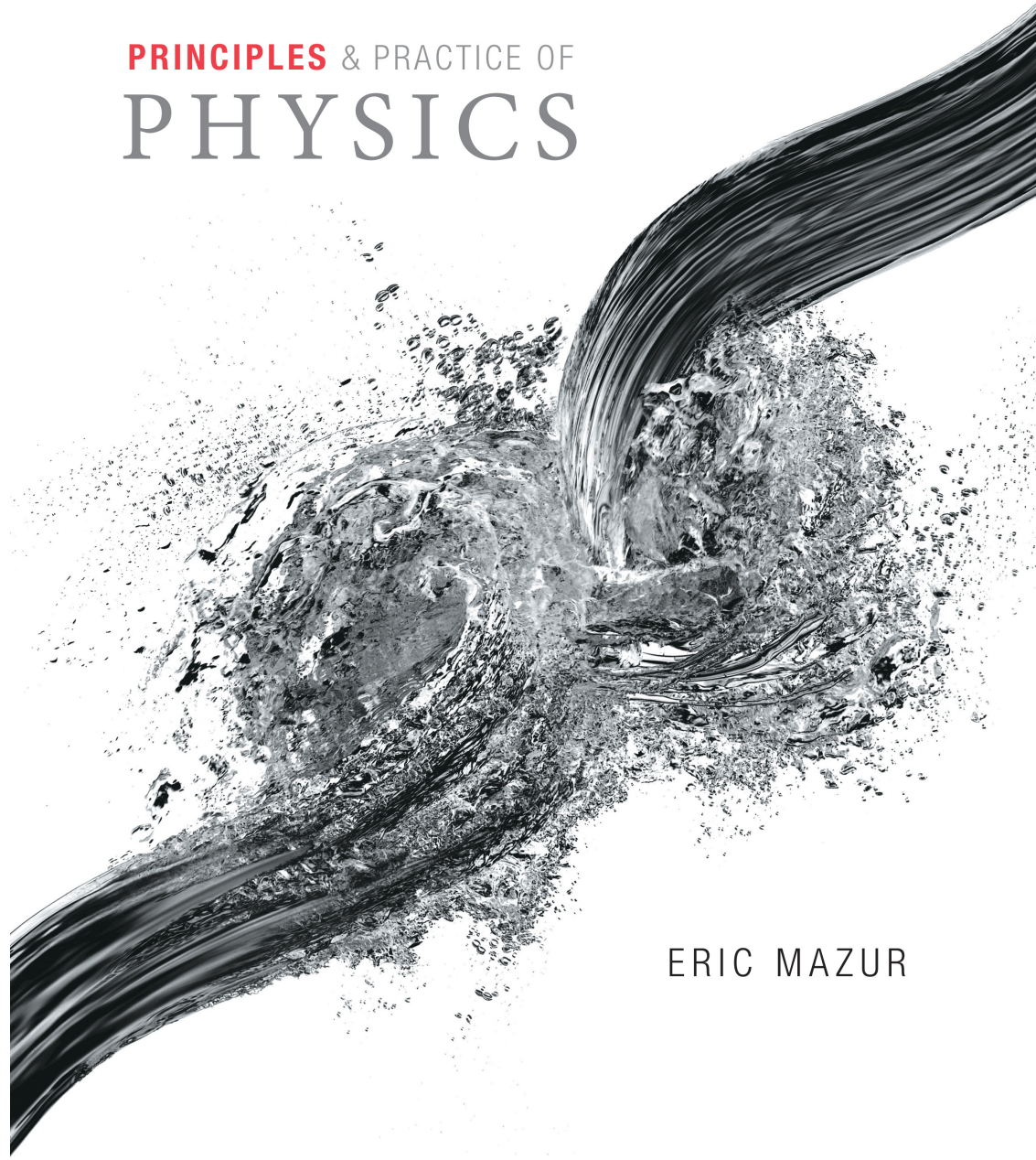


PRINCIPLES & PRACTICE OF  
PHYSICS

Chapter 4  
Momentum

ERIC MAZUR



# Things

## Various & Sundry topics

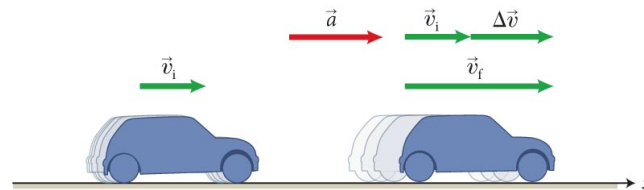
- ODS – reminder. Particularly testing-related
- travel ...
- reading quiz – delayed until 3:45pm
- labs
  - read procedure carefully – what is being asked?
  - “human error” is not a thing. do it again 😊
  - there *are* important reasons for inaccuracy
- office hours
  - you can show up. you are not bothering me

# Chapter 4 Preview

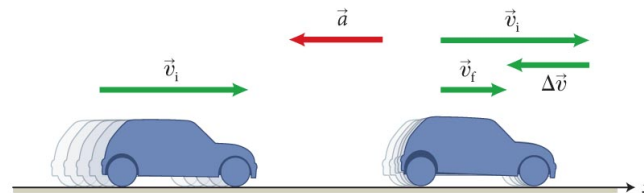
## Looking Back: Changes in velocity

- The non-uniform motion of an object can be described by the concept of acceleration.

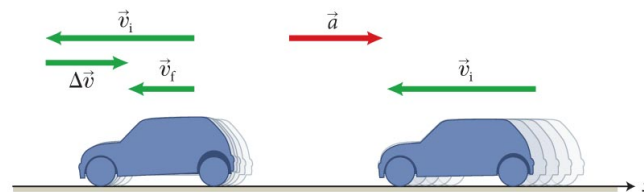
(a) Car speeds up in positive  $x$  direction



(b) Car slows down in positive  $x$  direction



(c) Car slows down in negative  $x$  direction

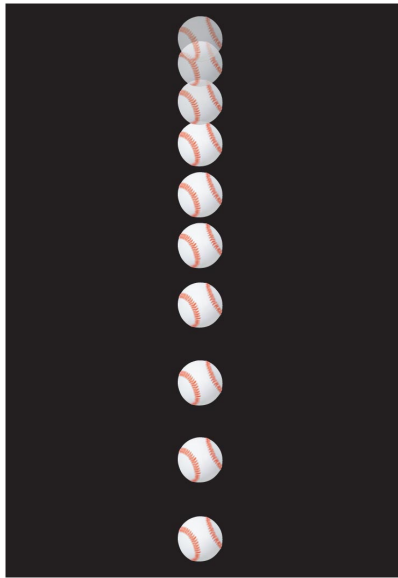


- In Chapter 3 you learned how to describe acceleration both graphically and mathematically.

# Chapter 4 Preview

## Looking Back: Acceleration due to gravity

- The photograph below shows the familiar pattern of an object speeding up as it falls.

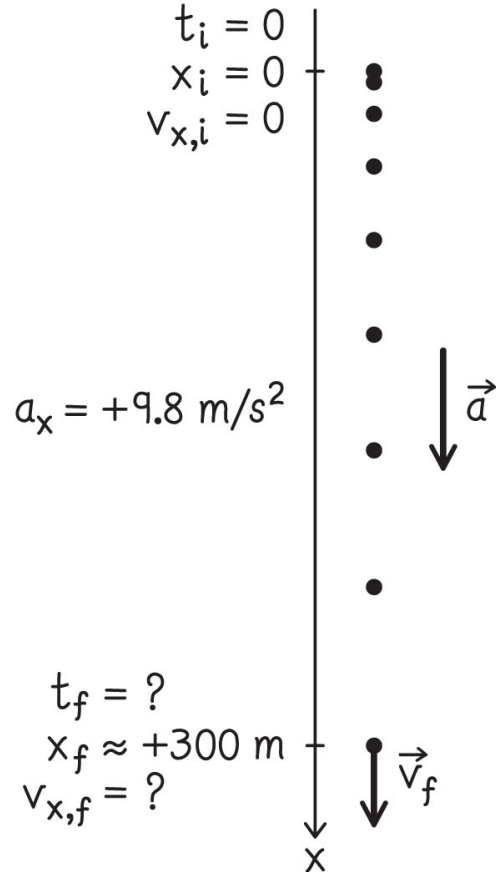


- Motion of objects near the Earth not purely horizontal are influenced by the force of gravity. Also, gravity is the main influence for astronomical objects moving through space.
- In Chapter 3 you learned how to account for the influence of gravity for objects moving near the surface of the Earth.

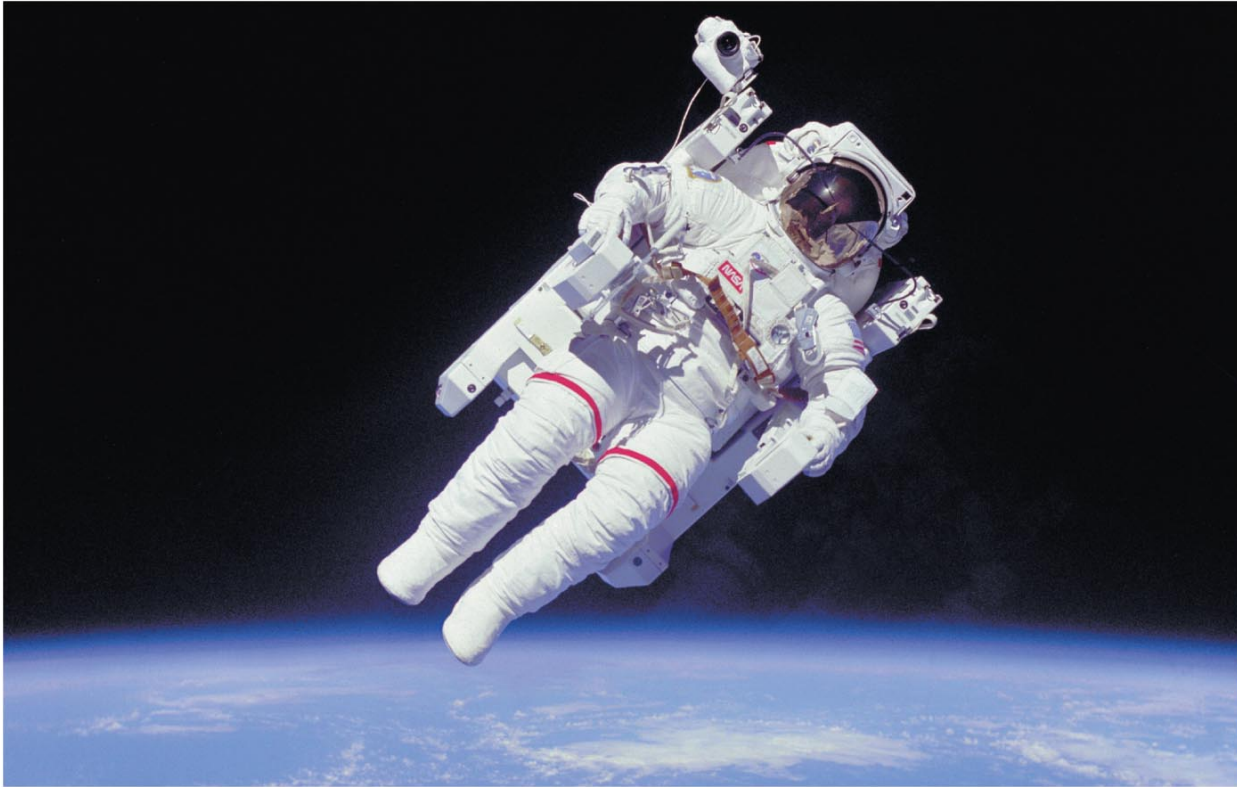
# Chapter 4 Preview

## Looking Back: Motion diagrams

- An important generalization of the “frame sequence” diagram from Chapter 2 was studied in Chapter 3. These representations were called **motion diagrams**.



# Chapter 4: Momentum



**Chapter Goal: To begin a theoretical analysis of motion using the concepts of inertia and momentum.**

# Chapter 4 Preview

## Looking Ahead: Inertia

- The tendency of an object to resist a change in its velocity is called **inertia**.



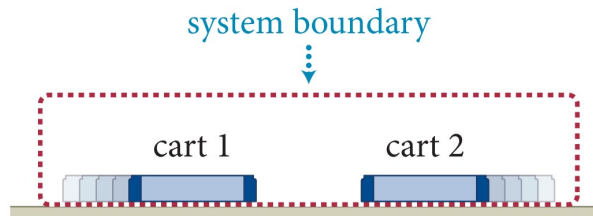
- You will learn that the **type of material and the volume** of an object is a measure of its inertia.

# Chapter 4 Preview

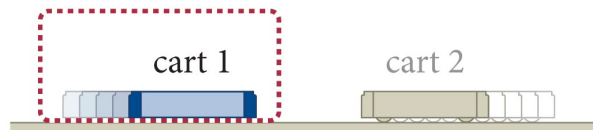
## Looking Ahead: Systems

- Most physical situations deal with multiple objects that interact with each other.

(a) Choice 1: system consists of both carts



(b) Choice 2: system consists of one cart



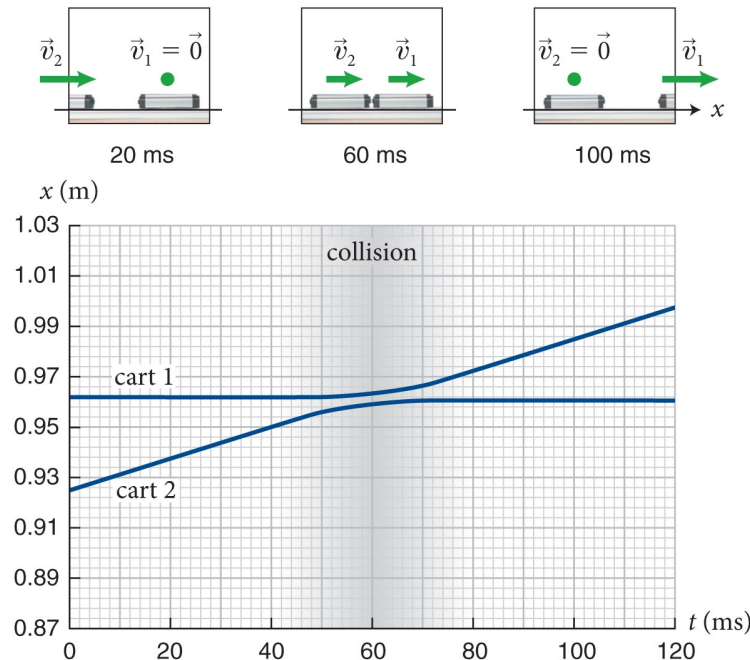
- Useful to separate the principle objects of interest (the **system**) from the rest of the universe (the **environment**).
- Learn how to pick the system and its environment depending on the physics of interest.



# Chapter 4 Preview

## Looking Ahead: Momentum

- The ability of an object to affect the motion of other objects in a collision is quantified by a concept called **momentum**.



- Momentum depends on an object's **amount of matter (inertia)** and **how fast it is moving (velocity)**.
- Learn how to calculate momentum for individual objects and systems of interacting objects.

# Chapter 4 Preview

## Looking Ahead: Conservation of momentum

- For **isolated** systems, with no external influences, momentum of the whole system is constrained
- The momentum of an isolated system does not change:  
**Momentum is conserved.**

**Table 4.3** Two choices of system for carts colliding

	Choice 1	Choice 2
System:	carts 1 & 2	cart 1
Environment:	track	cart 2 & track
Interactions:	internal	external
System isolated?	yes	no
Momentum changing?	no	yes

- You will learn how to apply the conservation of momentum in order to predict the kinematics of interacting objects.

# Chapter 4: Momentum

## Concepts

# Chapter 4: Momentum

**Remember, we do concepts first and worry  
about math later**

**If you don't have an idea how it works  
already, math won't help**

**(and you have to practice)**

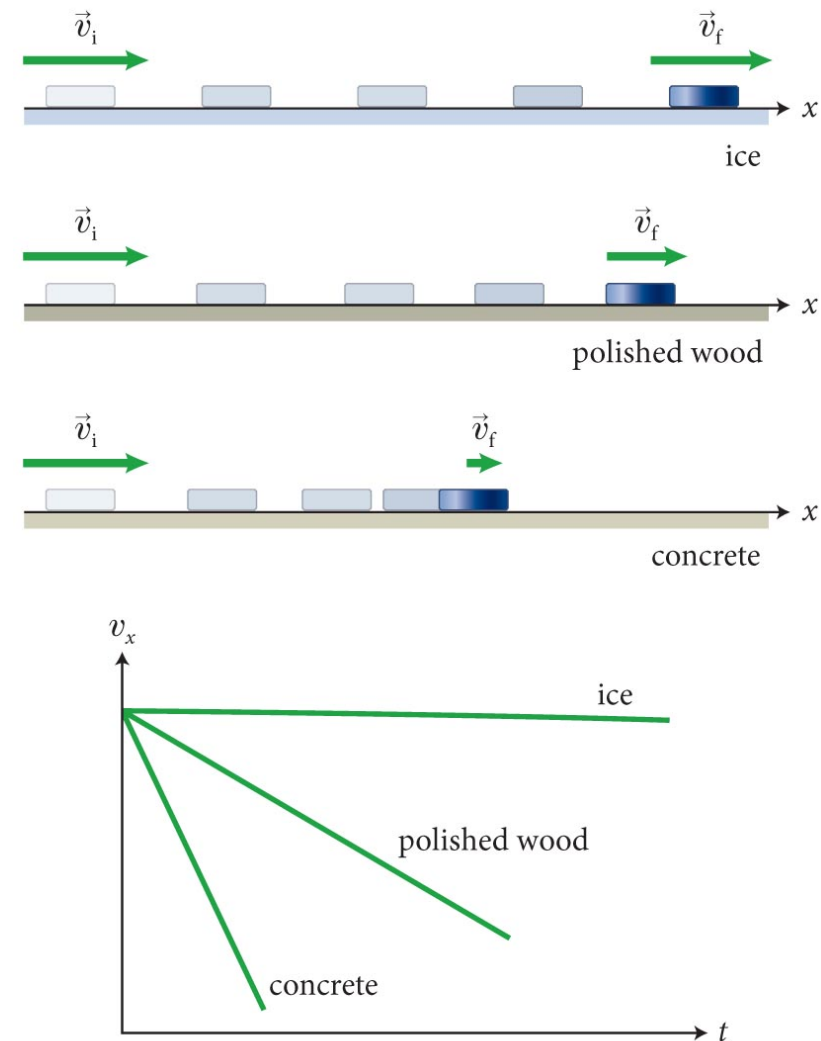
# Section 4.1: Friction

## Section Goals

- Identify the effects of **friction** on kinematics
- Generalize the situation of vanishing friction  
fundamental vs circumstantial
- In the absence of friction, objects moving along a horizontal surface keep moving without slowing down.
- *Your intuition includes friction*

# Section 4.1: Friction

- wooden block on three different surfaces.
  - The slowing down is due to **friction**—the resistance to motion that one surface (or object) encounters when moving over another.
  - The block slides easily over ice because there is little friction between the two surfaces.
  - The lower the friction, the longer it takes for the block to come to rest.



# Section 4.1: Friction

- What happens if there is no friction?
  - **In the absence of friction, objects moving along a horizontal track keep moving without slowing.**
  - There are no totally frictionless surfaces.
  - However, using an air track, we can minimize the friction to the point where it can be ignored during an experiment.

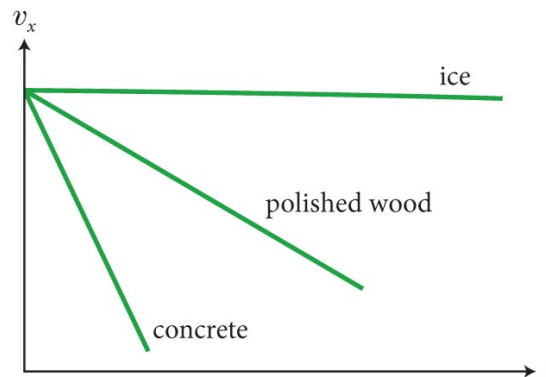
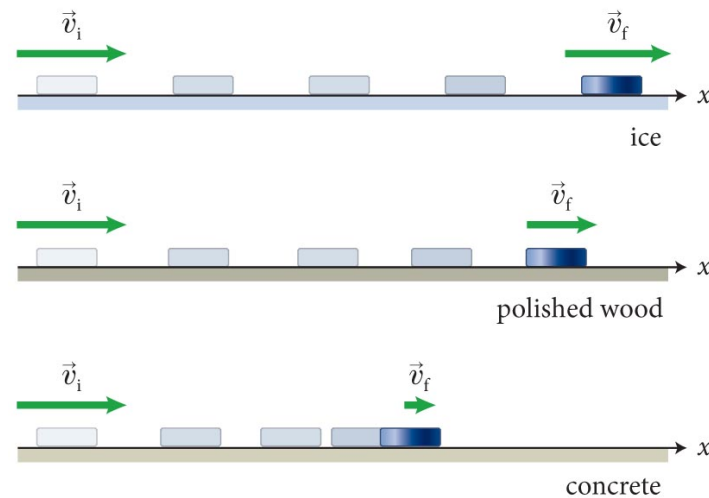


# Checkpoint



Based on the  $v_x(t)$  curves, what specifically is friction doing?

What else gives a linear decrease in velocity?





# Checkpoint



Velocity decreases at a constant rate?

this means friction causes a *constant acceleration*

it is in the direction opposing motion

Later: this means friction is a *constant force*

How does it work?

interaction of surfaces ... irregularities ... van der Waals

What does it depend on?

everything. materials, surface finish, air ...

