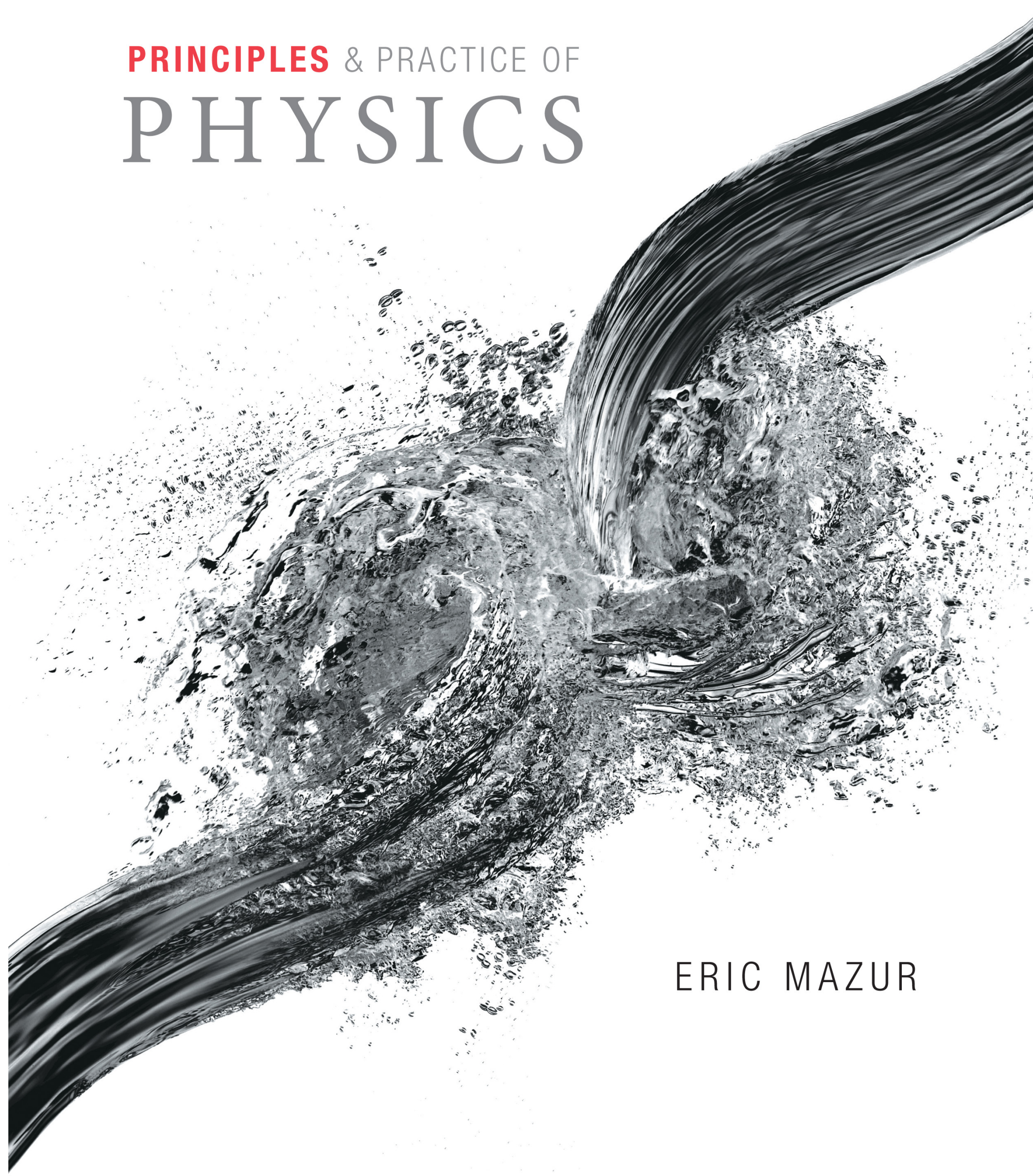
Chapter 8
Force



Concepts

Section 8.1: Momentum and force

Section Goals

You will learn to

- Determine that the **force** exerted on a single object is equal to its time rate of change in its **momentum.**
- Represent the force-momentum relationship mathematically.

Section 8.1: Momentum and force

- To relate the intuitive concept of force to momentum, consider the case of a moving object slamming into (a) a concrete wall and (b) a mattress. We can discover that
  - The force of impact is governed by the speed and inertia of the object.
  - In the case of the mattress, the momentum change occurs at a slow rate, compared to the concrete wall.
  - The longer the impact (interaction) time interval, the smaller the force of impact.

