Preface: Newton's Second Law of Motion

Note: this laboratory experiment was written with another textbook in mind, the one the other PH105 sections are using. Their textbook covers things in a slightly different order than ours, so this lab on Newton's second law (regarding forces) seems to come a bit early. In fact, all the essential physics of Newton's second law we already know from our discussion of momentum and impulse, we really only miss one thing: at what rate *does momentum change? Here we provide a short preface which should make the rest of the lab more comprehensible.*

The acceleration of an object is defined as the rate of change of its velocity. If the velocity changes by an amount $\Delta v = v_f - v_i$ in a time $t_f = t_f - t_i$, then the average acceleration during this time is

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} \qquad (1)$$

A nonzero acceleration necessarily implies a change in velocity. For an object of constant mass m, this necessarily implies a change in the object's momentum ($p = mv$). Acceleration and changes in momentum must be related! If the acceleration is caused by an external agent, outside our system, we would say that the resulting change in momentum is an *impluse* $J = \Delta p = m\Delta v$. An impulse J supplied by an external agent causes a change in momentum of an object within the system Δp. Using Eqn. 1, we now note

$$J = \Delta p = m\Delta v = ma\Delta t \qquad \text{or} \qquad a = \frac{J}{m\Delta t} \qquad (2)$$

This tells us two things: first, the impulse depends on the acceleration a and its duration Δt; second, looking at it another way the acceleration the object experiences depends on the impulse per unit time ($J/\Delta t$) divided by the object's mass. The second point means that the time over which the impulse is applied is important. Think of decelerating in a car from 100 km/h to 0 km/h - would you rather do it in 1 s or 10 s? The impulse is the same in either case, but the same impulse over a shorter time interval is "harder" somehow. The rate of change of the external impulse, and therefore rate of change of momentum, can be found by re-arranging Eq. 2 in a third way:

$$\frac{\Delta J}{\Delta t} = \frac{\Delta p}{\Delta t} = \frac{ma\Delta t}{\Delta t} = ma \equiv F \qquad (3)$$

The rates of change are proportional the object's mass and its resulting acceleration. This is Newton's second law, which we will attempt to verify with the present experiment. We identify $\Delta J/\Delta t$, the rate at which an external agent applies an impulse, with our everyday concept of *force*, F, and thus it is a force F applied for a duration Δt that causes an object's change in momentum $\Delta p = F\Delta t$. Force is the rate at which momentum is transferred from one agent to another, and we have also found that it must be equal to mass times acceleration.