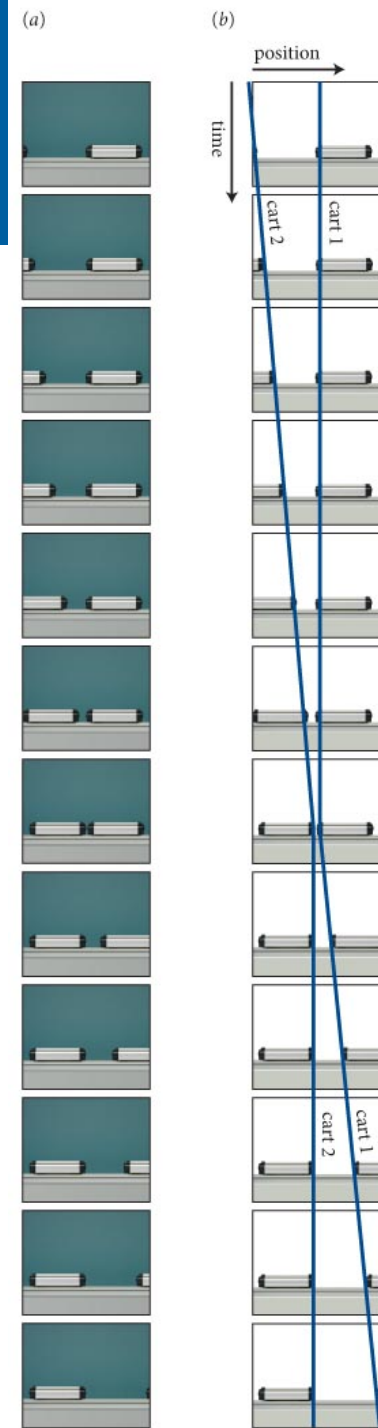
# Section 4.2: Inertia

## Section Goals

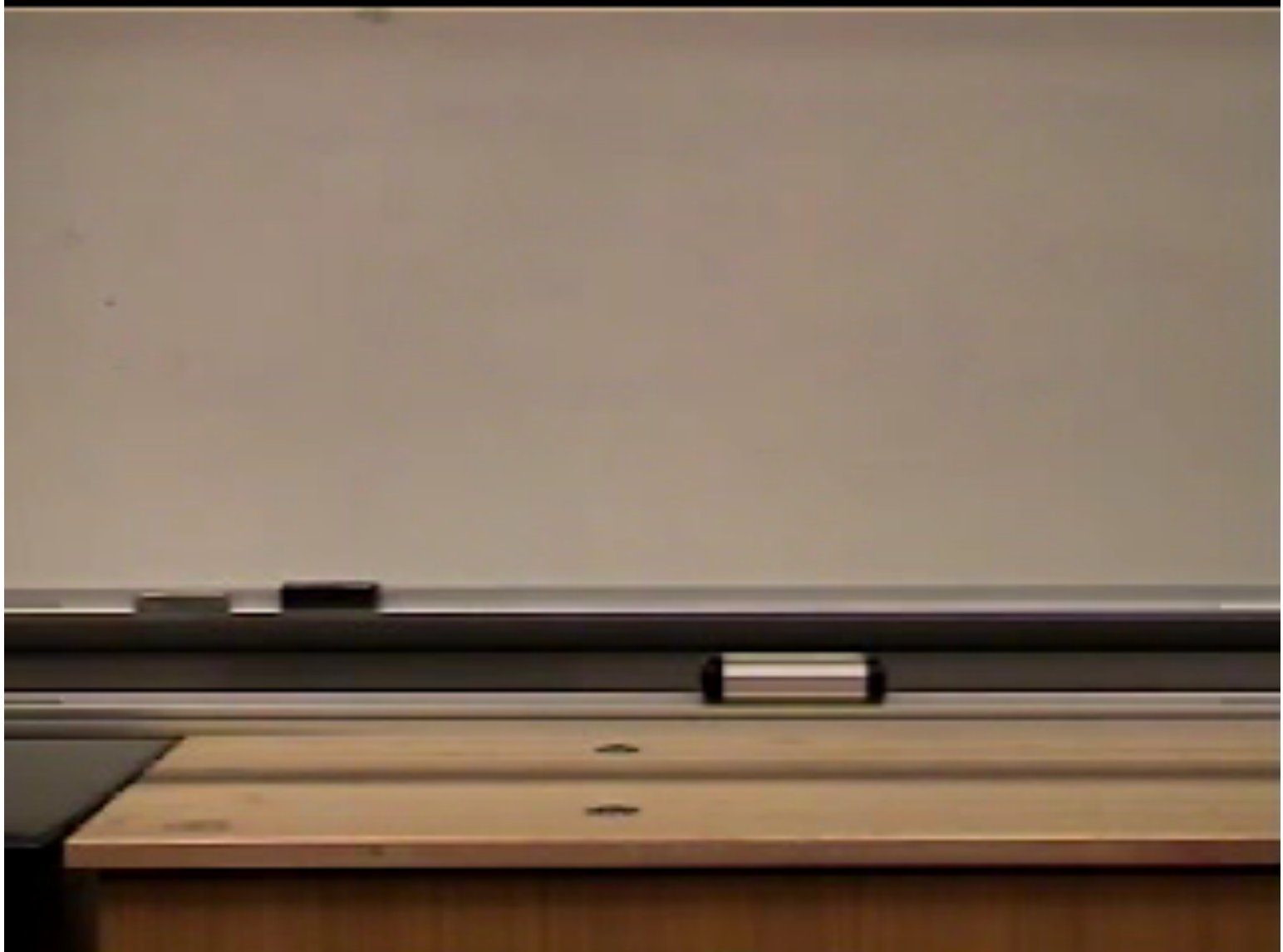
- Determine the changes in velocity of two colliding objects in the absence of friction.
- Display the collision process between two objects using motion diagrams, before-and-after diagrams, and motion graphs.
- Equate **inertia** as a measure of an object's tendency to resist any changes in its velocity.

# Section 4.2: Inertia

- We can discover one of the most fundamental principles of physics by studying how the velocities of two low-friction carts change when they collide →
- Start with two identical carts
- first cart stops, second starts *as though continuing motion of first*
- like something was transferred ...



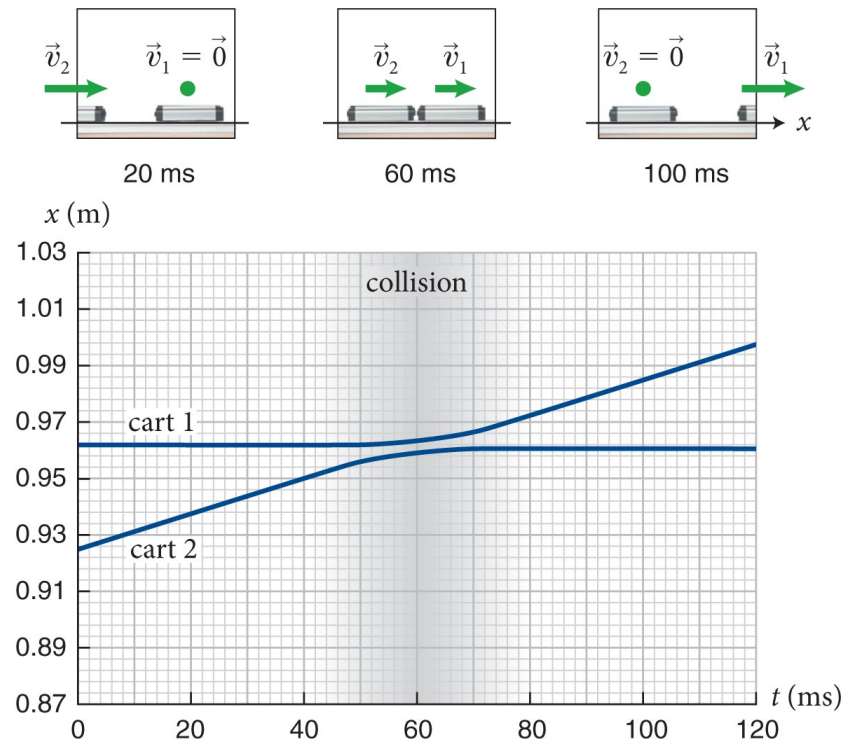
# Reality



# Section 4.2: Inertia

## Experiment 1

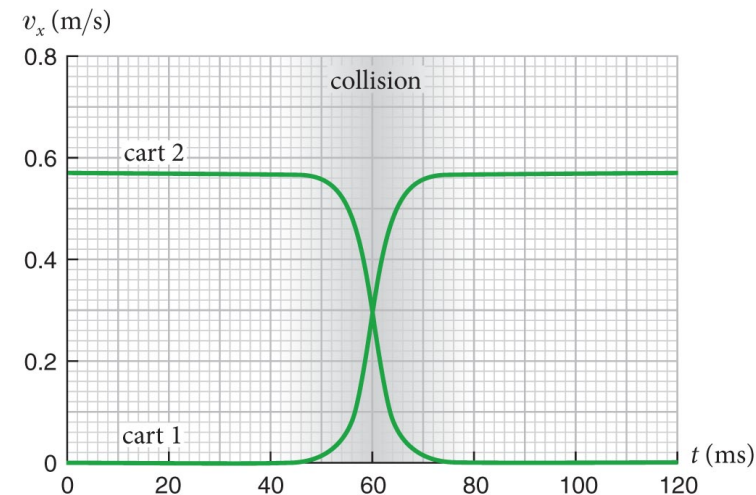
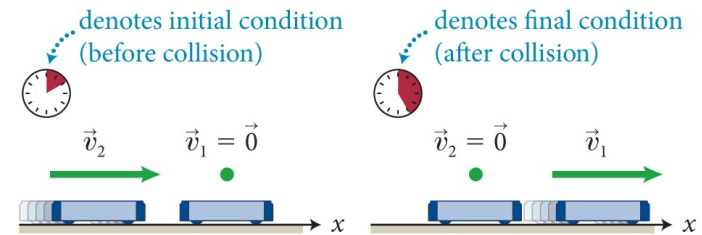
- Cart 1 initially at rest and Cart 2 moving toward it with velocity  $v_2$ .
  - measure  $x(t)$  for both
  - collision causes Cart 1 to move to the right, Cart 2 to come to a full stop.
- as though Cart 1 continues Cart 2's motion after the collision



# Section 4.2: Inertia

## Experiment 1 (cont.)

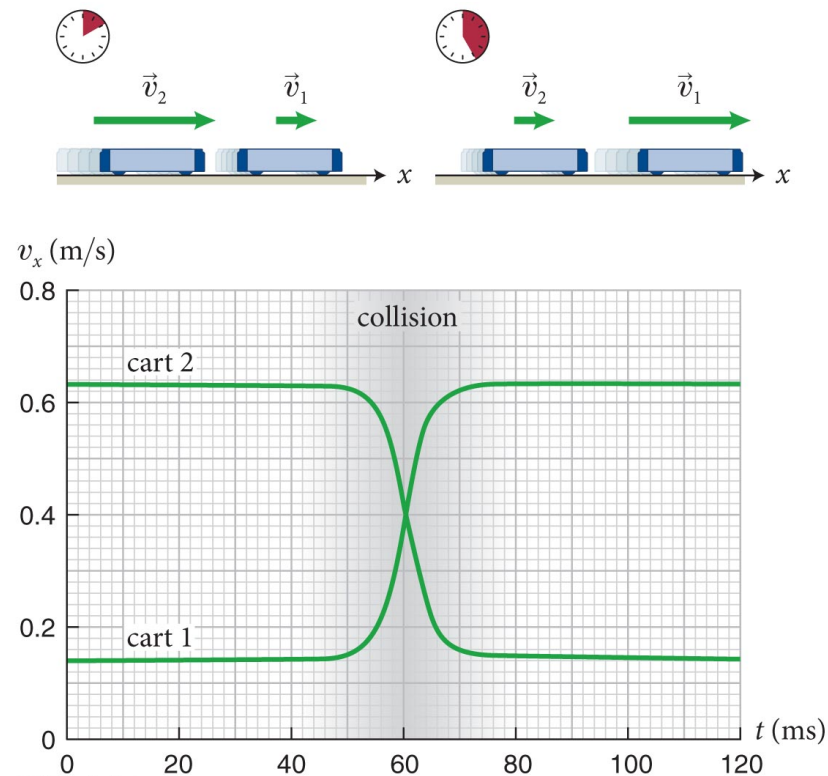
- Same deal, how about velocity?
  - $v(t)$  for both
  - *velocities are interchanged during the collision.*
  - Further experimentation: *no matter what the initial velocity of Cart 2, the collision always interchanges the two velocities.*
- again, like something is transferred



# Section 4.2: Inertia

## Experiment 2

- Now let both carts move to the right. Cart 2 is faster than Cart 1
  - The velocities are once again interchanged during the collision.
  - Repeating this experiment with different initial velocities will yield the same result.
- something is transferred or *conserved* during the collision



# Checkpoint

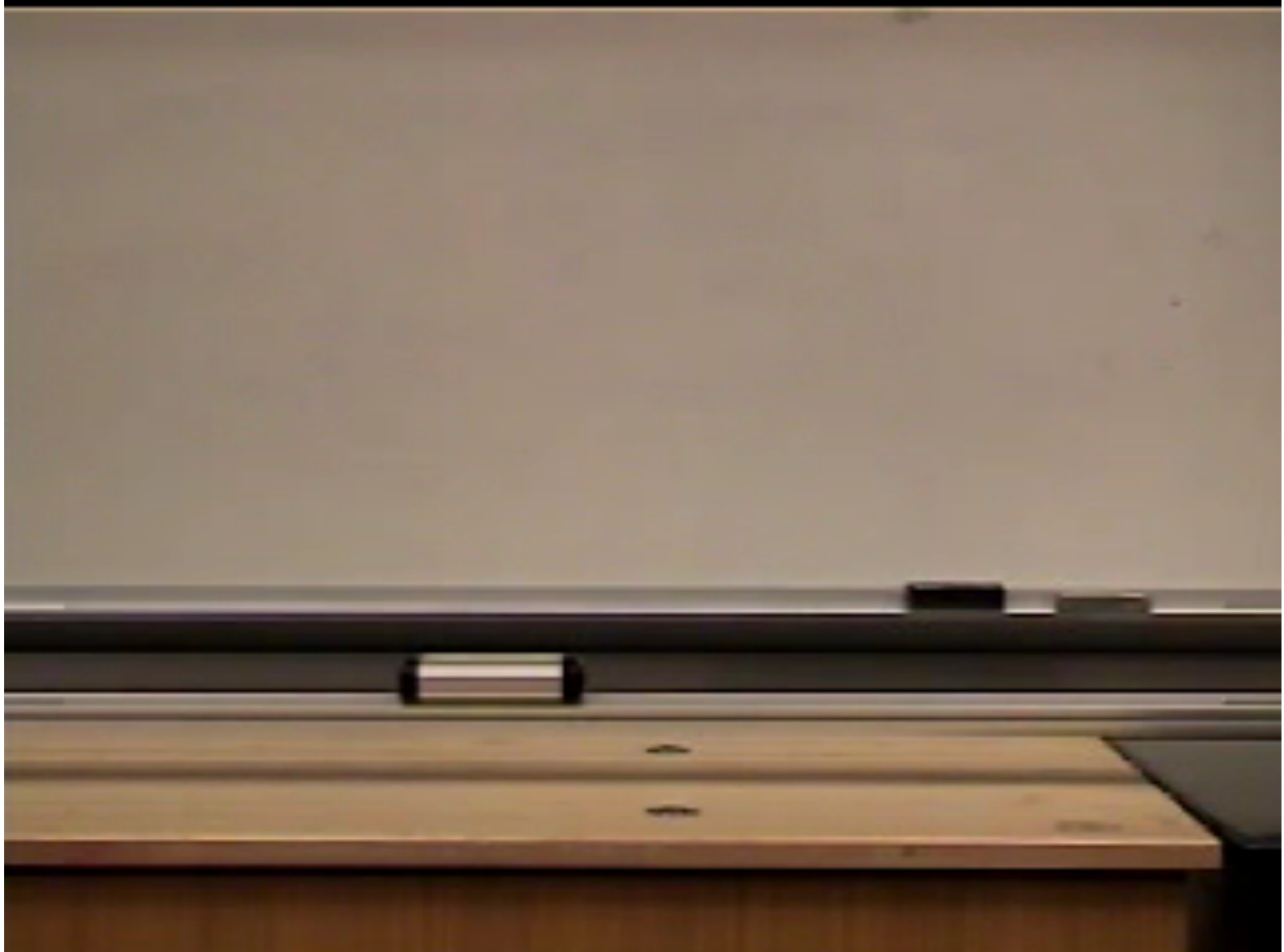


What would happen if we play the video in reverse?  
what if we watched it in a mirror?

Certain *symmetries* about the collision are implied



# Is this flipped or played in reverse?



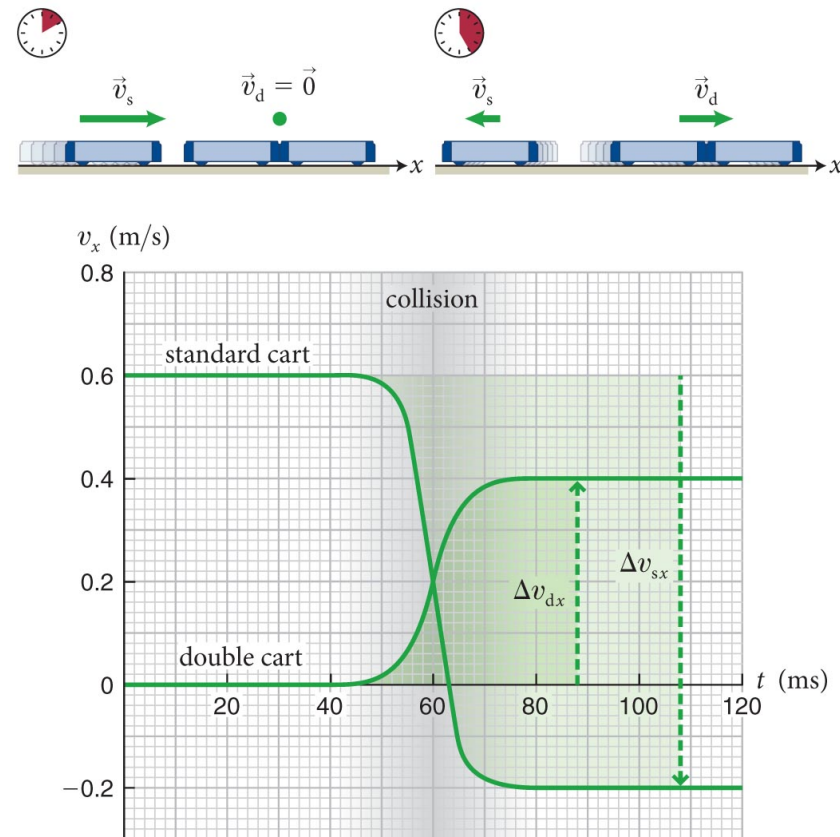
# It was flipped

- flipped and played backwards look the same
  - (except for the erasers)
- the system has mirror symmetry
  - left and right are the same
- the system has time reversal symmetry
  - forwards and backwards look the same
  - *reversible* phenomenon

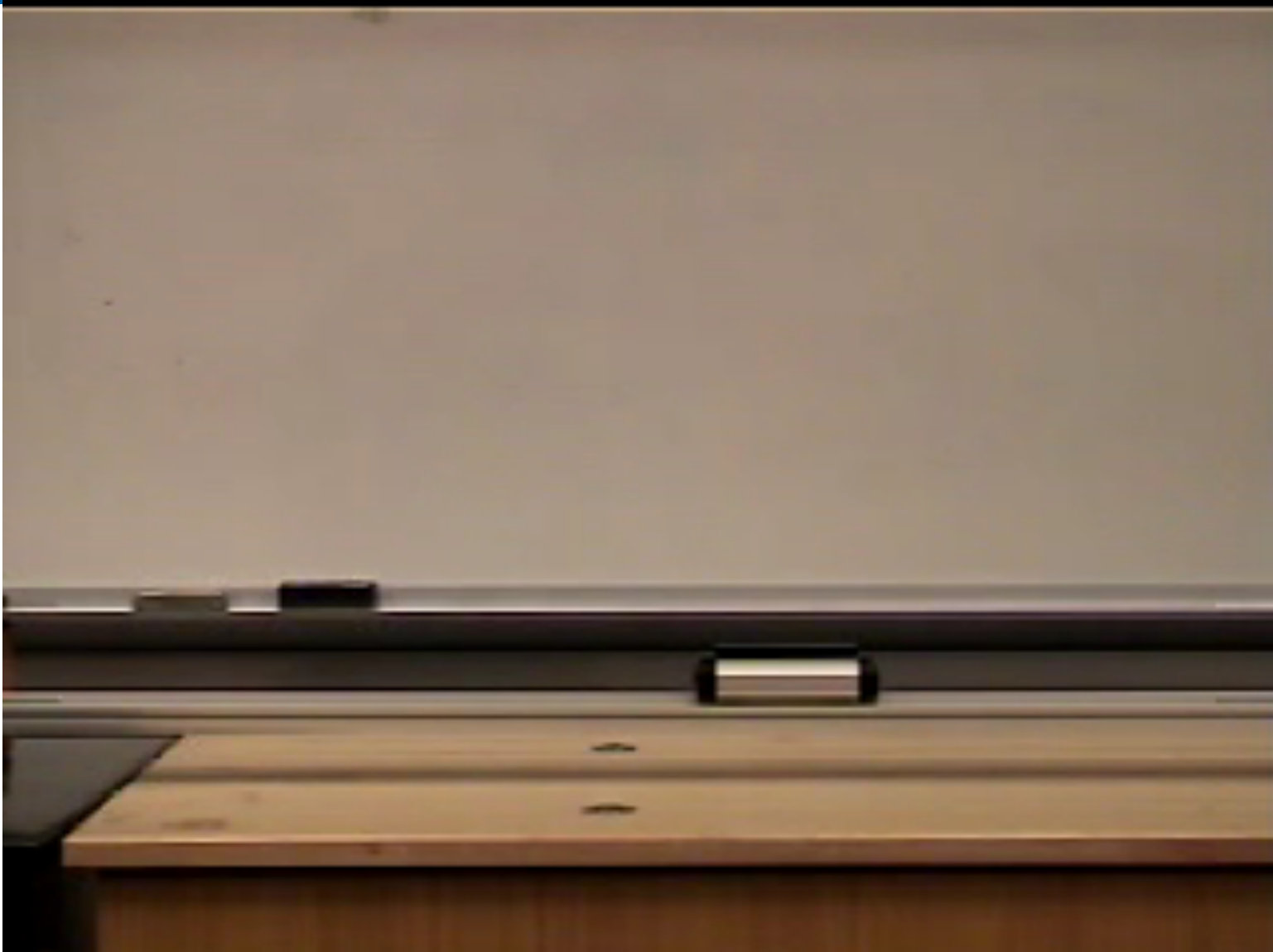
# Section 4.2: Inertia

## Experiment 3

- Now a standard cart moving towards a double cart at rest.
  - The moving cart reverses direction after the collision.
  - The change in velocity of the double cart is half that of the standard cart.



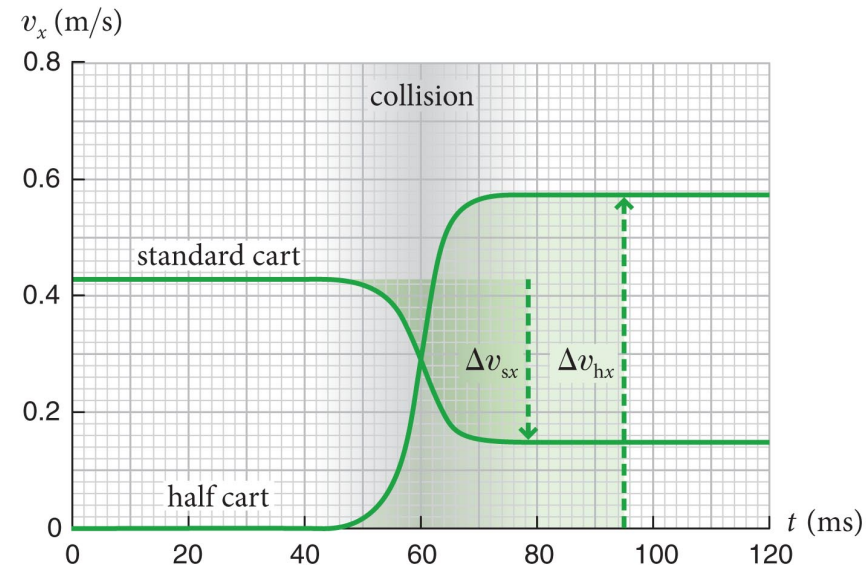
# Reality



# Section 4.2: Inertia

## Experiment 4

- Now a standard cart moving toward a *half* cart at rest.
  - The change in velocity of the half cart is twice that of the standard cart.
  - It seems that the amount of material that makes up each cart does affect the motion.



