this to find the time, and then use the cop's position and the time to find acceleration.

$$x_{\mathrm{car}} = 30 + 30t = 300 \qquad \Longrightarrow \qquad t = 9 \tag{13}$$

$$x_{\mathrm{cop}} = \frac{1}{2}at^2 = \frac{1}{2}a(9)^2 = 300 \qquad \Longrightarrow \qquad a \approx 7.41\,\mathrm{m/s^2} \tag{14}$$

12. B. Since we know the initial velocity is zero, we can write the position of the object as

$$x(t) = x_i + \frac{1}{2}at^2 \tag{15}$$

We know the objects displacement between two times as well:

$$\Delta x = x(t_2) - x(t_1) = \frac{1}{2}at_2^2 - \frac{1}{2}at_1^2 = \frac{1}{2}a\left(t_2^2 - t_1^2\right) \tag{16}$$

$$a = \frac{2\Delta x}{t_2^2 - t_1^2} \approx 8.00\,\mathrm{m/s^2} \tag{17}$$

13. A. We know the initial velocity zero), the displacement (1/4 mi), and the time. We can write down the dragster's displacement and time to find the acceleration, and then calculate the velocity from that and the time.

$$\Delta x = \frac{1}{2}at^2 \qquad \Longrightarrow \qquad a = \frac{2\Delta x}{t^2} \tag{18}$$

$$v_f = at = \frac{2\Delta x}{t} \approx 269\,\text{mi/h} \tag{19}$$

Remember to convert seconds to hours when plugging in the final numbers!

14. A. We know the initial and final velocities as well as the acceleration, and we want a displacement.

$$v_f^2 - v_i^2 = 2a\Delta x \tag{20}$$

$$\Delta x = \frac{v_f^2 - v_i^2}{2a} \approx 133\,\text{m} \tag{21}$$

15. A. Dropped from rest, the height an object falls is $h = \frac{1}{2}gt^2$. Thus, with $t_B = 2t_A$,

$$\frac{h_A}{h_B} = \frac{\frac{1}{2}gt_A^2}{\frac{1}{2}gt_B^2} = \left(\frac{t_A}{t_B}\right)^2 = \frac{1}{4} \tag{22}$$

16. B. If we call the ground position $x = 0$, the position of the ball as a function of time is

$$x(t) = v_i t - \frac{1}{2}gt^2 \tag{23}$$

We know the ball has the same position at times t_1 and t_2, so

$$x_2 - x_1 = 0 = v_i t_2 - \frac{1}{2}gt_2^2 - v_i t_1 + \frac{1}{2}gt_1^2 \tag{24}$$

$$v_i(t_2 - t_1) = \frac{1}{2}g(t_2^2 - t_1^2) = \frac{1}{2}g(t_2 - t_1)(t_2 + t_1) \tag{25}$$

$$v_i = \frac{1}{2}g(t_2 + t_1) \approx 22.6\,\text{s} \tag{26}$$

Given the initial velocity, we can find the height from either x_2 or x_1:

$$h = x_2 = v_i t_2 - \frac{1}{2}gt_2^2 = \frac{1}{2}g(t_2 + t_1)t_2 - \frac{1}{2}gt_2^2 = \frac{1}{2}gt_1 t_2 \approx 10\,\text{m} \tag{27}$$

17. B. We know both masses, both initial velocities, and the final velocity of the heavier car. Conservation of momentum is enough to find the remaining velocity.

$$p_i = p_f \tag{28}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \tag{29}$$

$$v_{2f} = \frac{m_1 v_{1i} + m_2 v_{2i} - m_1 v_{1f}}{m_2} \approx 13.6\,\text{m/s} \tag{30}$$

18. D. Again, conservation of momentum is enough. We note that the initial velocity of the second object v_{2i} is negative since it is moving in the opposite direction.

$$p_i = p_f \tag{31}$$
$$mv - mv_{2i} = (2m)v_f = 0.100(2m)v \tag{32}$$
$$mv_{2i} = mv - 0.100(2m)v \tag{33}$$
$$v_{2i} = 0.800v \tag{34}$$

19. C. The impulse for an object is its change in momentum. Since the two colliding objects form a closed syste, the magnitude of their momentum changes, and hence their impulses, are the same.

20. C. Conservation of momentum is enough. Let the 45.0 kg skater be skater 1. Since neither skater is moving at the beginning, the starting momentum is zero.

$$p_i = p_f \tag{35}$$
$$0 = m_1 v_{1f} + m_2 v_{2f} \tag{36}$$
$$v_{2f} = -\frac{m_1}{m_2} v_{1f} \approx -0.281\,\text{m/s} \tag{37}$$