

Laboratory experiment: coefficient of restitution

Needed:

1. Rubber ball
2. Meter stick
3. Stopwatch (online)
4. Group of 2-3 students
5. One lab report per group (handwritten or electronic)

1 Introduction

The coefficient of restitution ϵ is a measure of how much energy is lost in a real collision. Specifically, it is the ratio of the speed of separation after a collision to the speed of approach before a collision:

$$\epsilon = \frac{v_{\text{after}}}{v_{\text{before}}} \quad (1)$$

For a collision with perfect conservation of mechanical energy (an “elastic collision”), $\epsilon = 1$. The coefficient of restitution is a convenient way of specifying how much energy in a collision is transferred to internal energy. In general it will depend on the specific materials which are colliding, but under experimentally realizable conditions, it is a constant factor representing the energy transfer from kinetic energy to internal energy.

In order to illustrate the utility of the coefficient of restitution, consider a ball of mass m dropped from rest at a height h_0 . Neglecting air resistance, we can readily find the speed of the ball just before it reaches the floor knowing it experiences an acceleration $-g$ through its displacement $\Delta x = h_0$:

$$v_0^2 = v_{\text{top}}^2 + 2a\Delta x = 0 - 2gh_0 \quad \implies \quad v_0 = \sqrt{2gh_0} \quad (2)$$

The coefficient of restitution defined above gives us the velocity after the ball rebounding from the ground once as $v_1 = \epsilon v_0 = \epsilon \sqrt{2gh_0}$. The maximum height after one rebound can then be calculated using the same procedure in reverse, since we know v_1 in terms of v_0 , and we know v_0 in terms of h_0 :

$$v_{\text{top}}^2 = 0 = v_1^2 - 2gh_1 = \epsilon^2 v_0^2 - 2gh_1 = \epsilon^2 (2gh_0) - 2gh_1$$

$$\epsilon = \frac{v_{\text{after}}}{v_{\text{before}}} = \sqrt{\frac{h_1}{h_0}}$$

One can repeat the same analysis, and show that the height after the second rebound is $h_2 = \epsilon^4 h_0$, and in general $h_n = \epsilon^{2n} h_0$ after n rebounds. Measuring the height after n rebounds should allow us to extract a value for ϵ . What is curious is that h_n does not reach zero for finite n , *i.e.*, *the ball bounces an infinite number of times*. (This is version of “Zeno’s paradox.”)

We can also calculate the time for each bounce and rebound. For the initial drop, $t_0 = \sqrt{\frac{2h_0}{g}}$, and each subsequent rebound and drop is calculated in the same way (counting twice for the round trip). For example, the time for the first two bounces is:

$$t_1 = 2\sqrt{\frac{2h_1}{g}} = 2\epsilon\sqrt{\frac{2h_0}{g}} = 2\epsilon t_0 \quad t_2 = 2\sqrt{\frac{2h_2}{g}} = 2\epsilon^2\sqrt{\frac{2h_0}{g}} = 2\epsilon^2 t_0 \quad (3)$$

Continuing this way, we can sum the time for each of the infinite number of bounces, noting that $t_n = 2\epsilon^n t_0$:

$$\begin{aligned} t_{\text{total}} &= t_0 + t_1 + t_2 + \dots \\ &= t_0 [1 + 2\epsilon + 2\epsilon^2 + 2\epsilon^3 + \dots] \\ &= t_0 [2 + 2\epsilon + 2\epsilon^2 + 2\epsilon^3 + \dots - 1] \\ &= t_0 \left[\sum_{n=0}^{\infty} 2\epsilon^n \right] - 1 = t_0 \left[\frac{2}{1-\epsilon} - 1 \right] = t_0 \left[\frac{1+\epsilon}{1-\epsilon} \right] \end{aligned}$$

Perhaps surprisingly at first, the series converges, and the total time for all bounces is finite, $t_{\text{total}} = t_0 \left(\frac{2}{1-\epsilon} - 1 \right)$. So even though the ball bounces an “infinite” number of times, it does so in a finite amount of time.ⁱ We see that the total time before the ball stops bouncing is proportional to the square root of the original height, which we can use as another way to determine ϵ : we can simply measure how long it takes the ball to stop bouncing when dropped from a known height.

2 Hypothesis:

The loss in height of a bouncing ball can be well-characterized by a single parameter ϵ representing the loss in speed after each bounce. This leads to a finite time for the ball to stop bouncing, also

ⁱOf course in reality there are not an infinite number of bounces.

characterized by ϵ .

3 Procedure

You will need a meter stick, a ball to bounce, and a stopwatchⁱⁱ. We will measure the coefficient of restitution ϵ in two different ways.

Method 1:

From the equations above, we note that the ball's height after n bounces is $h_n = \epsilon^{2n} h_0$, or, the ratio of the height of the n^{th} bounce to the original release height will be ϵ^{2n} . Thus, a plot of $\log \frac{h_n}{h_0}$ (on the y axis) should be linear in n .

1. Drop your ball vertically from a given height (~ 1 m) next to an upright meter stick. This is h_0 . Take care not to spin the ball while releasing it.
2. Measure the rebound height after a certain number of bounces.
3. Drop your ball again from *the same height*, and measure the rebound height after a different number of bounces.
4. Do this for several values of n , the number of bounces, and plot $\log \frac{h_n}{h_0}$ (as y) versus n (as x) in Excel.
5. The slope of this plot should give you $2 \log \epsilon$, from which you can determine ϵ .

Method 2:

From the equations above, plotting the time to stop bouncing should scale as the square root of the initial height. Plotting the total bouncing time (on the y axis) versus the square root of the initial height (on the x axis) should give a straight line of slope:

$$\text{slope} = \sqrt{\frac{2}{g}} \left(\frac{1 + \epsilon}{1 - \epsilon} \right) \quad (4)$$

Thus,

$$\epsilon = \frac{(\text{slope}) \sqrt{\frac{g}{2}} - 1}{(\text{slope}) \sqrt{\frac{g}{2}} + 1} \quad (5)$$

1. As above, drop the ball next to the meter stick from a known height h_0 .
2. Using a stopwatch, measure how long the ball takes to stop bouncing by some reasonable and reproducible definition (e.g., when the sound of distinct bounces is no longer discernible.)

ⁱⁱDo a Google search for "stopwatch." You are feeling lucky.

3. Repeat this for a number of initial heights (*measured in meters*).
4. Plot the total time of bouncing versus $\sqrt{h_0}$ for several initial heights (say, 5). The coefficient of restitution can be found from the slope of this plot, as noted above.

If you finish early, you may want to repeat the above experiment with either a different ball (different material) or a different surface to bounce it off of. The value of ϵ will in general be different for different material combinations.

4 Questions

1. Report your values for ϵ for both methods and an estimate of your error.
2. Do the results agree from both methods agree, within the stated error?
3. Show that $\log \frac{h_n}{h_0}$ should be linear in n based on the model presented in the introduction.
4. In method 2, what is the intercept on the plot of total bouncing time versus $\sqrt{h_0}$?

5 On your lab report

- Your plots (2)
- The values of ϵ you determined, along with an estimate of error
- Your answers to the questions above

Further Reading:

N. Farkas and R.D. Ramsier, *Physics Education* **41**, 73 (2006)

<http://scienceworld.wolfram.com/physics/CoefficientofRestitution.html>