# Reality




# Section 4.2: Inertia

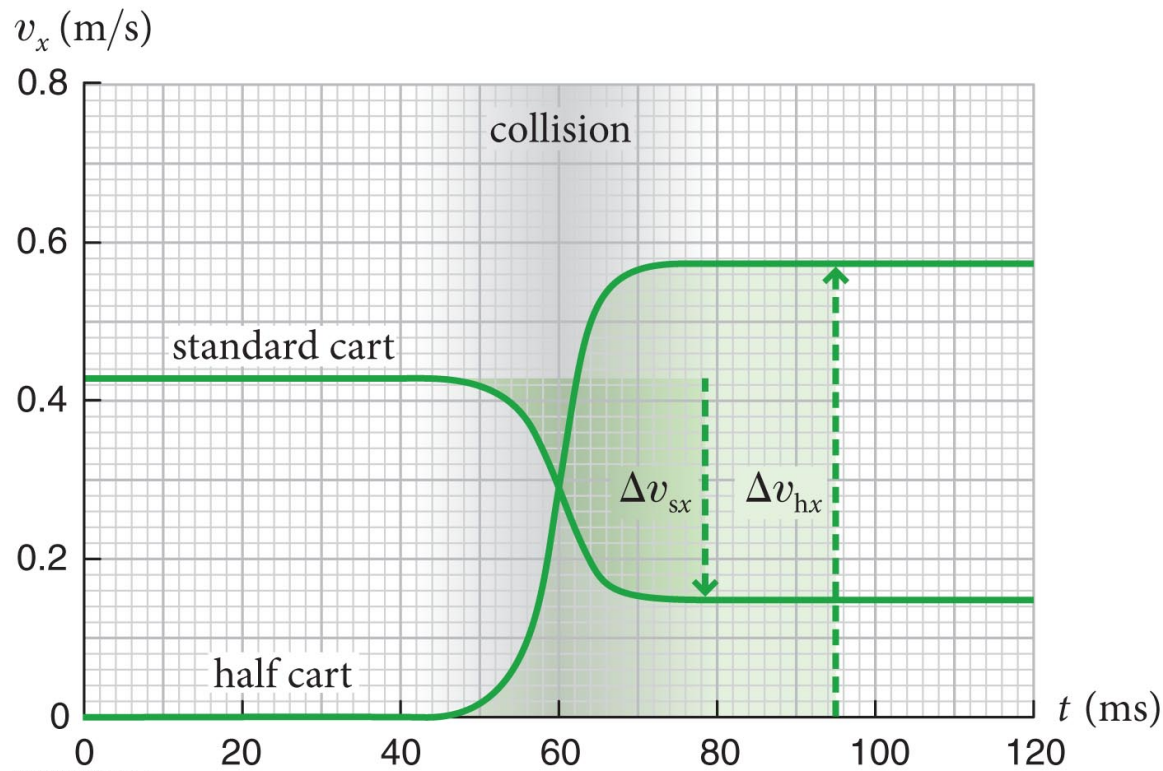
- **Conclusion from our experiments:** For objects made of the same material, the motion of larger objects are harder to change than smaller objects.
- **Inertia** is a measure of an object's tendency to resist any changes in its velocity.
  - The results of our experiments are summarized in Table 4.1.

**Table 4.1** Ratio of Velocity Changes in Collisions Between Two Carts

Experiment	Cart 1	Cart 2	$ \Delta v_{1x}  :  \Delta v_{2x} $
1	standard	standard	1
2	double	standard	0.5
3	half	standard	2

# Checkpoint 4.3

 **4.3** The  $x$  component of the final velocity of the standard cart is positive. Can you make it negative or zero by adjusting this cart's initial speed (while still keeping the half cart initially at rest)?





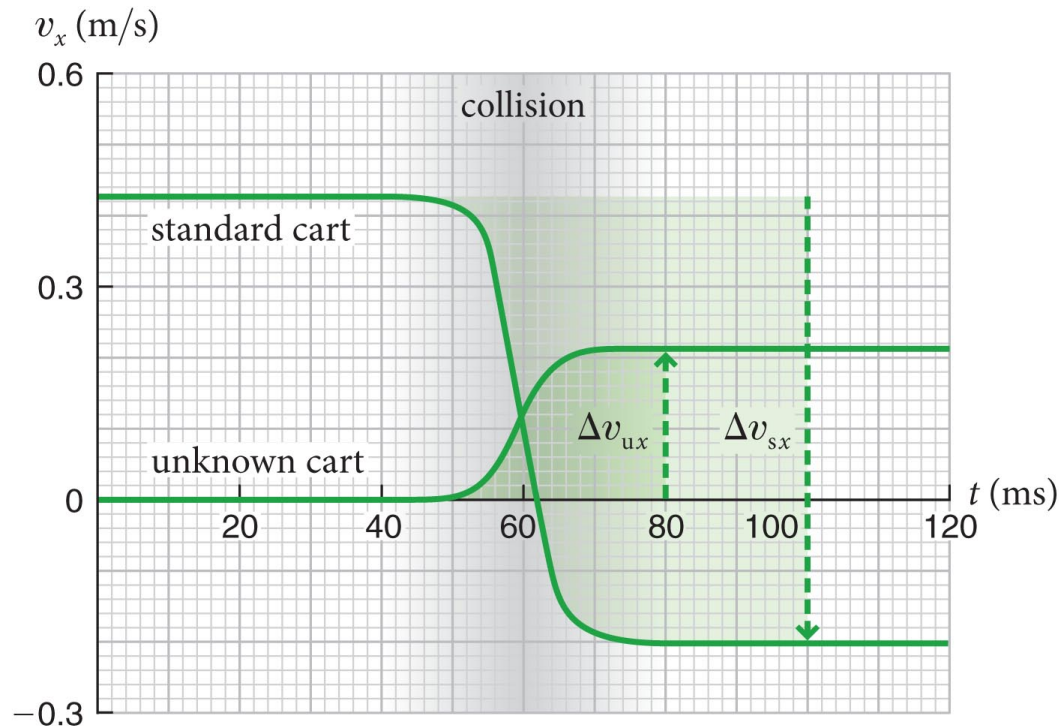


No – the standard cart always has half the velocity change of the half cart.

That means if you decrease the standard cart's velocity, you just shrink the whole graph vertically. The standard cart's velocity remains positive after the collision.

# Checkpoint 4.4

- The figure shows a graph of the collision between some unknown cart and a standard cart made of the same material.
- We see that  $|\Delta v_{ux}|/|\Delta v_{sx}| \approx 1/3$  for the two carts
- We can conclude that the unknown cart's inertia is three times that of the standard cart.



## Section 4.2

### Question 1

Carts A and B collide on a horizontal, low-friction track. Cart A has twice the inertia of cart B, and cart B is initially motionless. How does the change in the velocity of A compare with that of B?

1.  $v_A/v_B = 1$

2.  $v_A/v_B = -1$

3.  $v_A/v_B = 1/2$

(direction matters!)

4.  $v_A/v_B = -1/2$

5.  $v_A/v_B = 2$

6.  $v_A/v_B = -2$

# Section 4.2


## Question 1

Carts A and B collide on a horizontal, low-friction track. Cart A has twice the inertia of cart B, and cart B is initially motionless. How does the change in the velocity of A compare with that of B?

1.  $v_A/v_B = 1$

2.  $v_A/v_B = -1$

3.  $v_A/v_B = 1/2$

 4.  $v_A/v_B = -1/2$  more inertia = harder to change  
changes in velocity are in opp dir

5.  $v_A/v_B = 2$

6.  $v_A/v_B = -2$

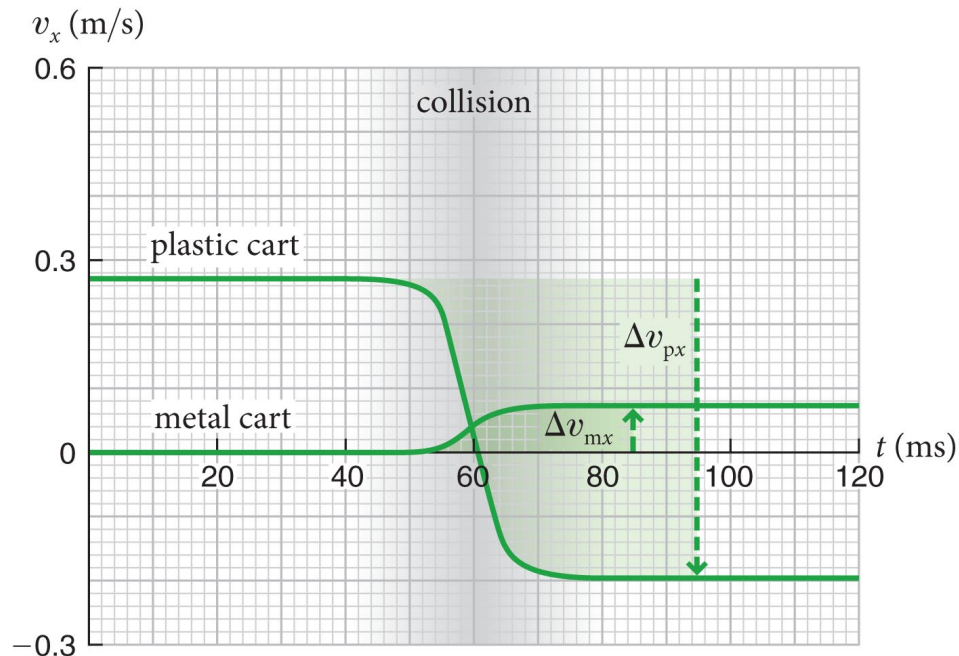
# Section 4.3: What determines inertia?

## Section Goals

- Extend the analysis of the collision between two objects that are composed of different materials and have different volumes.
- Recognize that the inertia of an object is determined by **the type of material and its volume.**

# Section 4.3: What determines inertia?

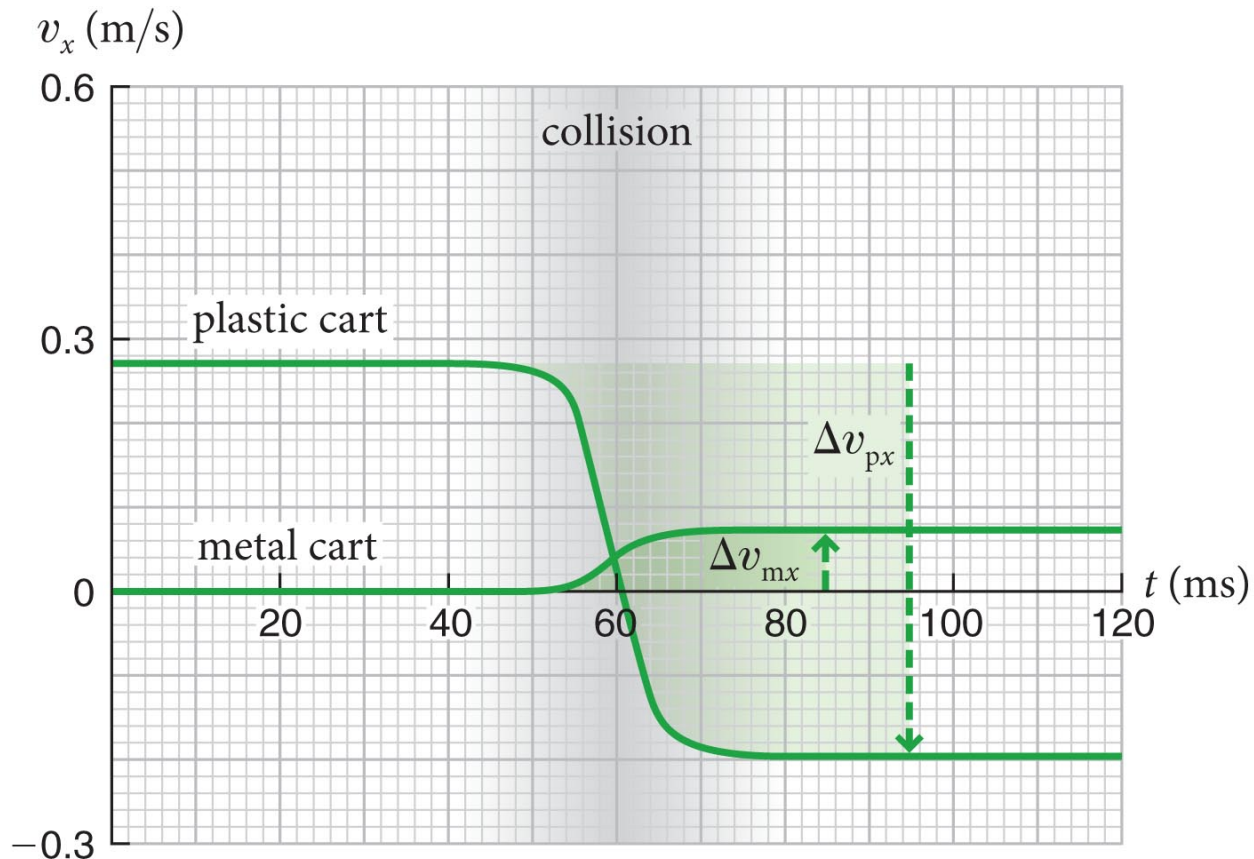
- Is it only the amount of material that determines inertia?
  - Repeat experiment 1 carried out for two carts, one made of plastic and the other metal, but both with an *identical amount* of material
  - The results shown in the figure are very different from the results for two identical carts.



# Checkpoint 4.5



- We find  $\Delta v_{px}/\Delta v_{mx} = -6.7$
- metal cart hardly budes!
- plastic cart rebounds with large fraction of original speed



## Section 4.3: What determines inertia?

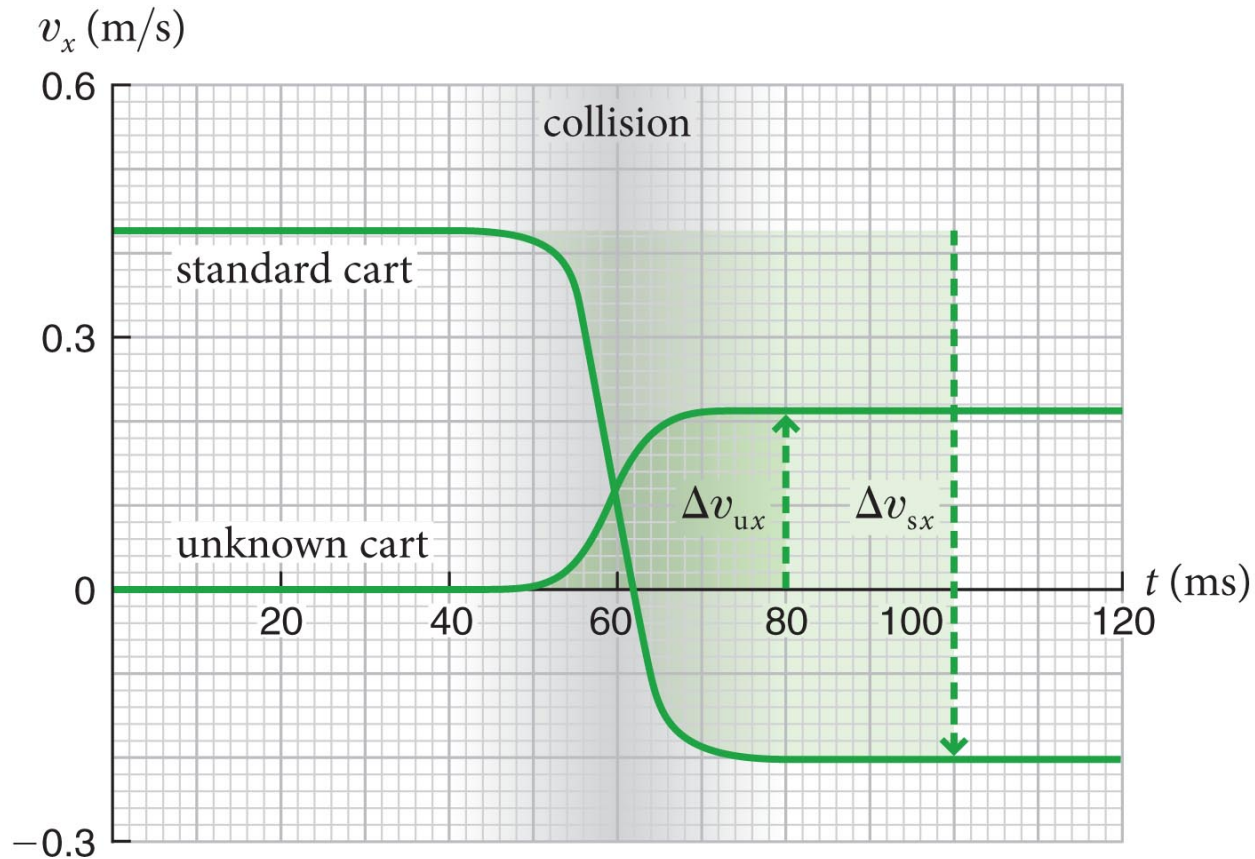
- **Conclusion:** The inertia of an object is determined entirely by the type of material of which the object is made and by the amount of that material contained in the object.
  
- So why were metal and plastic so different?



# Checkpoint 4.6

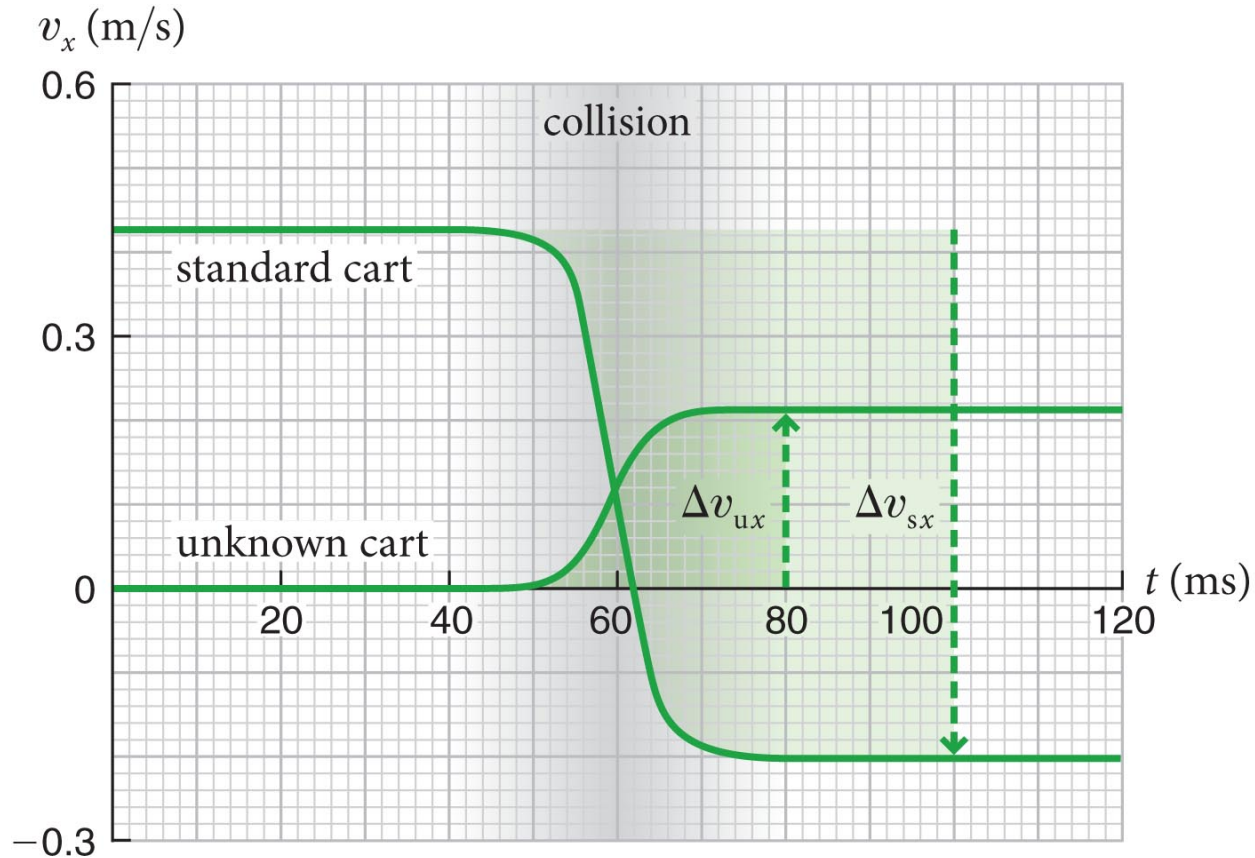


**4.6** Is the inertia of the cart of unknown inertia in the figure greater or less than that of the standard cart?



# Checkpoint 4.6

 **4.6** The unknown has greater inertia, since its change in velocity is much less



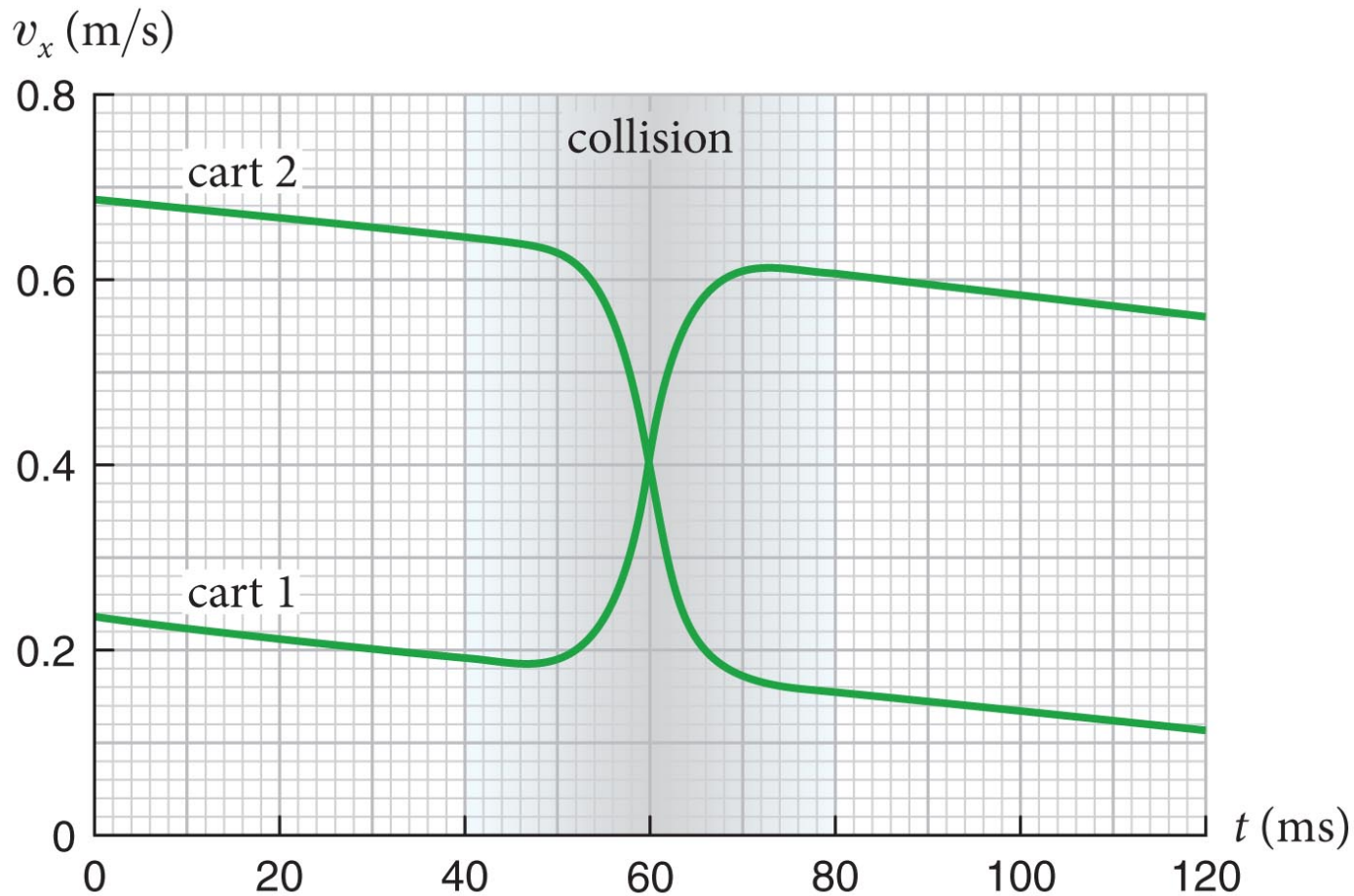
# Section 4.3: What determines inertia?

## Example 4.1 Friction and collisions

- The figure on the next slide shows the  $v_x(t)$  curves for a collision between two identical carts
- Now they are moving on a rough surface, and friction affects their motion.
- Are the changes in the velocity of the carts caused by the collision still equal in magnitude?
- what physics has really changed?

# Chapter 4.3: What determines inertia?

## Example 4.1 Friction and collisions (cont.)



# Section 4.3: What determines inertia?

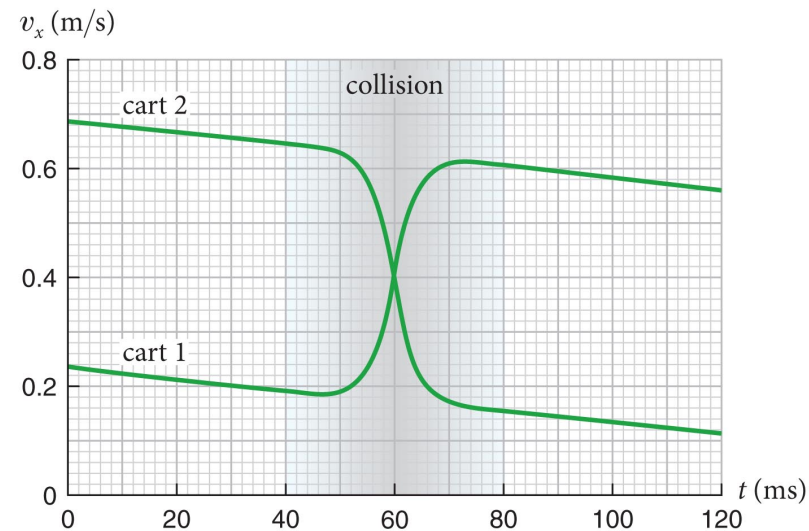
## Example 4.1 Friction and collisions (cont.)

### ① GETTING STARTED

We have two bits of physics now:

- collision
  - results in velocity exchange
- friction
  - results in linearly decreasing  $v$

reasonable to expect superposition  
and that's what it looks like ...



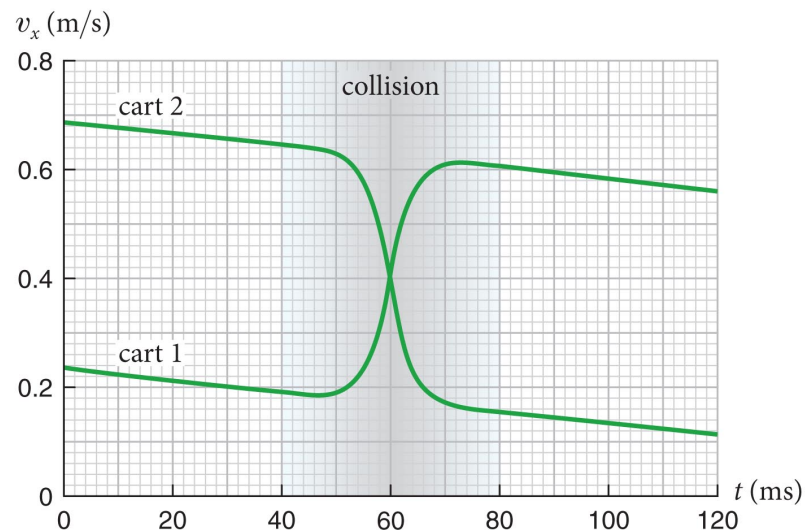
# Section 4.3: What determines inertia?

## Example 4.1 Friction and collisions (cont.)

### 1 GETTING STARTED

Shape suggests friction acts throughout the motion

If so, how do we figure the change in velocity?



# Section 4.3: What determines inertia?

## Example 4.1 Friction and collisions (cont.)

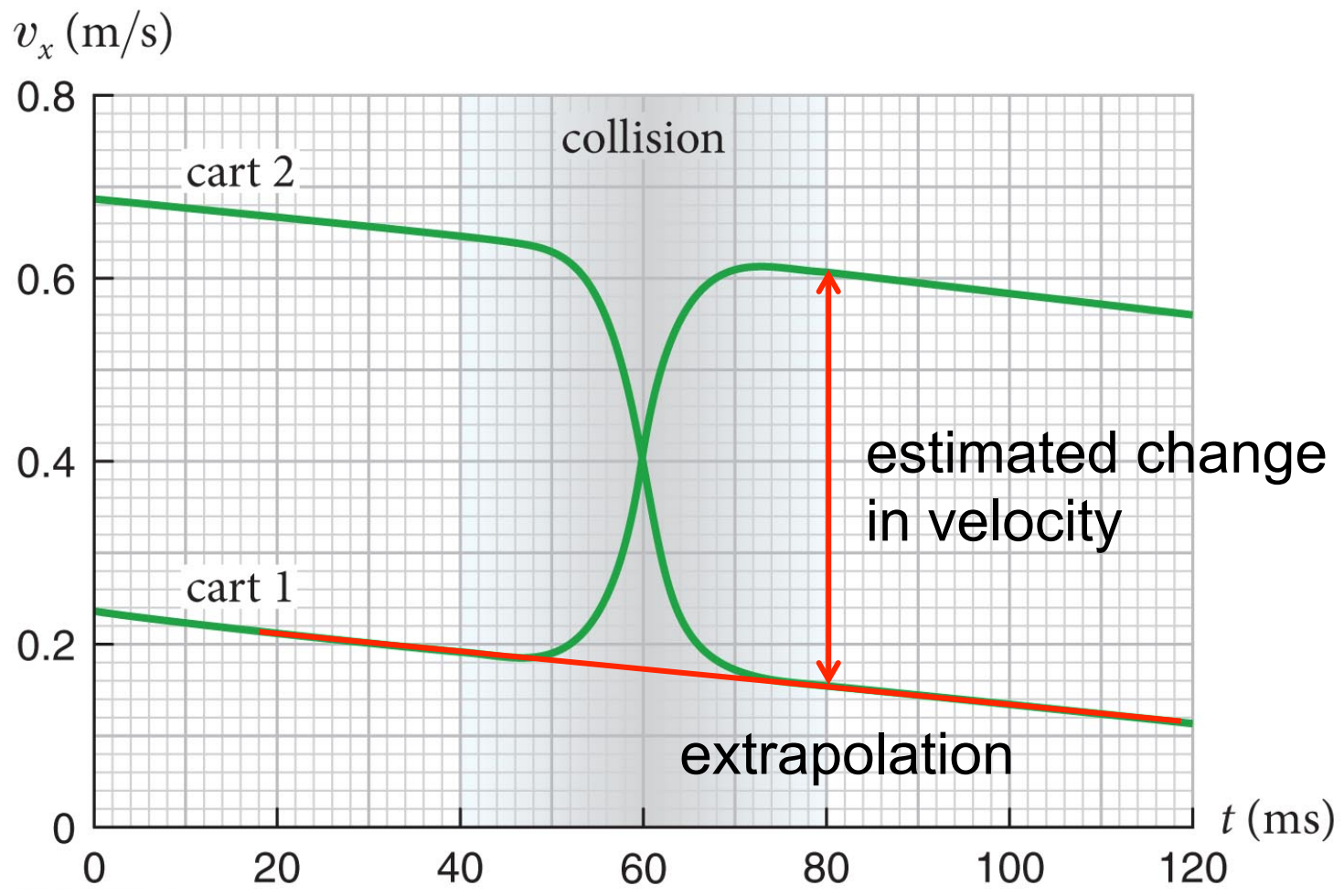
### ② DEVISE PLAN

We can extrapolate the  $v_x(t)$  curve for cart 1 to calculate what its velocity *would* have been at  $t = 80$  ms if the collision had not taken place.

To determine the change in velocity due to the collision, read off the actual value of the velocity of cart 1 at  $t = 80$  ms and subtract the two velocities. Repeat for cart 2.

# Section 4.3: What determines inertia?

## That sounds complicated, but it isn't





## Section 4.3: What determines inertia?

### Example 4.1 Friction and collisions (cont.)

#### ③ EXECUTE PLAN

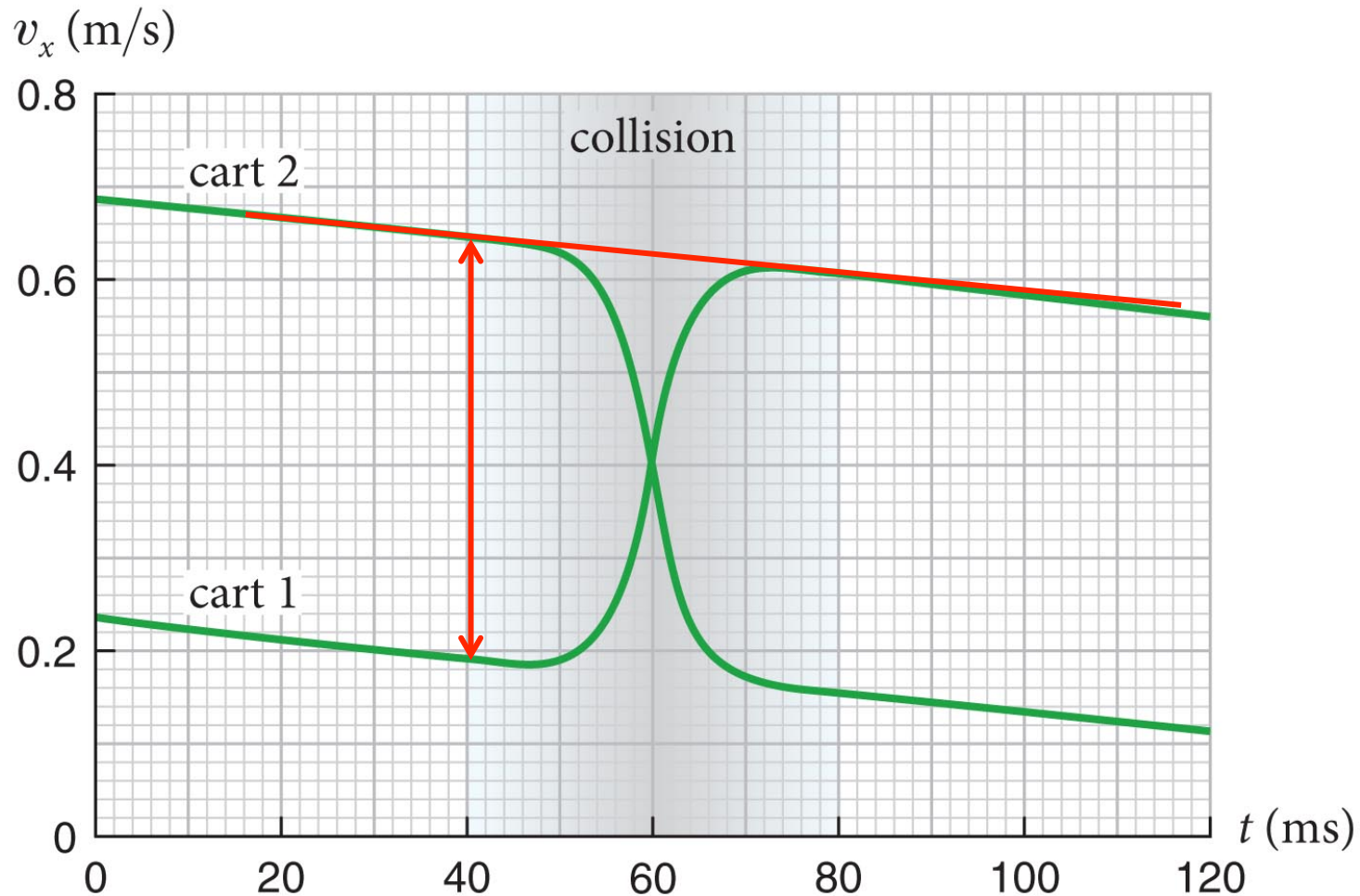
Doing this for cart 1 at  $t = 80$  ms gives  $+0.15$  m/s.

Repeating this procedure for cart 2, about  $+0.60$  m/s.

Do the same just *before* the collision (40 ms) to get velocities before the collision

# Section 4.3: What determines inertia?

## Now before the collision



# Section 4.3: What determines inertia?

## Example 4.1 Friction and collisions (cont.)

### ③ EXECUTE PLAN

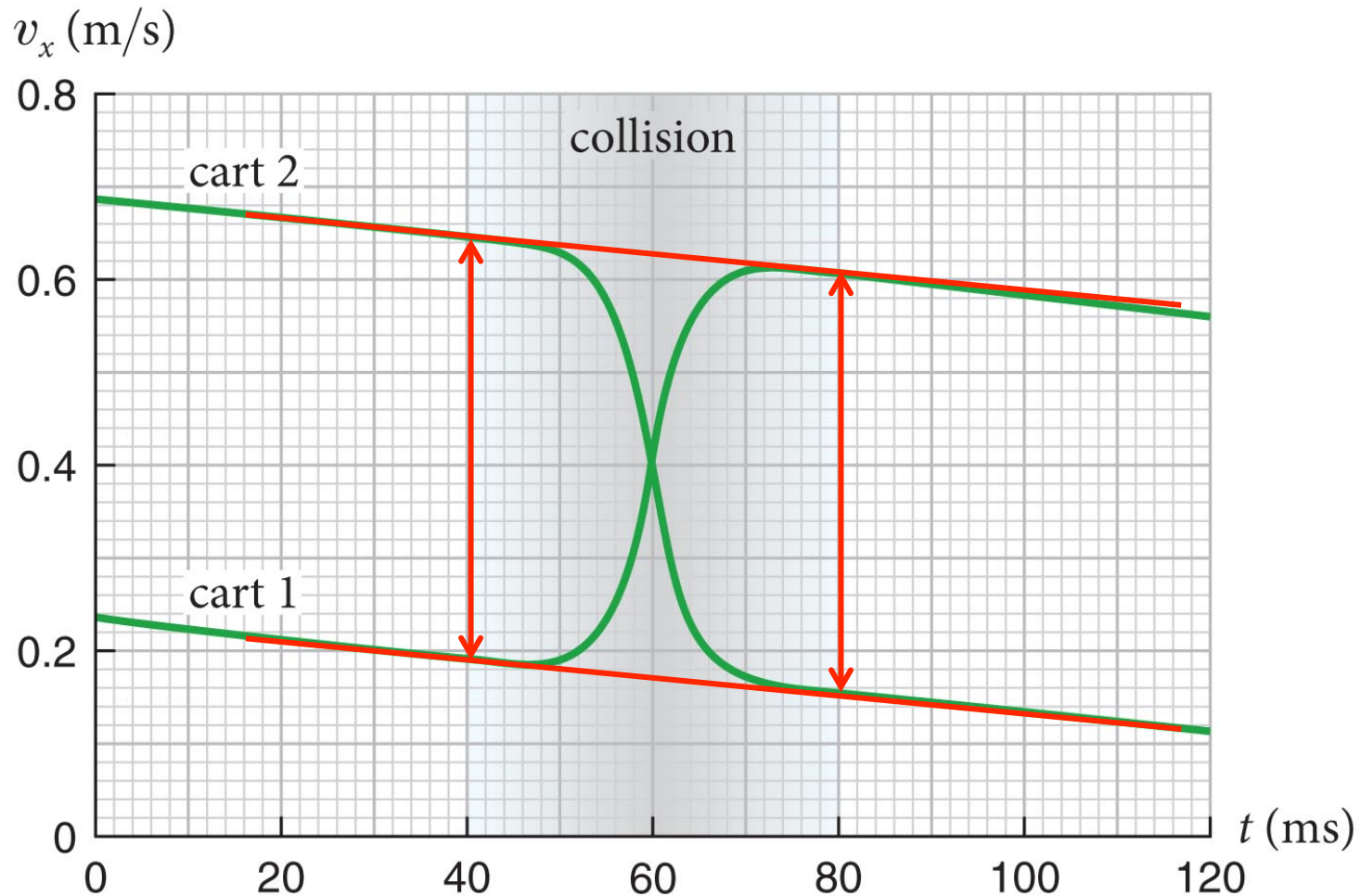
Knowing (extrapolated) initial and final velocities, we can find the change in velocity.

We find 0.45 m/s for both carts. ✓

- ❖ This tells us that friction is an additive effect, does not change the underlying fundamental physics.
- ❖ Whatever friction is, it is an *independent* interaction

# Section 4.3: What determines inertia?

**Arrows are the same size.**



# Section 4.3

## Question

Two objects of identical volume and shape are made of different materials: iron and wood. How do their inertias compare?

1. Inertia of iron = inertia of wood
2. Inertia of iron  $>$  inertia of wood
3. Inertia of iron  $<$  inertia of wood

# Section 4.3

## Clicker Question 2

Two objects of identical volume and shape are made of different materials: iron and wood. How do their inertias compare?

1. Inertia of iron = inertia of wood
- ✓ 2. Inertia of iron > inertia of wood
3. Inertia of iron < inertia of wood

What word are we dancing around by saying “inertia”?  
Why are we being careful?

# Section 4.4: Systems

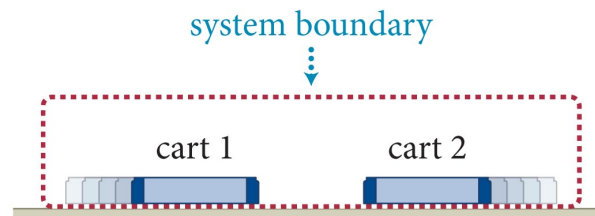
## Section Goals

- Classify physical situations as being composed of a **system** of objects of interest and the rest of the universe, **the environment**.
- Choose the appropriate system and environment for physical situations depending on the physics of interest.
- Decide if the two need to interact at all  
(Hint: you don't want them to)

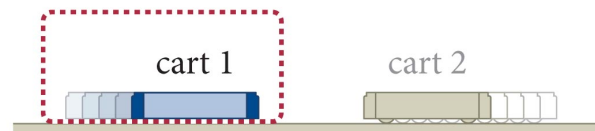
# Section 4.4: Systems

- The first step in the analysis: separate the object(s) of interest from the rest of the universe:
  - Any object or group of objects that we can separate, in our minds, from the surrounding environment is a system.

(a) Choice 1: system consists of both carts



(b) Choice 2: system consists of one cart

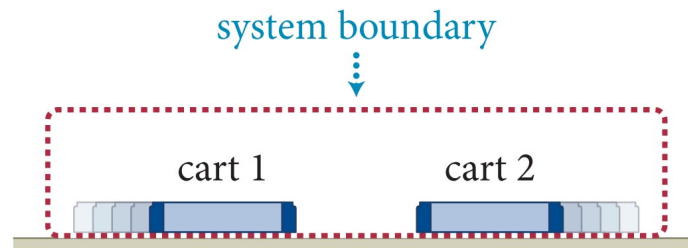




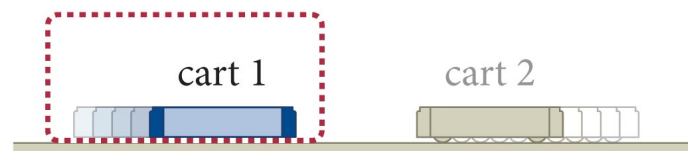
# Section 4.4: Systems

- Which choice might be more clever here?
- Why? You want to isolate interactions ...
- What would make both choices inconvenient?

(a) Choice 1: system consists of both carts



(b) Choice 2: system consists of one cart



# Section 4.4: Systems

## Exercise 4.2 Choosing a system

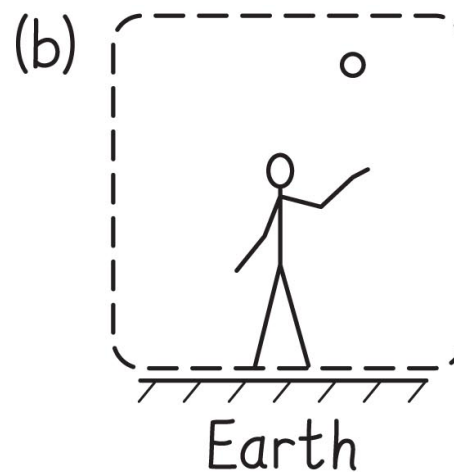
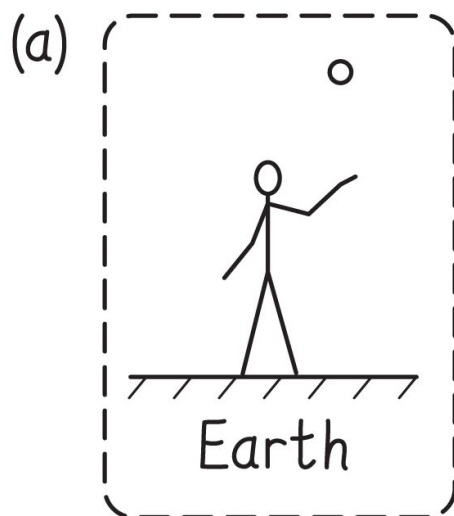
Indicate at least two possible choices of system in each of the following two situations. For each choice, make a sketch showing the system boundary and state which objects are inside the system and which are outside.

- (a) After you throw a ball upward, it accelerates downward toward Earth.
- (b) A battery is connected to a light bulb that illuminates a room.

# Section 4.4: Systems

## Exercise 4.2 Choosing a system (cont.)

**SOLUTION** (a) The description of the situation mentions three objects: the ball, Earth, and you. One option is to include all three of them in the system. As a second choice, include you and the ball in the system. ✓



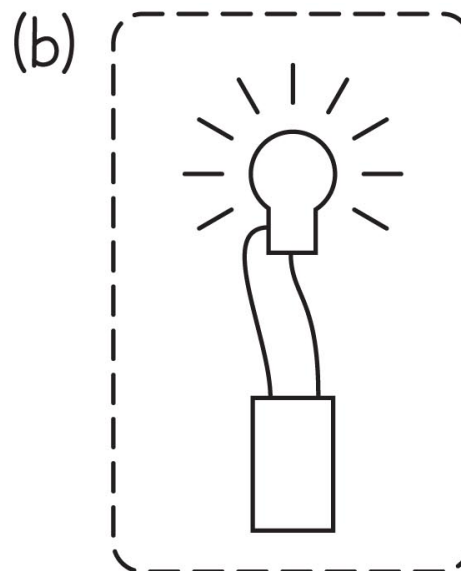
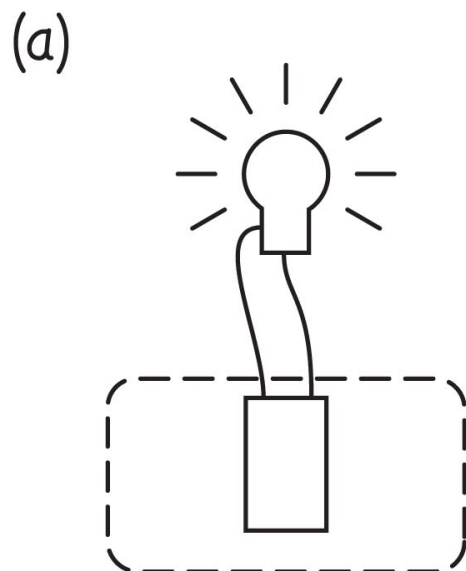
# Section 4.4: Systems

## Exercise 4.2 Choosing a system (cont.)

SOLUTION (b) 3 objects: battery, light bulb, and room.

I can choose just one of them—the battery—as my system or two of them—the battery and the light bulb. ✓

Which is “better” depends on what you’re trying to find!



# Section 4.4: Systems

## Exercise 4.2 Choosing a system (cont.)

### SOLUTION

- Choices of system are arbitrary.
- Nothing in the problem prescribes a choice of system.
- If you tried this problem on your own and made different choices, then your answer is just as “correct”!

(But some “correct” choices are easier than others.)

# Section 4.4: Systems

- Once we have chosen a system, we can study how certain quantities associated with the system change over time:
  - **Extensive quantities:** quantities whose value is proportional to the size of the system.
    - examples: volume, energy, length, mass, *momentum*
  - **Intensive quantities:** quantities that do not depend on the extent of the system.
    - examples: temperature, pressure, density

# Section 4.4: Systems

- An **intensive property** is a **bulk property**
  - e.g., when a diamond is cut, pieces retain hardness
- An **extensive property** an additive effect
  - depends on amount of material
  - mass and volume depend on amount of stuff
- Ratio of two extensive properties is intensive!
  - e.g., ratio of extensive mass and volume gives *intensive*
  
- Who. Cares?

## Section 4.4: Systems

- intensive properties are *invariant*, and in some way more fundamental
- extensive properties depend on exact details; circumstantial
- we want to find and first explain *intensive* properties
  - figure out what is fundamental vs circumstantial
  - figure out how to combine parameters to make something intensive



# Section 4.4: Systems

- Only four processes can change the value of an extensive quantity: input, output, creation, destruction.
- Then, the change of a certain quantity over a time interval is given by

$$\textit{change} = \textit{input} - \textit{output} + \textit{creation} - \textit{destruction}$$

(a)



Extensive quantities can be changed by input, output, creation, and destruction.

# Section 4.4: Systems

- More simply:

$$\text{change} = (\text{net flux into system}) + (\text{net appearance of new stuff})$$

(a)



Extensive quantities can be changed by input, output, creation, and destruction.

# Section 4.4: Systems

- If there is no transfer of an extensive quantity across the boundary of the system (no flux in or out)

$$\text{change} = \text{creation} - \text{destruction}$$

- Any extensive quantity that cannot be created or destroyed is said to be **conserved**:

- The change in the value of a conserved quantity is

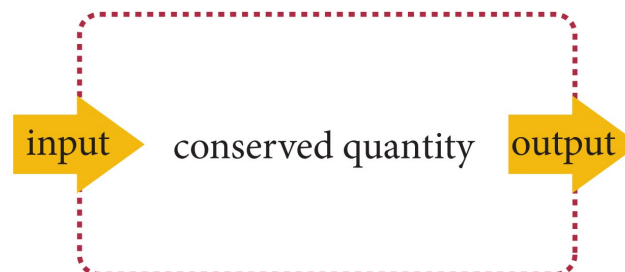
$$\text{change} = \text{input} - \text{output}$$

(b)



If there is no input or output, the quantity can be changed only by creation or destruction.

(c)



If an extensive quantity cannot be created or destroyed, it is said to be conserved.

# Reading quiz for today

## question 1

A freight car moves along a frictionless level railroad track at constant speed. The freight car is open on the top. A large load of sand is suddenly dumped into the freight car. What happens to the speed of the freight car?

- The speed of the freight car decreases.
- The speed of the freight car remains the same.
- The speed of the freight car increases.
- It is impossible to determine

*discuss for a bit ...*

# Chapter 4: Momentum

## Quantitative Tools

# Section 4.5: Inertial standard

## Section Goals

You will learn to

- Represent the **standard quantity of inertia** by the kilogram in the metric system.
- Compare the inertia of any object with the kilogram standard of inertia.

## Section 4.5: Inertial standard

- The inertia of an object is represented by the symbol  $m$ :
  - $m$  is for mass, a concept related to inertia.
- The basic SI unit of inertia is the kilogram (kg):
  - The inertial standard is a platinum-iridium cylinder
- Using the definition of the ratio of inertias of colliding carts and the inertial standard ( $m_s$ ), we can find inertia of any object ( $m_u$ ):

$$\frac{m_u}{m_s} \equiv - \frac{\Delta v_{sx}}{\Delta v_{ux}}$$
$$m_u \equiv - \frac{\Delta v_{sx}}{\Delta v_{ux}} m_s$$

- The minus sign is because the velocity of one of the carts will decrease during collision.

# Section 4.5

- This implies

$$m_u \Delta v_{u,x} = -m_s \Delta v_{s,x}$$

- *or ...*

$$m_u \Delta v_{u,x} + m_s \Delta v_{s,x} = 0$$

- *which implies no net change for the whole system*
- further, noting definition of  $\Delta v$ :

$$m_u v_{u,xi} + m_s v_{s,xi} = m_u v_{u,xf} + m_s v_{s,xf}$$

- total of the product of inertia and velocity is constant
- find  $mv$  for all objects in closed system. add together.
  - sum never changes – problem solving!



## Section 4.5

- the quantity  $mv$  is an important one
  - $mv = \mathbf{momentum}$
- its sum for the whole system is a constant
- solving problems: write it down before & after!
- think back to identical carts collision ...
  - it is the  $mv$  that is exchanged by carts
  - in total, the amount of  $mv$  stays constant

# Reading quiz for today

## question 2

When a person steps forward out of a small boat onto a dock, the boat recoils backward in the water. Why does this occur?

- The total mechanical energy of the system is conserved.
- The momentum of the person is greater than the momentum of the boat
- The momentum of the boat is greater than the momentum of the person.
- The energy of the person is greater than the energy of the boat.
- The total momentum of the system is conserved.
- The energy of the boat is greater than the energy of the person.

*discuss for a bit ...*

# Reading quiz for today

## question 3

- When a constant force acts on an object, what does the object's change in momentum depend upon?
- The change in momentum depends upon the change in the velocity of the object.
- ... depends upon the mass of the object.
- ... depends upon the time interval during which the force acts.
- ... depends upon the change in the position of the object.
- ... depends upon the magnitude of the force.

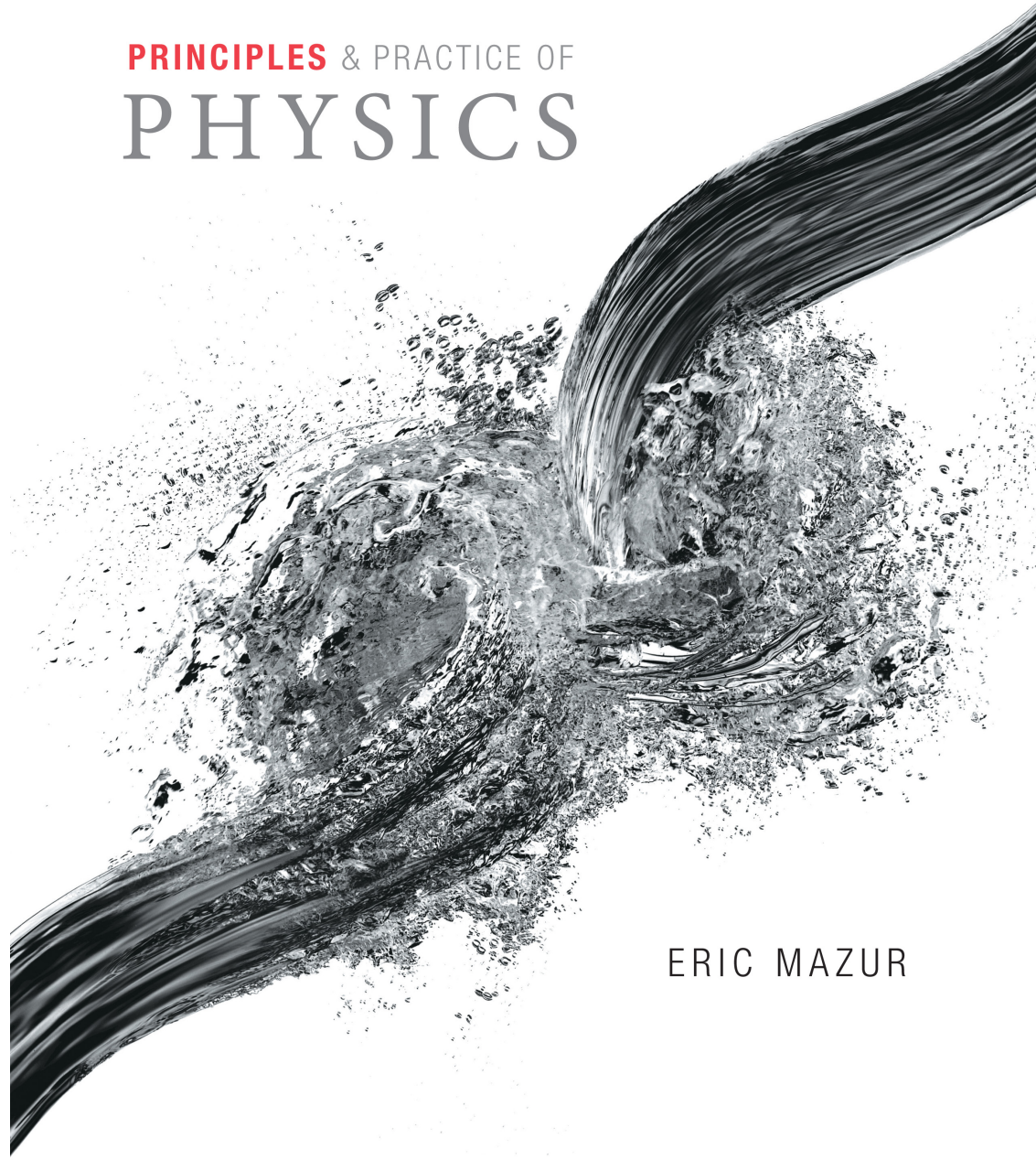
momentum is  $mv$  – two clearly important variables

if you push something, does it matter how long?

does it matter where you do it?

PRINCIPLES & PRACTICE OF  
PHYSICS

Momentum



ERIC MAZUR

# Various

- lab this week
  - just posted it. pretty simple
  - will have extra time for HW questions
- today
  - finish up momentum
  - more problems
- Thursday
  - most of the energy chapter
  - exam 1 details

# Reading quiz

Which of the following statements are true? Select all correct responses.

**Choose all that apply.**

- The impulse delivered to a particle equals the change of momentum of the particle.
- The total momentum of any number of particles is equal to the vector sum of the momenta of the individual particles.
- The impulse delivered to a particle equals the change of velocity of the particle.
- The total momentum of any system is constant.
- The total momentum of any number of particles is equal to the algebraic sum of the magnitudes of the momenta of the individual particles.
- The total momentum of an isolated system is constant.

# Reading quiz

Which of the following statements are true? Select all correct responses.

**Choose all that apply.**

- **The impulse delivered to a particle equals the change of momentum of the particle.**
- **The total momentum of any number of particles is equal to the vector sum of the momenta of the individual particles.**
- The impulse delivered to a particle equals the change of velocity of the particle.
- The total momentum of any system is constant.
- The total momentum of any number of particles is equal to the algebraic sum of the magnitudes of the momenta of the individual particles.
- **The total momentum of an isolated system is constant.**

# Reading quiz: water rocket

- Suppose we repeat the experiment from the video, but this time we use a rocket three times as inertia as the one in the video, and in place of water we use a fluid that is twice as inertia (dense) as water. If the new fluid leaves the rocket at the same speed as the water in the video, what will be the ratio of the horizontal speed of our rocket to the horizontal speed of the rocket in the video after all the fluid has left the rocket? (Ignore air resistance.)



# Reading quiz: water rocket

- The rocket's speed is proportional to the ratio of the fluid's inertia to the rocket's inertia.

(twice the fluid inertia)/(three times the rocket inertia) =  $2/3$

# Reading quiz

- A red ball with a velocity of  $+3.0$  m/s collides head-on with a yellow ball of equal mass moving with a velocity of  $-2.0$  m/s. What is the velocity of the two balls after the elastic collision?

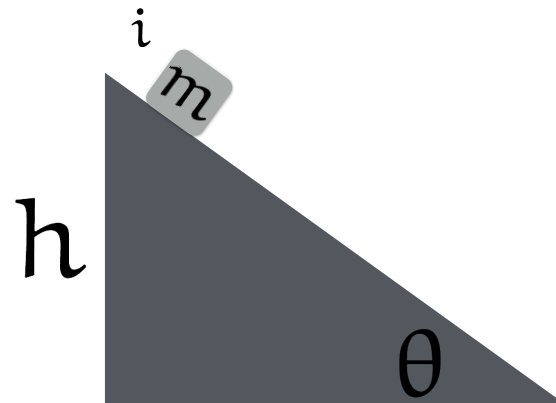
# Reading quiz

- A red ball with a velocity of  $+3.0$  m/s collides head-on with a yellow ball of equal mass moving with a velocity of  $-2.0$  m/s. What is the velocity of the two balls after the elastic collision?
- Change in momentum is the same for both
- Identical masses *exchange velocity* after an ideal collision
- Red:  $-2.0$  m/s, yellow:  $+3.0$  m/s

# Review problem

## Sliding down a ramp

- A block slides down a ramp without friction, starting from rest
- what is its speed at the bottom?



# Review problem

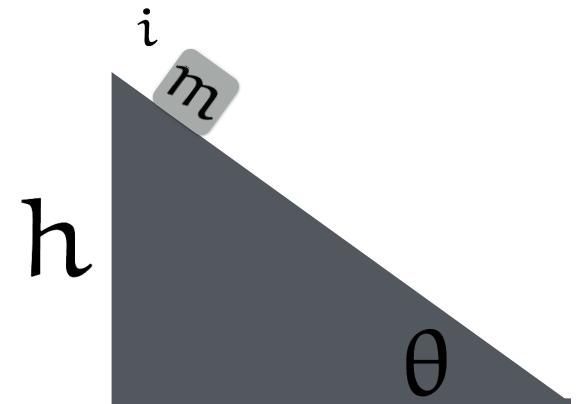
## Sliding down a ramp

- We know the initial speed, the displacement along the ramp, and acceleration.
- Let  $+x$  be down the ramp

$$\sin \theta = h/\Delta x \quad \text{so} \quad \Delta x = h/\sin \theta$$

$$\mathbf{a} = +g \sin \theta$$

$$\mathbf{v}_i = \mathbf{0}$$



- We want speed, we don't know time

$$v_f^2 = v_i^2 + 2a\Delta x$$

# Review problem

## Sliding down the ramp

- Putting it together:

$$\sin \theta = h/\Delta x \quad \text{so} \quad \Delta x = h/\sin \theta$$

$$a = -g \sin \theta$$

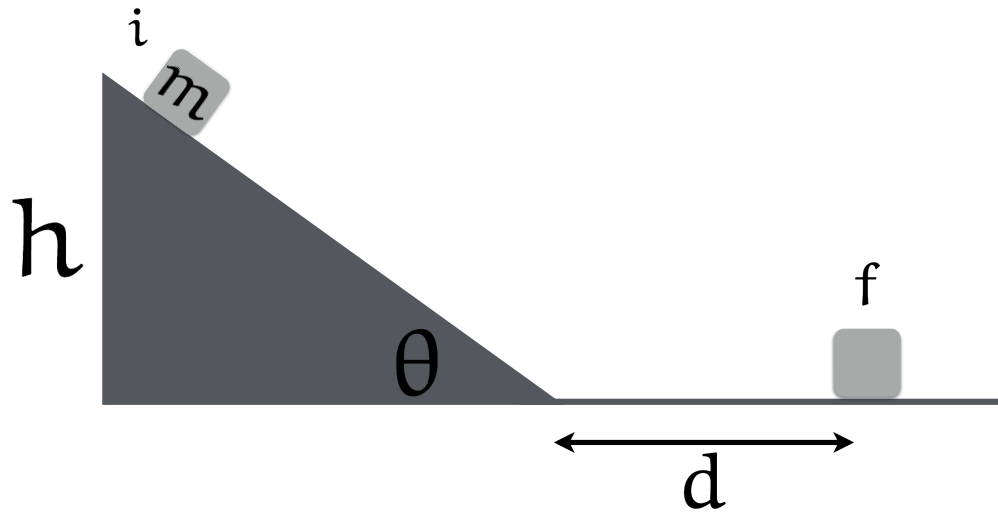
$$v_f^2 = v_i^2 + 2a\Delta x = 2g \sin \theta (h/\sin \theta) = 2gh$$

- Curiously, speed does not depend on angle!
  - Only height of ramp matters

# Review problem

## Sliding down a ramp

- If the flat surface has an acceleration due to friction of  $\mu$ , how far does the block slide before stopping?



# Review problem

## Sliding down a ramp

- We know the initial velocity from the earlier part
- We know the acceleration
- We know the final velocity is zero
- Let  $+x$  be to the right

$$a = -\mu$$

$$v_i = \sqrt{2gh}$$

$$v_f = 0$$

- Want to find  $d$



# Review problem

## Sliding down a ramp

- Same as last part, only the unknown changed.

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$0 = 2gh - 2\mu d$$

$$\mu d = gh$$

$$d = \frac{gh}{\mu}$$

# Review problem

## Sliding down a ramp

- Sensible?
  - ✓ higher the ramp, faster at the bottom, farther it goes
  - ✓ depends on how strong  $g$  is
  - ✓ if friction goes away,  $\mu \rightarrow 0$  and  $d \rightarrow \infty$ 
    - ✓ (the object doesn't stop if there is no friction)

# Section 4.6: Momentum

## Section Goals

You will learn to

- Identify **momentum** as the ability of an object to affect the motion of other objects in a collision.
- Calculate momentum for individual objects and systems of interacting objects

## Section 4.6: Momentum

- recall:

$$m_u \Delta v_{u x} + m_s \Delta v_{s x} = 0$$

- Using  $\Delta v_x = v_{x,f} - v_{x,i}$ , we get

$$m_u v_{u x,f} - m_u v_{u x,i} + m_s v_{s x,f} - m_s v_{s x,i} = 0$$

- This equation suggests that the product of inertia and velocity is an important quantity referred to as ***momentum***:

$$\vec{p} \equiv m\vec{v}$$

where the  $x$  component of momentum is

$$p_x \equiv mv_x$$

## Section 4.6: Momentum

- With this definition of momentum, we can rewrite in the form

$$\Delta p_{ux} + \Delta p_{sx} = 0$$

where  $\Delta p_x = p_{x,f} - p_{x,i}$ .

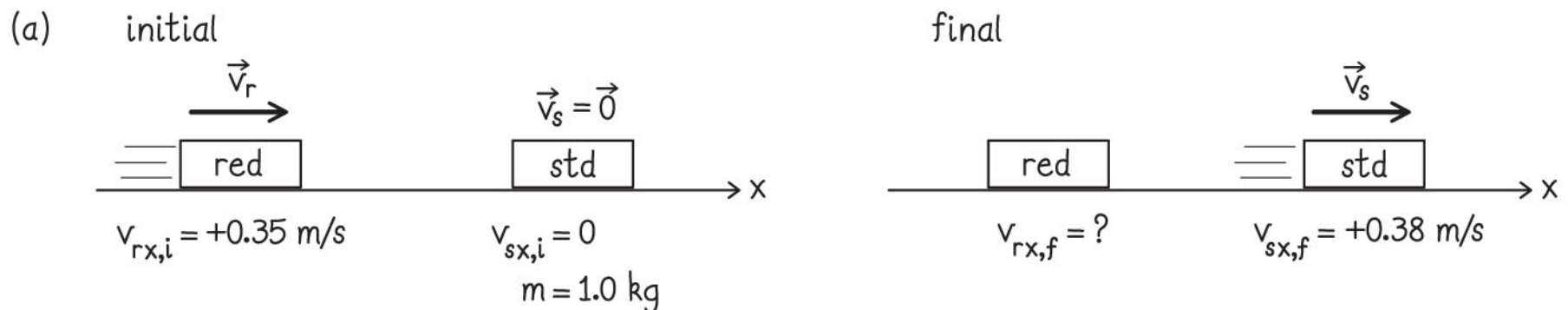
- This equation can be rewritten in vectorial form as

$$\Delta \vec{p}_u + \Delta \vec{p}_s = \vec{0}$$

## Section 4.6: Momentum

### Example 4.6 Collisions and momentum changes

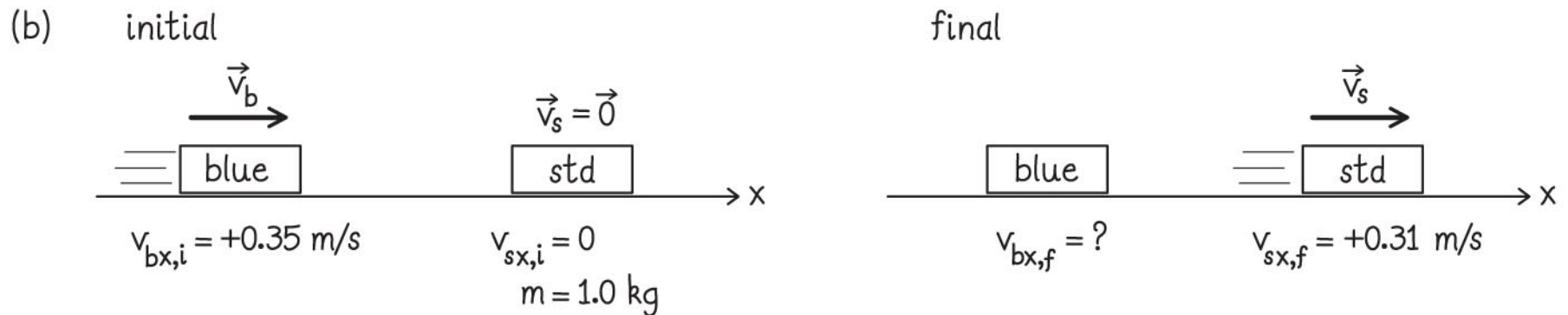
(a) A red cart with an initial speed of 0.35 m/s collides with a stationary standard cart ( $m_s = 1.0$  kg). After the collision, the standard cart moves away at a speed of 0.38 m/s. What is the momentum change for each cart?



# Section 4.6: Momentum

## Example 4.6 Collisions and momentum changes (cont.)

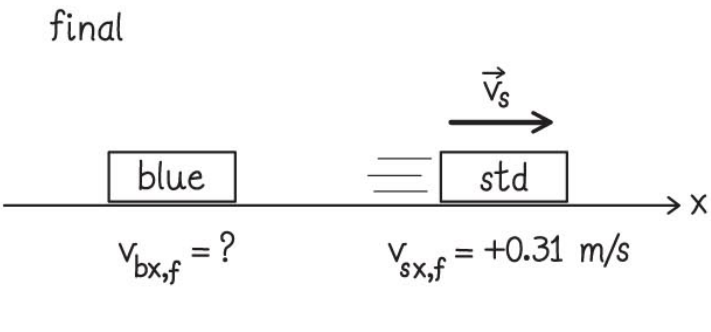
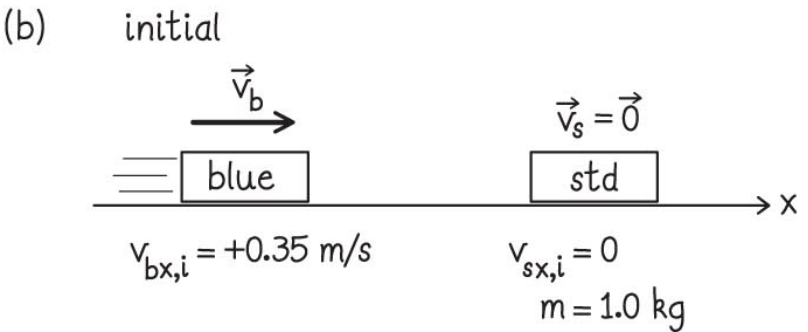
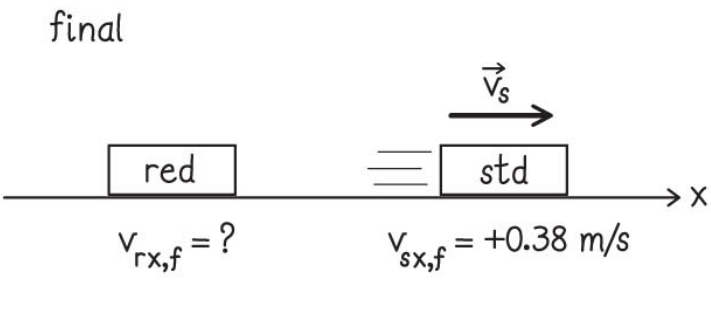
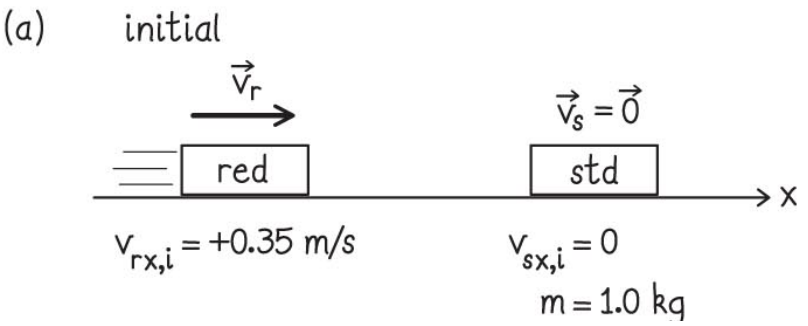
(b) The experiment is repeated with a blue cart, and now the final speed of the standard cart is 0.31 m/s. What is the momentum change for each cart in this second collision?



# Section 4.6: Momentum

## Example 4.6 Collisions and momentum changes (cont.)

(c) If in the collisions  $v_{rx,f} = +0.032 \text{ m/s}$  and  $v_{bx,f} = -0.039 \text{ m/s}$ , what are the inertias of the red and the blue carts?







## Section 4.6: Momentum

### Example 4.6 Collisions and momentum changes (cont.)

② **DEVISE PLAN** In each collision, I know the inertia and the  $x$  components of the initial and final velocities of the standard cart, so I can readily determine the change in the  $x$  component of its momentum.

To determine the change in momentum for the other carts, I know the changes in the  $x$  components of the momenta of the two carts must add up to zero.

## Section 4.6: Momentum

### Example 4.6 Collisions and momentum changes (cont.)

② **DEVISE PLAN** For part *c*, I can use the definition of momentum change, together with the values for the red and blue carts' initial and final velocities and their changes in momenta to determine the inertias.

## Section 4.6: Momentum

### Example 4.6 Collisions and momentum changes (cont.)

#### ③ EXECUTE PLAN

(a) Because it initially is at rest, the standard cart's initial momentum is zero. The  $x$  component of its final momentum is  $p_{s,x,f} = (1.0 \text{ kg})(+0.38 \text{ m/s}) = +0.38 \text{ kg}\cdot\text{m/s}$ , and so the  $x$  component of its change in momentum is

$$\Delta p_{s,x} = +0.38 \text{ kg}\cdot\text{m/s} - 0 = +0.38 \text{ kg}\cdot\text{m/s}. \quad \checkmark$$

## Section 4.6: Momentum

### Example 4.6 Collisions and momentum changes (cont.)

#### ③ EXECUTE PLAN

Because the  $x$  components of the momentum changes of the two carts must add up to zero, the corresponding  $x$  component of the red cart's momentum change is

$$\Delta p_{rx} = -0.38 \text{ kg}\cdot\text{m/s.} \quad \checkmark$$

# Section 4.6: Momentum

## Example 4.6 Collisions and momentum changes (cont.)

### ③ EXECUTE PLAN

(b) Applying the same procedure for this second collision

$$\Delta p_{s_x} = (1.0 \text{ kg})(+0.31 \text{ m/s}) - 0 = +0.31 \text{ kg}\cdot\text{m/s};$$

$$\Delta p_{b_x} = -0.31 \text{ kg}\cdot\text{m/s}. \checkmark$$

## Section 4.6: Momentum

### Example 4.6 Collisions and momentum changes (cont.)

**3** EXECUTE PLAN (c) The change in momentum for the red cart is  $\Delta p_{rx} = p_{rx,f} - p_{rx,i}$ . I can write this change in momentum as

$$\Delta p_{rx} = m_r v_{rx,f} - m_r v_{rx,i} = m_r (v_{rx,f} - v_{rx,i})$$

and so

$$m_r = \frac{\Delta p_{rx}}{v_{rx,f} - v_{rx,i}} = \frac{-0.38 \text{ kg} \cdot \text{m/s}}{+0.032 \text{ m/s} - 0.35 \text{ m/s}}$$
$$= 1.2 \text{ kg. } \checkmark$$

## Section 4.6: Momentum

### Example 4.6 Collisions and momentum changes (cont.)

③ EXECUTE PLAN Likewise, for the blue cart:

$$m_b = \frac{\Delta p_{b,x}}{v_{bx,f} - v_{bx,i}} = \frac{-0.31 \text{ kg} \cdot \text{m/s}}{-0.039 \text{ m/s} - 0.35 \text{ m/s}}$$
$$= 0.80 \text{ kg. } \checkmark$$



# Section 4.6: Momentum

## Example 4.6 Collisions and momentum changes (cont.)

④ EVALUATE RESULT If either the red or the blue cart had an inertia of 1.0 kg, then the cart would come to rest in the collision. My work shows that the inertia of the red cart is greater than that of a standard cart.

Indeed, after the collision it still moves in the same direction it initially moved, as it should for a cart of greater inertia colliding with one of lesser inertia.

In contrast, the inertia of the blue cart is less than that of the standard cart. After the collision it bounces back and travels in the opposite direction, as I would expect.

# Section 4.6

## Question

Consider these situations:

- (i) a ball moving at speed  $v$  is brought to rest;
- (ii) the same ball is projected from rest so that it moves at speed  $v$ ;
- (iii) the same ball moving at speed  $v$  is brought to rest and then projected backward to its original speed.

In which case(s) does the ball undergo the largest change in momentum?

1. (i)
2. (i) and (ii)
3. (i), (ii), and (iii)
4. (ii)
5. (ii) and (iii)
6. (iii)

# Section 4.6

## Question 5

Consider these situations:

- (i) a ball moving at speed  $v$  is brought to rest;
- (ii) the same ball is projected from rest so that it moves at speed  $v$ ;
- (iii) the same ball moving at speed  $v$  is brought to rest and then projected backward to its original speed.

In which case(s) does the ball undergo the largest change in momentum?

1. (i)
2. (i) and (ii)
3. (i), (ii), and (iii)
4. (ii)
5. (ii) and (iii)
6. (iii)



# Section 4.7: Isolated systems

## Section Goals

You will learn to

- Develop an accounting scheme for the **total momentum** of a system of interacting objects.
- Identify that when a system interacts with the environment, the momentum of the system changes.
- Recognize that for **isolated** systems, that is, ones that have no external influences on the environment, the momentum of the system **does not change**.

## Section 4.7: Isolated systems

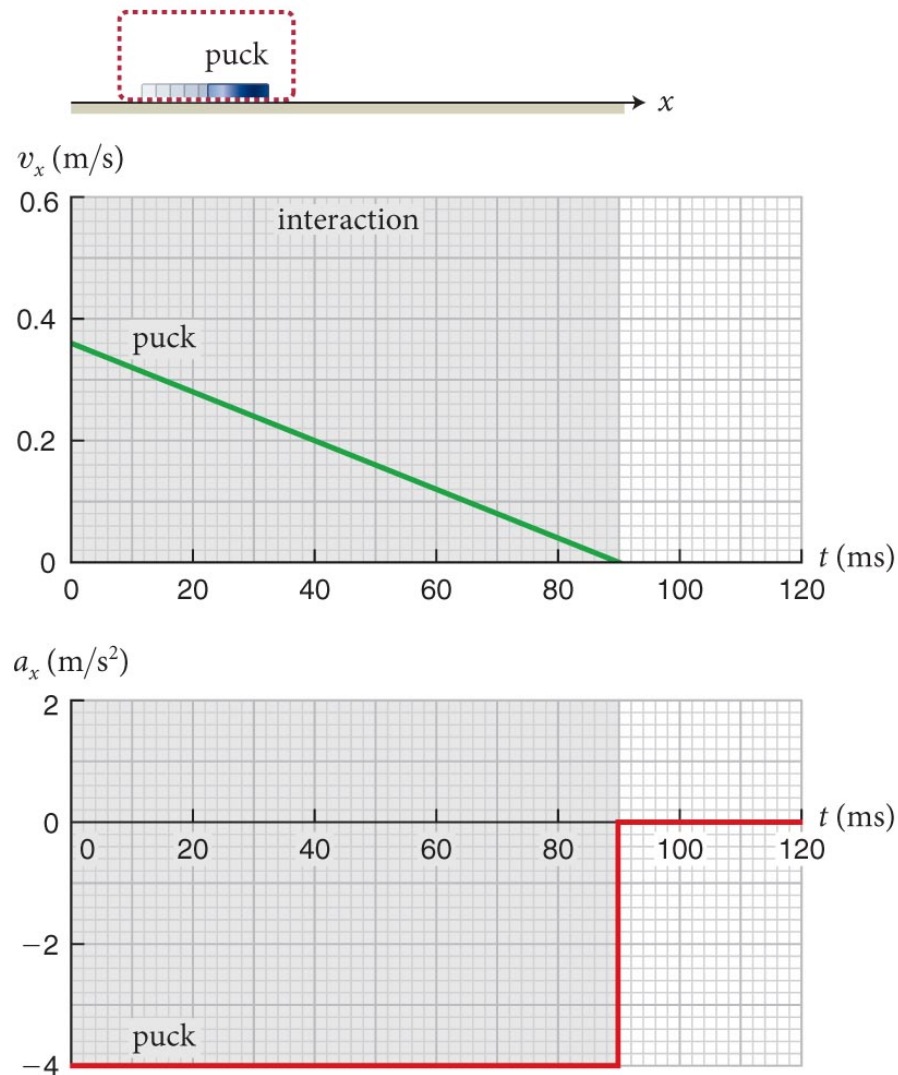
- You can add up the momentum of all objects in a system to obtain the *momentum of the system*. Therefore, the momentum of a system of two moving carts is

$$\vec{p} \equiv \vec{p}_1 + \vec{p}_2$$

- With this definition we can begin to develop an accounting scheme for the momentum of a system.
- Let us begin by examining the four specific situations seen in the figure on the next four slides.

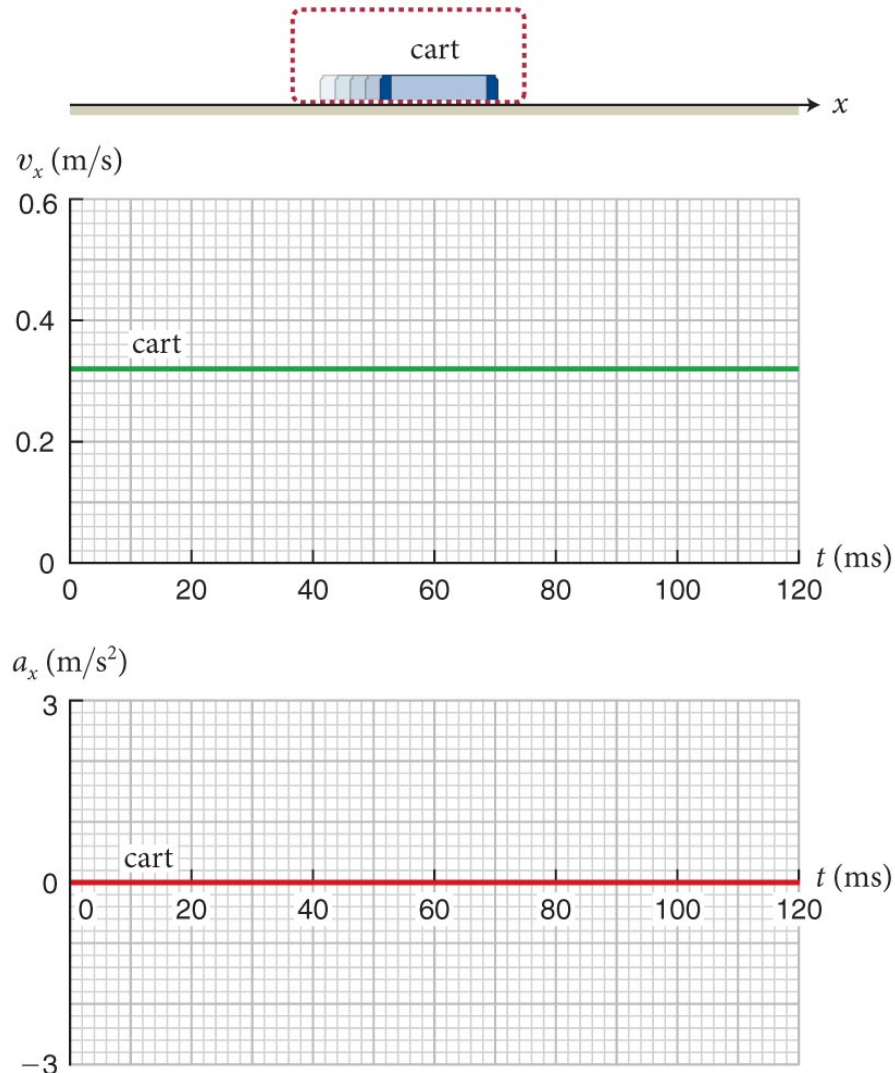
# Section 4.7: Isolated systems

(a) Puck slows to a halt



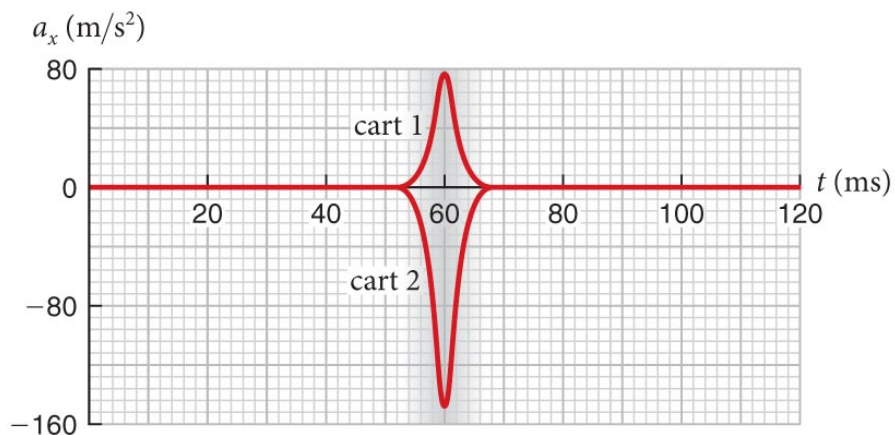
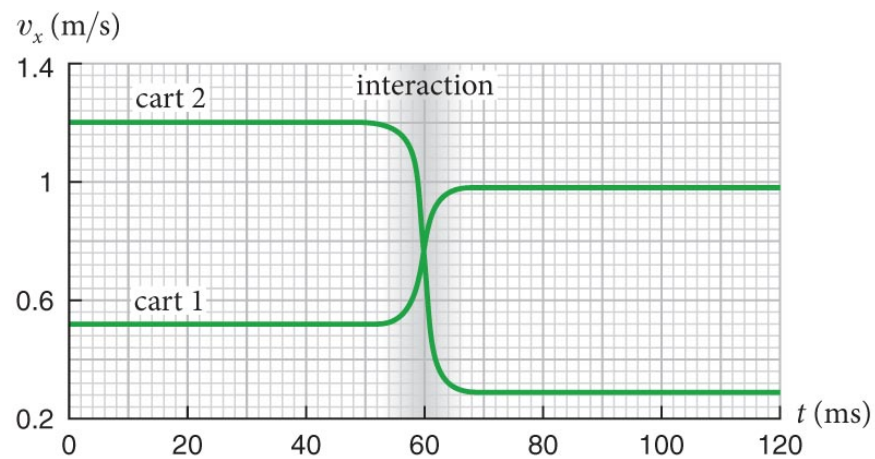
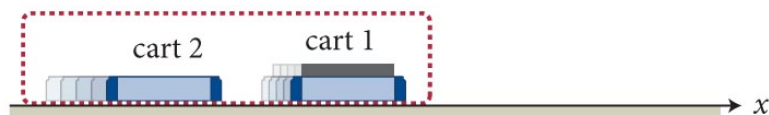
# Section 4.7: Isolated systems

(b) Cart moves at constant velocity



# Section 4.7: Isolated systems

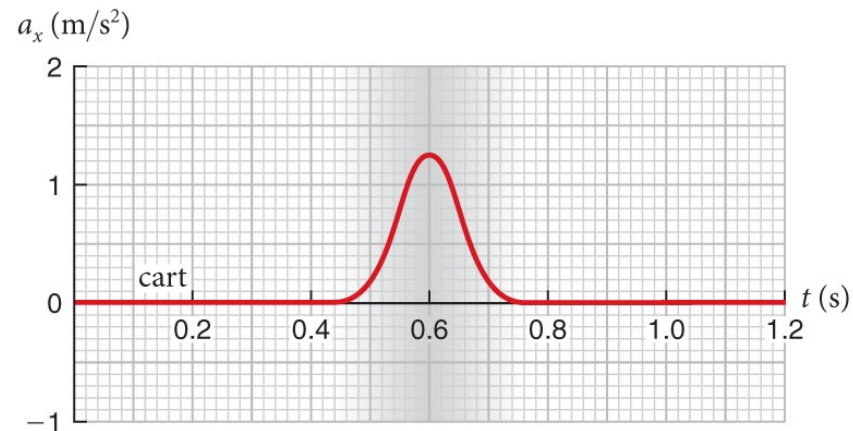
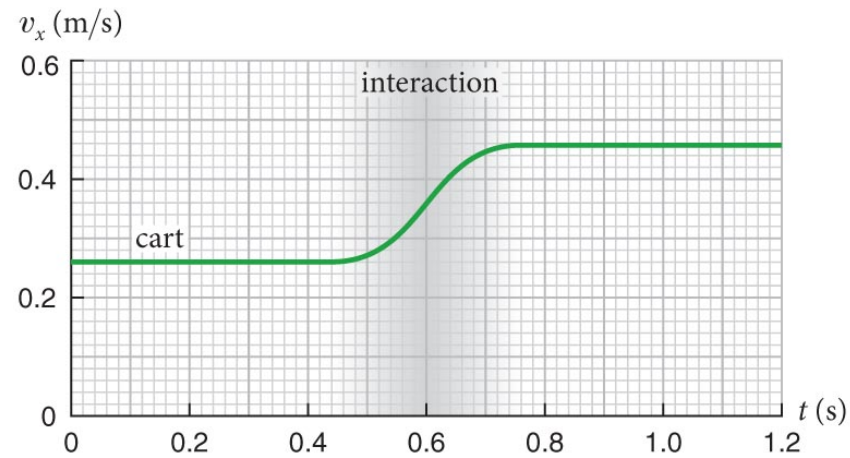
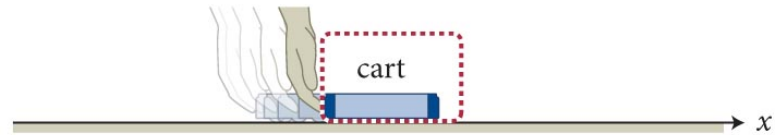
(c) Standard cart collides with cart of unknown inertia





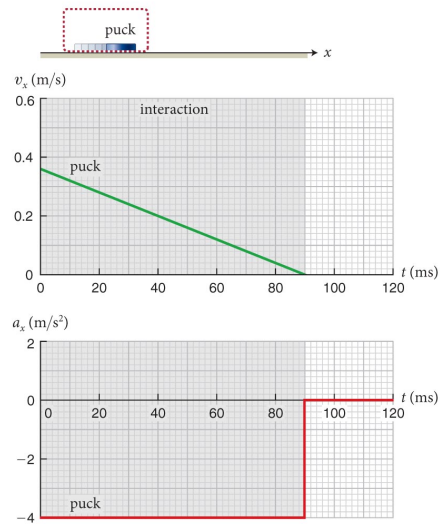
# Section 4.7: Isolated systems

(d) Cart moving at constant velocity is given a shove

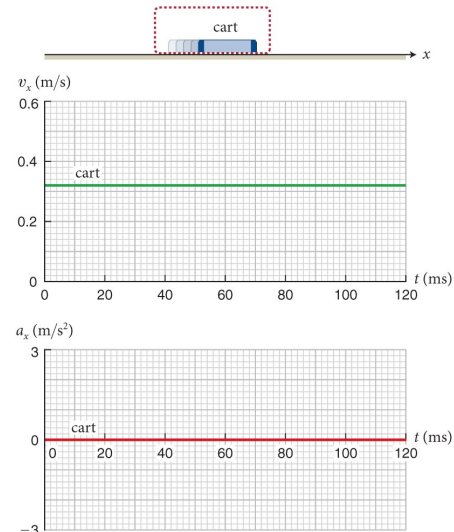


# Which are isolated? Interact with environment?

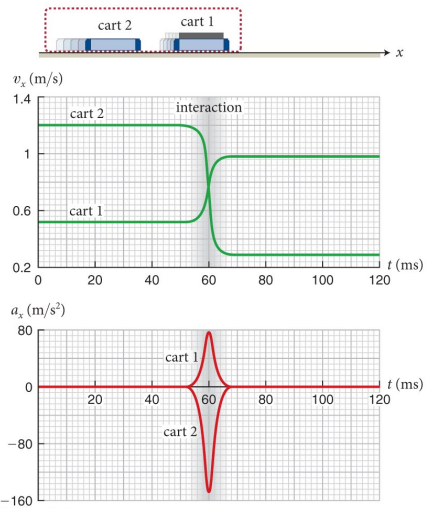
(a) Puck slows to a halt



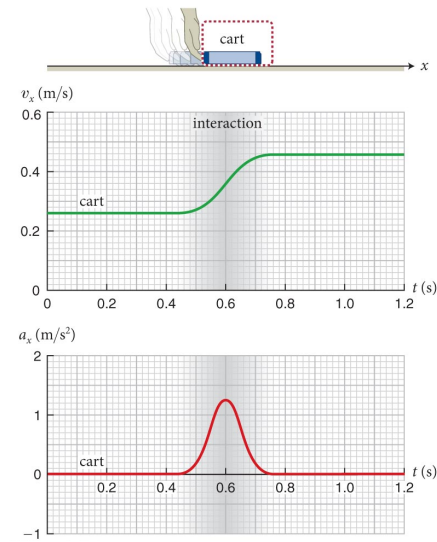
(b) Cart moves at constant velocity



(c) Standard cart collides with cart of unknown inertia



(d) Cart moving at constant velocity is given a shove



# Section 4.7: Isolated systems

**Table 4.2** Interactions and momentum changes in Checkpoint 4.11

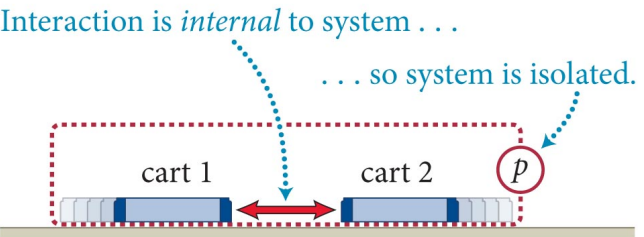
Situation	Interacting objects	System	System interacting?	$\Delta\vec{p}$
<i>a</i>	floor $\leftrightarrow$ puck	puck	yes	nonzero
<i>b</i>	none	cart	no	$\vec{0}$
<i>c</i>	cart 1 $\leftrightarrow$ cart 2	cart 1	yes	nonzero
<i>d</i>	cart 1 $\leftrightarrow$ cart 2	carts 1 & 2	no	$\vec{0}$
<i>e</i>	hand $\leftrightarrow$ cart	cart	yes	nonzero

- Note that whenever the system interacts with the environment, the momentum changes.
- A system for which there are no external interactions is said to be **isolated**:
  - For such systems,  $\Delta\vec{p} = \vec{0}$ .

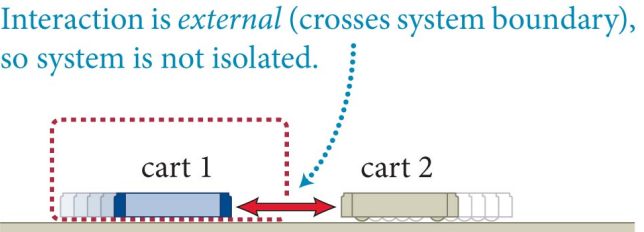
# Section 4.7: Isolated systems

- The figure and table below illustrate two system choices for studying a collision.
- The system containing both cars is isolated and  $\Delta \vec{p} = \vec{0}$ .
- The system containing just one cart is not isolated and  $\Delta \vec{p} \neq \vec{0}$ .

(a) Choice 1: system = both carts



(b) Choice 2: system = cart 1



**Table 4.3** Two choices of system for carts colliding

	Choice 1	Choice 2
System:	carts 1 & 2	cart 1
Environment:	track	cart 2 & track
Interactions:	internal	external
System isolated?	yes	no
Momentum changing?	no	yes

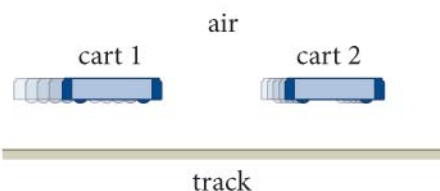
# Section 4.7: Isolated systems

- In the figure, the procedure for choosing an isolated system for a collision between two carts is illustrated.

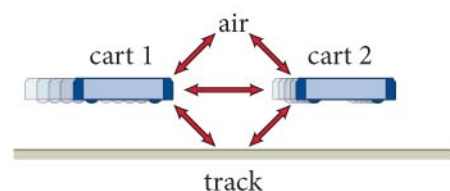
Situation



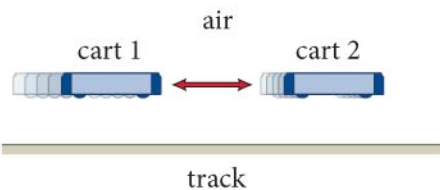
1 Separate objects from each other.



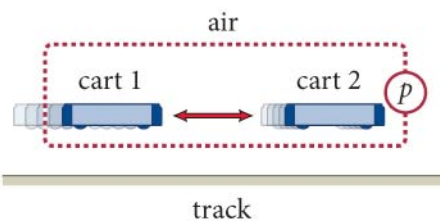
2 Identify all interactions.



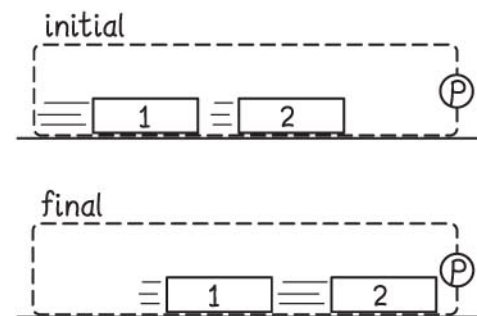
3 Eliminate interactions that do not accelerate interacting objects.



4 Choose isolated system.



5 Draw system diagrams.



# Section 4.7: Isolated systems

## Procedure: Choosing an isolated system

When you analyze momentum changes in a problem, it is convenient to choose a system for which no momentum is transferred into or out of the system (an isolated system). To do so, follow these steps:

# Section 4.7: Isolated systems

## Procedure: Choosing an isolated system (cont.)

1. Separate all objects named in the problem from one another.
2. Identify all possible interactions among these objects and between these objects and their environment (the air, Earth, etc.).

# Section 4.7: Isolated systems

## Procedure: Choosing an isolated system (cont.)

3. Consider each interaction individually and determine whether it causes the interacting objects to accelerate. Eliminate any interaction that does not affect (or has only a negligible effect on) the objects' accelerations during the time interval of interest.



# Section 4.7: Isolated systems

## Procedure: Choosing an isolated system (cont.)

4. Choose a system that includes the object or objects that are the subject of the problem (for example, a cart whose momentum you are interested in) in such a way that none of the remaining interactions cross the system boundary. Draw a dashed line around the objects in your choice of system to represent the system boundary. None of the remaining interactions should cross this line.

# Section 4.7: Isolated systems

## Procedure: Choosing an isolated system (cont.)

5. Make a system diagram showing the initial and final states of the system and its environment.

# Section 4.7

## Question

Which of these systems is isolated?

1. While slipping on a patch of ice, a car collides totally inelastically with another car. *System*: both cars
2. Same situation as in A. *System*: slipping car
3. A single car slips on a patch of ice. *System*: car
4. A car makes an emergency stop on a road. *System*: car
5. A ball drops to Earth. *System*: ball
6. A billiard ball collides elastically with another billiard ball on a pool table. *System*: both balls

# Section 4.7

## Clicker Question 6

Which of these systems is isolated?

1. While slipping on a patch of ice, a car collides totally inelastically with another car. *System*: both cars
2. Same situation as in A. *System*: slipping car
3. A single car slips on a patch of ice. *System*: car
4. A car makes an emergency stop on a road. *System*: car
5. A ball drops to Earth. *System*: ball
- ✓ 6. A billiard ball collides elastically with another billiard ball on a pool table. *System*: both balls

# Problem

An archer stands on a frozen pond. Her inertia is 60kg.  
She fires a 0.5kg arrow at 50 m/s  
What is her velocity afterwards?

# Problem

Can neglect friction on ice  $\rightarrow$  isolated system

That means:

interactions are internal

momentum does not change

total momentum is thus constant

# Problem

- Momentum is conserved
- Initially: zero
- After?

still zero! If arrow moves in  $+x$ , she moves in  $-x$

- Set up conservation. Arrow has  $v_a$ , girl  $v_g$

$$p_i = p_f = 0$$

$$p_f = m_g v_{g,f} + m_a v_{a,f} = 0$$

$$v_{g,f} = - \left( \frac{m_a}{m_g} v_{a,f} \right)$$

$$v_{g,f} \approx -0.42 \text{ m/s}$$

# Section 4.7: Isolated systems

## Exercise 4.7 Who's pulling whom?

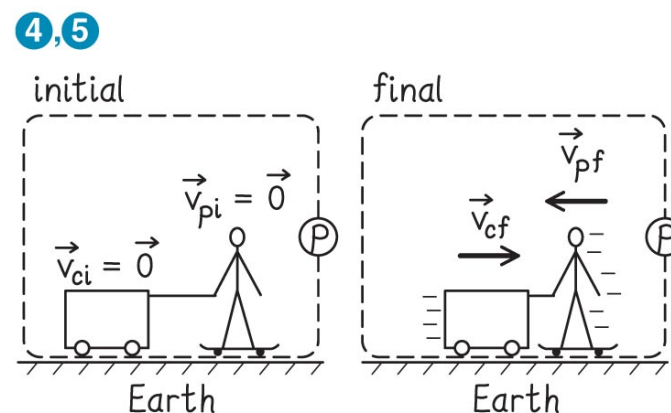
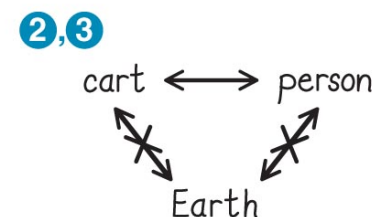
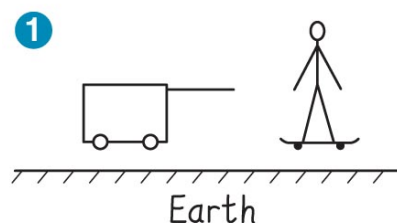
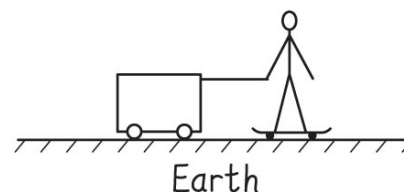
A person standing on a skateboard on horizontal ground pulls on a rope fastened to a cart. Both the person and the cart are initially at rest. Identify an isolated system and make a system diagram.



# Section 4.7: Isolated systems

## Exercise 4.7 Who's pulling whom? (cont.)

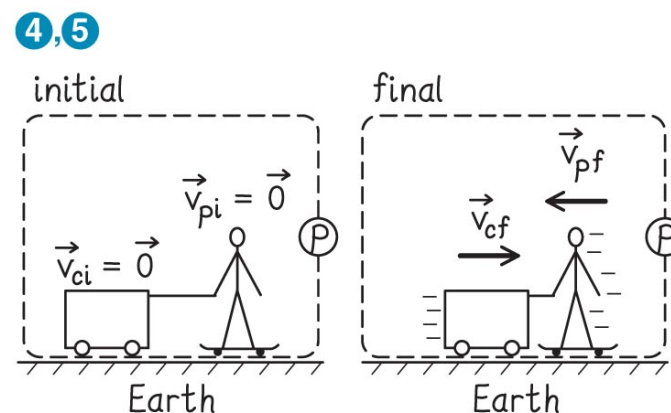
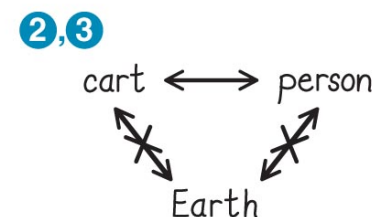
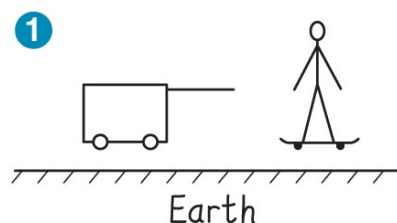
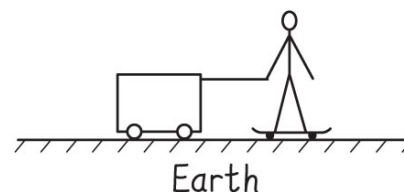
**SOLUTION** Begin by separating the objects in the problem: the person, the cart, and Earth. (I could always go into more detail—include the air, the rope, and the skateboard—but it pays to keep things as simple as possible. For that reason, I consider the skateboard to be part of the person and the rope to be part of the cart.)



# Section 4.7: Isolated systems

## Exercise 4.7 Who's pulling whom? (cont.)

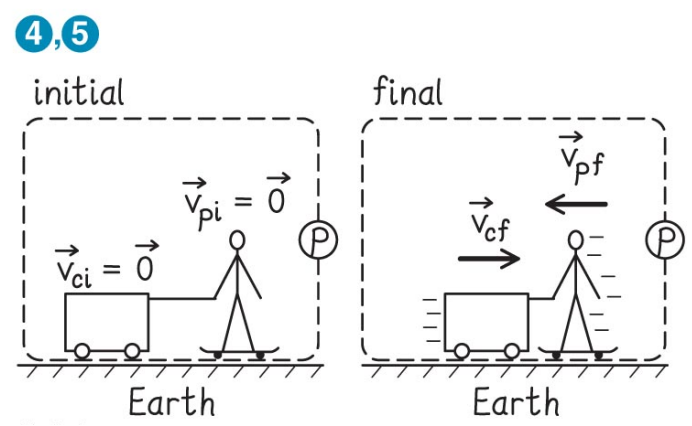
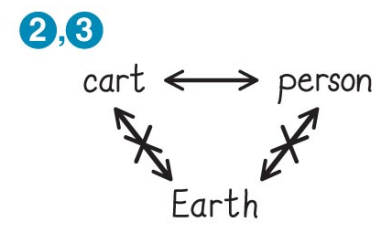
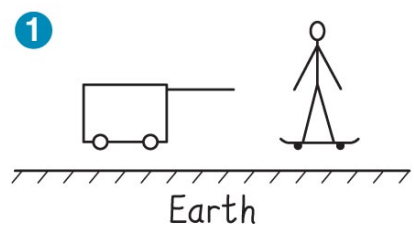
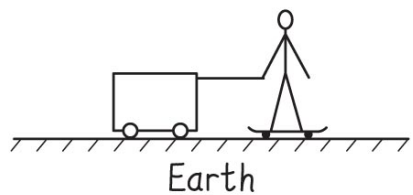
**SOLUTION** The cart interacts with Earth and the person; the person interacts with the cart and Earth. Ignoring friction in the wheels of the cart, I know that the interaction between the cart and Earth has no effect on any motion, and so I can eliminate it from the analysis. The same holds for the interaction between the person (the skateboard) and Earth.



# Section 4.7: Isolated systems

## Exercise 4.7 Who's pulling whom? (cont.)

SOLUTION I then draw a boundary around the person and the cart, making the interaction between the two internal. Because there are no external interactions, this system is isolated. Finally I draw a system diagram showing the initial and final conditions of the system with the person and cart initially at rest and then moving. ✓



# Section 4.8: Conservation of momentum

## Section Goals

You will learn to

- Recognize that for an isolated system there is **no creation or destruction** of momentum inside the system, only transfer between the objects of the system.
- Calculate the kinematics of isolated systems using the conservation of momentum law.

## Section 4.8: Conservation of momentum

- We saw that

$$\Delta \vec{p} = \vec{0} \text{ (isolated system)}$$

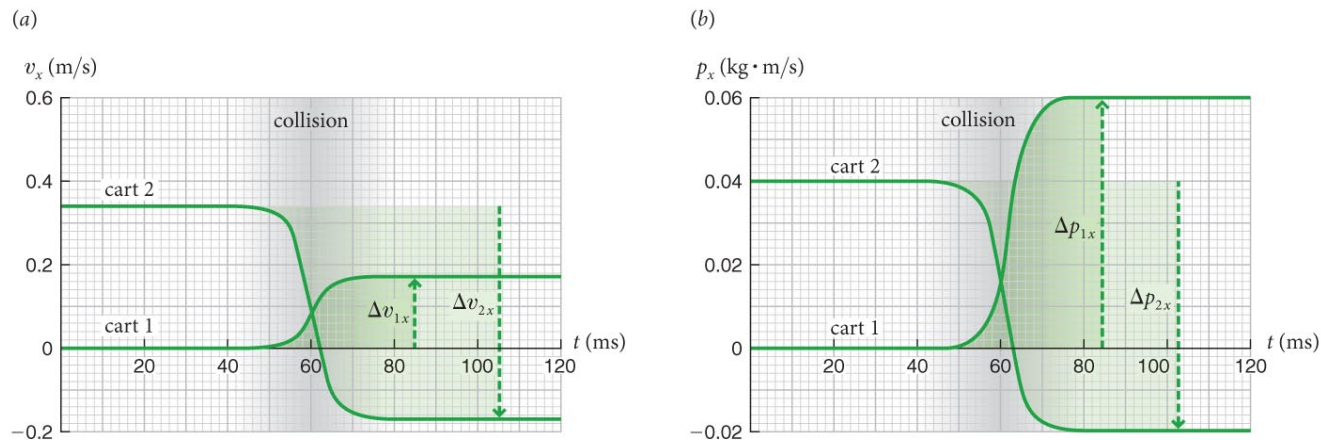
- So, for any two objects colliding should we expect to get

$$\Delta \vec{p}_1 + \Delta \vec{p}_2 = \vec{0}?$$

- The only way to verify this is to do an experiment.

# Section 4.8: Conservation of momentum

- The figure shows the results of a collision experiment.



- Cart 1:  $m_1 = 0.36$  kg,  $v_{1x,i} = 0$  m/s,  $v_{1x,f} = +0.17$  m/s:

$$\Delta p_{1x} = m_1(v_{1x,f} - v_{1x,i}) = 0.0061 \text{ kg} \cdot \text{m/s}$$

- Cart 2:  $m_2 = 0.12$  kg,  $v_{2x,i} = +0.34$  m/s,  $v_{2x,f} = -0.17$  m/s:

$$\Delta p_{2x} = m_2(v_{2x,f} - v_{2x,i}) = -0.0061 \text{ kg} \cdot \text{m/s}$$

- Consequently the momentum of the systems does not change:

$$\Delta \vec{p} \equiv \Delta \vec{p}_1 + \Delta \vec{p}_2 = \vec{0}$$

# Section 4.8: Conservation of momentum

- Repeating the experiment with any other pair of objects yields the same result.
- We must therefore conclude that **momentum can be transferred from one object to another, but it cannot be created or destroyed.**
- This statement is referred to as the **conservation of momentum**, and for isolated systems this means

$$\Delta\vec{p} \equiv \Delta\vec{p}_1 + \Delta\vec{p}_2 = \vec{0}$$

or

$$\vec{p}_f = \vec{p}_i \text{ (isolated system)}$$

- For systems that are not isolated, we have the **momentum law**:

$$\Delta\vec{p} = \vec{J}$$

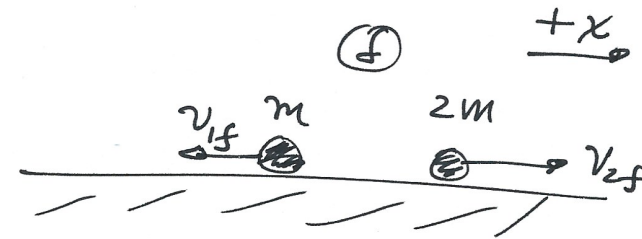
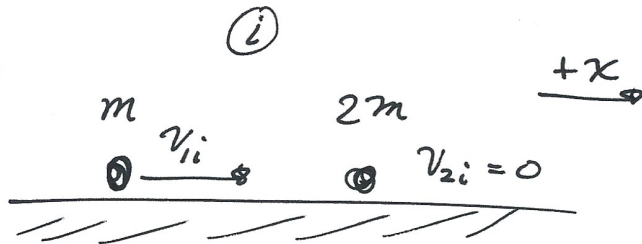
- where  $\vec{J}$  represents the momentum transfer from the environment to the system; this is called the **impulse**.

# Collision!

- Two identical balls collide. One is at rest with inertia  $2m$ , the other with inertia  $m$  is moving toward it with speed  $v$ .
- If the first ball rebounds with half its initial velocity, what is the velocity of the second ball after the collision?



# Collision!



$$P_i = m v_{1i}$$

choice of axes  
↓

$$P_f = -m v_{1f} + 2m v_{2f}$$

$$P_i = P_f$$

$$m v_{1i} = -m v_{1f} + 2m v_{2f}$$

$$v_{1i} = -v_{1f} + 2v_{2f}$$

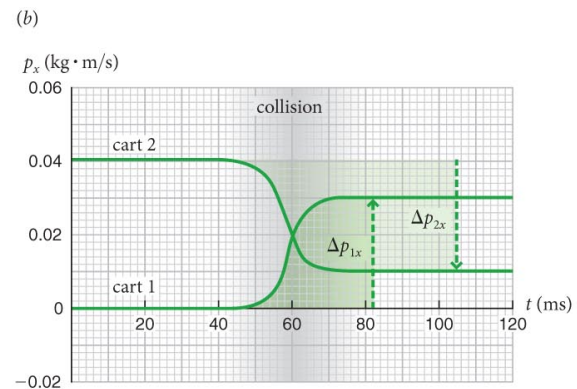
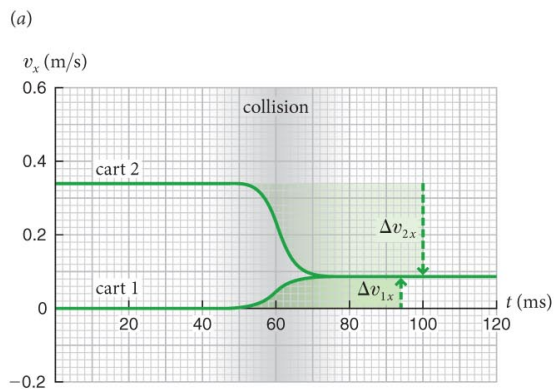
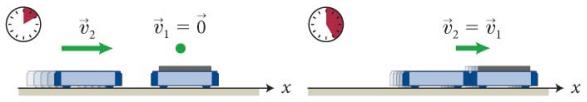
$$v_{2f} = (v_{1i} + v_{1f}) \cdot \frac{1}{2} \quad v_{1i} = v, \quad v_{1f} = -\frac{1}{2}v$$

$$v_{2f} = \frac{3}{4}v \quad \checkmark$$

# Section 4.8: Conservation of momentum

- Let us look at a somewhat different experiment:
  - Let carts 1 and 2 collide with the same initial velocities as before, but this time they **stick together** after the collision.
  - The two carts will have the same final velocity.
  - Very different sort of collision ...
  - The data confirms the momentum of the system is unchanged:

$$\vec{p}_i = \vec{p}_{1,i} + \vec{p}_{2,i} + \vec{p}_{3,i} = \vec{p}_{1,f} + \vec{p}_{2,f} + \vec{p}_{3,f} = \vec{p}_f \text{ (isolated system)}$$



# Section 4.8

## Question

The impulse delivered to a body by a force is

1. defined only for interactions of short duration.
2. equal to the change in momentum of the body.
3. defined only for elastic collisions.

# Section 4.8

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# Section 4.8

## Question


Which is true? Conservation of the total momentum of a system

1. holds only when mechanical energy is conserved.
2. holds for any system.
3. follows from Newton's second law.
4. is equivalent to Newton's third law.

# Section 4.8

## Question

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# Section 4.8

## Question


A rocket is propelled forward by ejecting gas at high speed. The forward motion is a consequence of

1. conservation of energy.
2. conservation of momentum.
3. both of the above.
4. neither of the above.

# Section 4.8

## Question

A rocket is propelled forward by ejecting gas at high speed. The forward motion is a consequence of

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# Section 4.8

## Question

Suppose the entire population of the world gathers in one spot and, at the sounding of a prearranged signal, everyone jumps up. While all the people are in the air, does Earth gain momentum in the opposite direction?

1. No: The inertial mass of Earth is so large that the planet's change in motion is imperceptible.
2. Yes: Because of its much larger inertial mass, however, the change in momentum of Earth is much less than that of all the jumping people.
3. Yes: Earth recoils, like a rifle firing a bullet, with a change in momentum equal to and opposite that of the people.
4. It depends.

# Section 4.8

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- ✓ 3. Yes: Earth recoils, like a rifle firing a bullet, with a change in momentum equal to and opposite that of the people.
4. It depends.

# How much

Mass of average human  $\sim 70\text{kg}$

Number of humans  $\sim 7 \times 10^9$

Total mass of humans  $\sim 5 \times 10^{11}$

Total mass of earth  $\sim 6 \times 10^{24}$

The ratio of velocity changes will be about  $10^{-13}$

Earth's orbital velocity  $\sim 3 \times 10^4$  m/s

Our velocity  $\sim 5$  m/s

*change of earth's speed is  $\sim 1$  part in  $10^{17}$*

# Section 4.8

## Question

A car accelerates from rest. In doing so the car gains a certain amount of momentum and Earth gains

1. more momentum.
2. the same amount of momentum.
3. less momentum.
4. The answer depends on the interaction between the two.

# Section 4.8

## Question

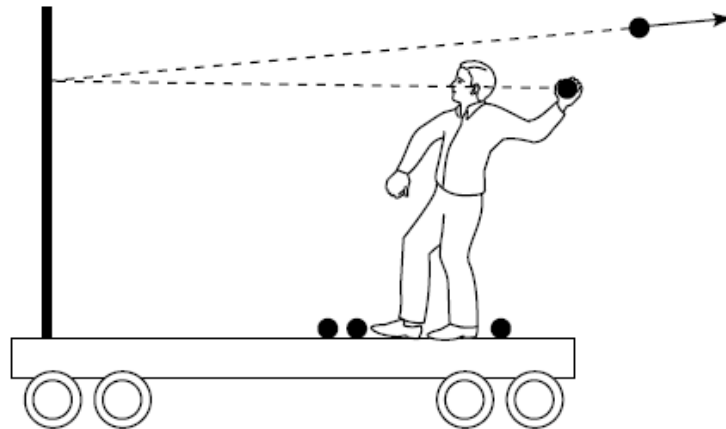
A car accelerates from rest. In doing so the car gains a certain amount of momentum and Earth gains

1. more momentum.
- ✓ 2. the same amount of momentum.
3. less momentum.
4. The answer depends on the interaction between the two.

# Section 4.8

## Question

Suppose you are on a cart that is initially at rest on a track with very little friction. You throw balls at a partition that is rigidly mounted on the cart. If the balls bounce straight back as shown in the figure, is the cart put in motion?

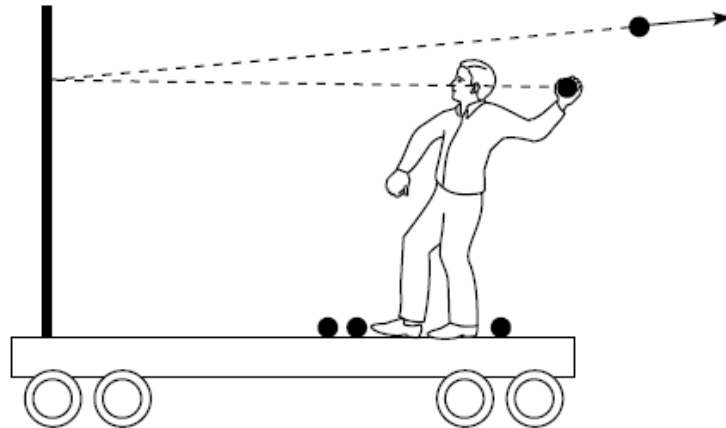


1. Yes, it moves to the right.
2. Yes, it moves to the left.
3. No, it remains in place.

# Section 4.8

## Clicker Question 12

Suppose you are on a cart that is initially at rest on a track with very little friction. You throw balls at a partition that is rigidly mounted on the cart. If the balls bounce straight back as shown in the figure, is the cart put in motion?



1. Yes, it moves to the right.
- ✓ 2. Yes, it moves to the left.
3. No, it remains in place.

# Section 4.8

## Question

Is it possible for a stationary object that is struck by a moving object to have a larger final momentum than the initial momentum of the incoming object?

1. Yes.
2. No, because such an occurrence would violate the law of conservation of momentum.




# Section 4.8

## Question

Is it possible for a stationary object that is struck by a moving object to have a larger final momentum than the initial momentum of the incoming object?

1. Yes.

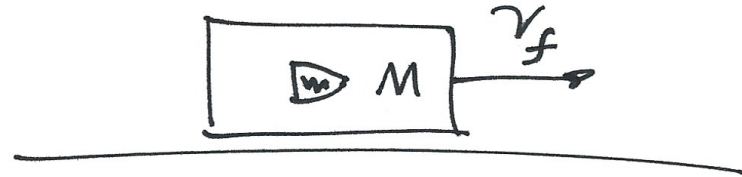
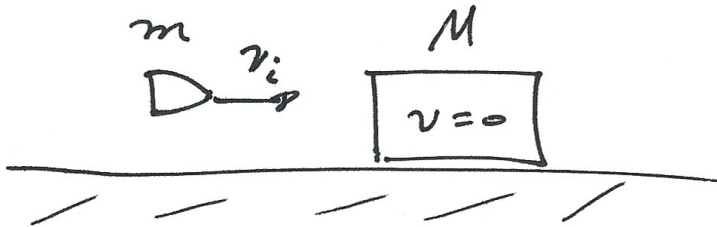
 2. No, because such an occurrence would violate the law of conservation of momentum.

# Problem

A bullet of mass  $m$  is fired at velocity  $v_i$  into a wooden block of mass  $M$  initially at rest. The bullet embeds itself in the block.

What is the velocity of the bullet & block after the collision?

# Problem



$$P_i = mv_i + M \cdot 0 = mv_i$$

$$P_f = mv_f + Mv_f = (m+M)v_f$$

$$mv_i = (m+M)v_f$$

$$v_f = \left( \frac{m}{m+M} \right) v_i \quad \checkmark$$

check:

units

$$m=M, v_f = v_i/2$$

$$M=0, v_f = v_i$$

# Section 4.8: Conservation of momentum

## Example 4.8 Bounce

A 0.20-kg rubber ball is dropped from a height of 1.0 m onto a hard floor and bounces straight up.

Assuming the speed with which the ball rebounds from the floor is the same as the speed it has just before hitting the floor, determine the impulse delivered by the floor to the rubber ball.

# Section 4.8: Conservation of momentum

## Example 4.8 Bounce (cont.)

**1** GETTING STARTED I define the ball to be my system in this problem. The impulse delivered to the ball is then given by the change in its momentum. I need to develop a way to determine this change in momentum.

## Section 4.8: Conservation of momentum

### Example 4.8 Bounce (cont.)

② **DEVISE PLAN** To solve this problem I need to first determine the velocity of the ball just before it hits the floor.

I therefore break the problem into two parts: the downward fall of the ball and its collision with the floor.

I can use 1D motion to determine the time interval it takes the ball to fall from its initial height (assuming the ball is initially at rest).

# Section 4.8: Conservation of momentum

## Example 4.8 Bounce (cont.)

② DEVISE PLAN As it falls, the ball undergoes constant acceleration, so I can use 1D motion to calculate its velocity just before it hits the floor.

Because its speed is not changing as it rebounds, I also know its velocity after it bounces up.

Knowing the velocities, I can calculate the ball's momentum before and after the bounce. The change in momentum then gives the impulse.

## Section 4.8: Conservation of momentum

### Example 4.8 Bounce (cont.)

③ EXECUTE PLAN From Eq. 3.15 I see that it takes an object  $t = \sqrt{2h/g} = \sqrt{2(1.0 \text{ m})/9.81 \text{ m/s}^2} \approx 0.45 \text{ s}$  to fall from a height of 1.0 m.

Choosing the positive  $x$  axis pointing upward, we can find the velocity

$$v_{x,f} = 0 + (-9.8 \text{ m/s}^2)(0.45 \text{ s}) \approx -4.4 \text{ m/s}$$

(note error in textbook)



## Section 4.8: Conservation of momentum

### Example 4.8 Bounce (cont.)

**3** EXECUTE PLAN Now that I know the ball's velocity just before it hits the ground I can obtain the  $x$  component of the momentum of the ball just before it hits the ground by multiplying the ball's velocity by its inertia:

$$p_{x,i} = (0.20 \text{ kg})(-4.4 \text{ m/s}) = -0.88 \text{ kg} \cdot \text{m/s}.$$

(I use the subscript  $i$  to indicate that this is the initial momentum of the ball before the collision.)

## Section 4.8: Conservation of momentum

### Example 4.8 Bounce (cont.)

③ EXECUTE PLAN If the ball rebounds with the same speed, then the  $x$  component of the momentum after the collision with the floor has the same magnitude but opposite sign:  $p_{x,f} = +0.88 \text{ kg} \cdot \text{m/s}$ . The change in the ball's momentum is thus

$$\begin{aligned}\Delta p_x &= p_{x,f} - p_{x,i} = +0.88 \text{ kg} \cdot \text{m/s} - (-0.88 \text{ kg} \cdot \text{m/s}) \\ &= +1.76 \text{ kg} \cdot \text{m/s}.\end{aligned}$$

## Section 4.8: Conservation of momentum

### Example 4.8 Bounce (cont.)

**3** EXECUTE PLAN The interaction with the ground changes the momentum of the ball, making it rebound. The ball does not constitute an isolated system, and the change in its momentum is due to an impulse delivered by Earth to the ball. To determine the impulse, I substitute the change in momentum of the ball into Eq. 4.18:

$$\vec{J} = \Delta\vec{p} = \Delta p_x \hat{i} = (+1.76 \text{ kg} \cdot \text{m/s}) \hat{i} \quad \checkmark$$

# Section 4.8: Conservation of momentum

## Example 4.8 Bounce (cont.)

### ④ EVALUATE RESULT

The  $x$  component of the velocity of the ball just before it hits the floor is negative because the ball moves downward, in the negative  $x$  direction.

After the collision it moves in the opposite direction, and consequently the  $x$  components of the changes in velocity and momentum are both positive.

# Section 4.8: Conservation of momentum

## Example 4.8 Bounce (cont.)

### ④ EVALUATE RESULT

Because the  $x$  component of the change in the ball's momentum is positive, the impulse is directed upward (in the positive  $x$  direction). This makes sense because this impulse changes the direction of travel of the ball from downward to upward. ✓

# Section 4.8: Conservation of momentum

## Worked Problem 4.5 Forensic physics

Your friend from law enforcement claims that, in certain cases, a piece of an object hit by gunfire can move toward the shooter rather than away from the shooter. You decide to investigate whether this is possible by firing a target rifle at some melons. Suppose that in one of your tests an 8.0-g bullet is fired at 400 m/s toward a 1.20-kg melon several meters away, splitting the melon into two pieces of unequal size. The bullet lodges in the smaller piece and propels it forward (that is, in the direction the bullet originally traveled) at 9.2 m/s. If the combined inertia of this piece and the lodged bullet is 0.45 kg, determine the final velocity of the larger piece.

## Section 4.8: Conservation of momentum

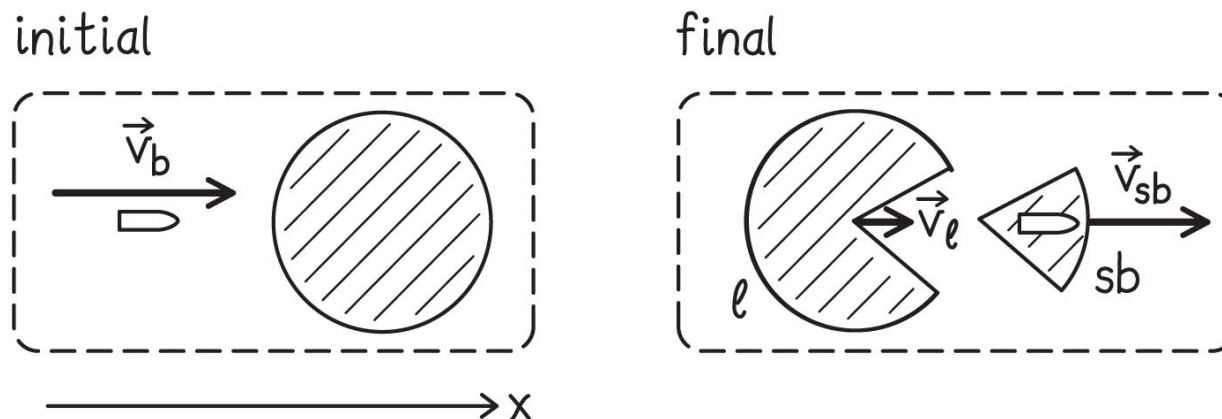
### Worked Problem 4.5 Forensic physics (cont.)

**1** GETTING STARTED As usual, we begin by identifying an isolated system. What about one made up of the bullet and melon? There may be some interaction with the ground or the support upon which the melon sits, but if we choose our time interval to start immediately before the bullet strikes the melon and end immediately after the smaller piece breaks loose, we can ignore this interaction. During this very short time interval, we can treat the system as isolated.

# Section 4.8: Conservation of momentum

## Worked Problem 4.5 Forensic physics (cont.)

**1** GETTING STARTED Next we sketch a system diagram showing the initial and final conditions of the system (Figure WG4.4). We arbitrarily choose the direction in which the smaller piece moves (to the right in the figure) as our positive  $x$  direction. We don't know which way the larger piece moves, and so we draw it as going forward, too. Our calculation will let us know whether this guess is right or wrong. The momentum of the system remains constant during this collision because the system is isolated.





## Section 4.8: Conservation of momentum

### Worked Problem 4.5 Forensic physics (cont.)

② **DEVISE PLAN** We can translate the information given into an equation that requires no change in the momentum of the system. There are two objects in the system in the final condition, but they are different from the ones in the initial condition. We designate the inertia of the bullet as  $m_b$  and the  $x$  component of its velocity as  $v_{b,x}$ , and we note that the intact melon had an initial speed  $v_{m,i} = 0$  and inertia  $m_m$ .

## Section 4.8: Conservation of momentum

### Worked Problem 4.5 Forensic physics (cont.)

② DEVISE PLAN We have

$$\Delta \vec{p} = 0 \Rightarrow \vec{p}_i = \vec{p}_f$$

$$m_b v_{bx,i} + m_m v_{mx,i} = (m_s + m_b) v_{sbx,f} + m_\ell v_{\ell x,f}$$

where the subscript  $s$  denotes the smaller piece, the subscript  $\ell$  denotes the larger piece, and the subscript  $sb$  denotes the combination of the bullet and the smaller piece. We are looking for  $v_{\ell x,f}$ , and because all our other variables have known values, the planning is finished.

## Section 4.8: Conservation of momentum

### Worked Problem 4.5 Forensic physics (cont.)

**3** EXECUTE PLAN We isolate the desired unknown and then insert numerical values. Recall that all of the velocity components in our diagram are in the positive direction.

$$m_\ell v_{\ell x,f} = m_b v_{bx,i} + m_m v_{mx,i} - (m_s + m_b) v_{sbx,f}$$

$$v_{\ell x,f} = \frac{m_b (+|\vec{v}_{b,i}|) - (m_s + m_b)(+|\vec{v}_{sb,f}|)}{m_\ell}$$

$$= \frac{(0.0080 \text{ kg})(400 \text{ m/s}) - (0.45 \text{ kg})(9.2 \text{ m/s})}{1.20 \text{ kg} - (0.45 \text{ kg} - 0.0080 \text{ kg})}$$

$$= -1.240 \text{ m/s} = -1.2 \text{ m/s.} \checkmark$$

## Section 4.8: Conservation of momentum

### Worked Problem 4.5 Forensic physics (cont.)

④ **EVALUATE RESULT** The negative sign means that the larger piece moves in the negative  $x$  direction, which means that our initial guess for the direction of this motion was wrong. Note, however, that the algebra nicely informed us of the incorrect assumption without pain or confusion. The larger piece really does move back toward the rifle. The magnitude of this velocity is fairly small, as we would expect, but it certainly would be noticeable.

# Chapter 4: Summary

## Concepts: Inertia

- *Friction* is the resistance to motion that one surface encounters when moving over another surface. In the absence of friction, objects moving along a horizontal track keep moving without slowing down.
- **Inertia** is a measure of an object's tendency to resist a change in its velocity. Inertia is determined entirely by the type of material of which the object is made and by the amount of that material contained in the object. Inertia is related to *mass*, and for this reason we use the symbol  $m$  to represent it. The SI unit of inertia is the **kilogram** (kg).

# Chapter 4: Summary

## Quantitative Tools: Inertia

- If an object of unknown inertia  $m_u$  collides with an inertial standard of inertia  $m_s$ , the ratio of the inertias is related to the changes in the velocities by

$$\frac{m_u}{m_s} \equiv -\frac{\Delta v_{sx}}{\Delta v_{ux}}$$

# Chapter 4: Summary

## Concepts: Systems and momentum

- A **system** is any object or group of objects that can be separated, in our minds, from the surrounding environment.
- The **environment** is everything that is not part of the system. You can choose the system however you want, but once you decide to include a certain object in the system, that object must remain a part of the system throughout your analysis.

# Chapter 4: Summary

## Concepts: Systems and momentum

- A system for which there are no external interactions is called an **isolated system**.
- An **extensive quantity** is one whose value is proportional to the size or “extent” of the system.
- An **intensive quantity** is one that does not depend on the extent of the system.
- A **system diagram** shows the initial and final conditions of a system.



# Chapter 4: Summary

## Quantitative Tools: Systems and momentum

- The **momentum**  $\vec{p}$  of an object is the product of its inertia and velocity:

$$\vec{p} \equiv m\vec{v}$$

- The momentum of a system of objects is the sum of the momenta of its constituents:

$$\vec{p} \equiv \vec{p}_1 + \vec{p}_2 + \cdots$$

# Chapter 4: Summary

## Concepts: Conservation of momentum

- Any extensive quantity that cannot be created or destroyed is said to be **conserved**, and the amount of any *conserved* quantity in an isolated system is *constant*.
- Momentum is a conserved quantity, and therefore the momentum of an isolated system is constant.
- The momentum can be transferred from one object to another in the system, but the momentum of the system cannot change.

# Chapter 4: Summary

## Quantitative Tools: Conservation of momentum

- The momentum of an isolated system is constant:

$$\Delta\vec{p} = \vec{0}$$

- Another way to say this is that for an isolated system, the initial momentum is equal to the final momentum:

$$\vec{p}_i = \vec{p}_f$$

- The **impulse**  $\vec{J}$  delivered to a system is equal to the change in momentum of the system:

$$\vec{J} = \Delta\vec{p}$$

- For an isolated system,  $\vec{J} = \vec{0}$ .