Section 8.1: Momentum and force

- The scenario mentioned in the previous slide illustrates two important aspects of forces:
  - Forces are manifestations of interactions.
  - For an object participating in one interaction only, we can quantitatively define **force:**
    - **The force exerted on the object is the time rate of change in the object's momentum.**

Checkpoint 8.1

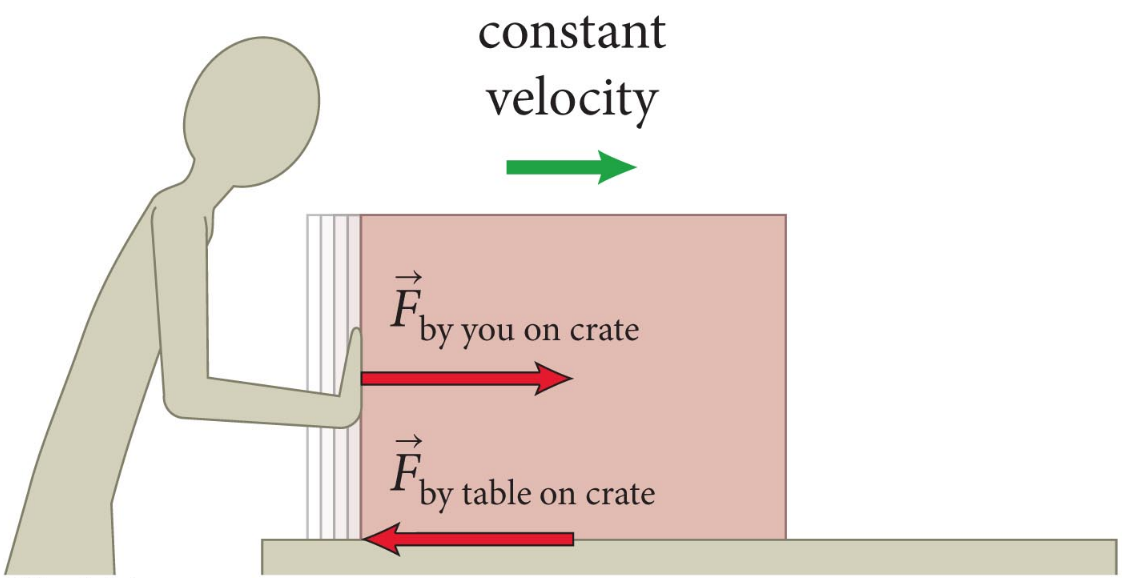
8.1 Imagine pushing a crate in a straight line along a surface at a steady speed of 1 m/s. What is the time rate of change in the momentum of the crate?

if its speed is constant along a straight line, its velocity is constant

this means its momentum is constant, and has **zero** time rate of change

Section 8.1: Momentum and force

- For an object interacting with more than one other object (as in the case of the crate shown in figure), we can experimentally discover that
  - **The vector sum of all forces exerted on an object equals the time rate of change in the momentum of the object.**



Checkpoint 8.2

8.2 Imagine pushing on a crate initially at rest so that it begins to move along a floor.

(a) While you are setting the crate in motion, increasing its speed in the desired direction of travel, what is the direction of the vector sum of the forces exerted on it?

(b) Suppose you suddenly stop pushing and the crate slows to a stop. While the crate slows down, what is the direction of the vector sum of the forces exerted on it?

(c) What is the direction of the vector sum of the forces once the crate comes to rest?

Checkpoint 8.2

(a) While you are setting the crate in motion, increasing its speed in the desired direction of travel, what is the direction of the vector sum of the forces exerted on it?

along the direction of travel (same as acceleration, *a*)

change in momentum is along direction of travel

(b) Suppose you suddenly stop pushing and the crate slows to a stop. While the crate slows down, what is the direction of the vector sum of the forces exerted on it?

opposite direction of travel (*a* has reversed direction)

change in momentum opposite direction of travel

(c) What is the direction of the vector sum of the forces once the crate comes to rest?

while at rest momentum doesn't change – zero net force

Section 8.1
Question 1

What are the magnitude and direction of the vector sum of the forces exerted on a car traveling on a straight downward slope of a highway at a constant 100 km/h?

1. An amount of force proportional to the speed down the slope
2. An amount of force proportional to the speed up the slope
3. Zero
4. Cannot be determined from the given information

Section 8.1
Question 1

What are the magnitude and direction of the vector sum of the forces exerted on a car traveling on a straight downward slope of a highway at a constant 100 km/h?

1. An amount of force proportional to the speed down the slope
2. An amount of force proportional to the speed up the slope
3. ✔ Zero
4. Cannot be determined from the given information

Force is not the same as motion

If there is a net force sum, the object is accelerating

its momentum changes in time

Sitting still or moving at constant velocity?

momentum constant in either case

defines forces summing to zero

a net force is not required for motion

only for *changes* in motion

E.g., car: still has force of gravity (weight), but force from road exactly cancels it.

Section 8.2: The reciprocity of forces

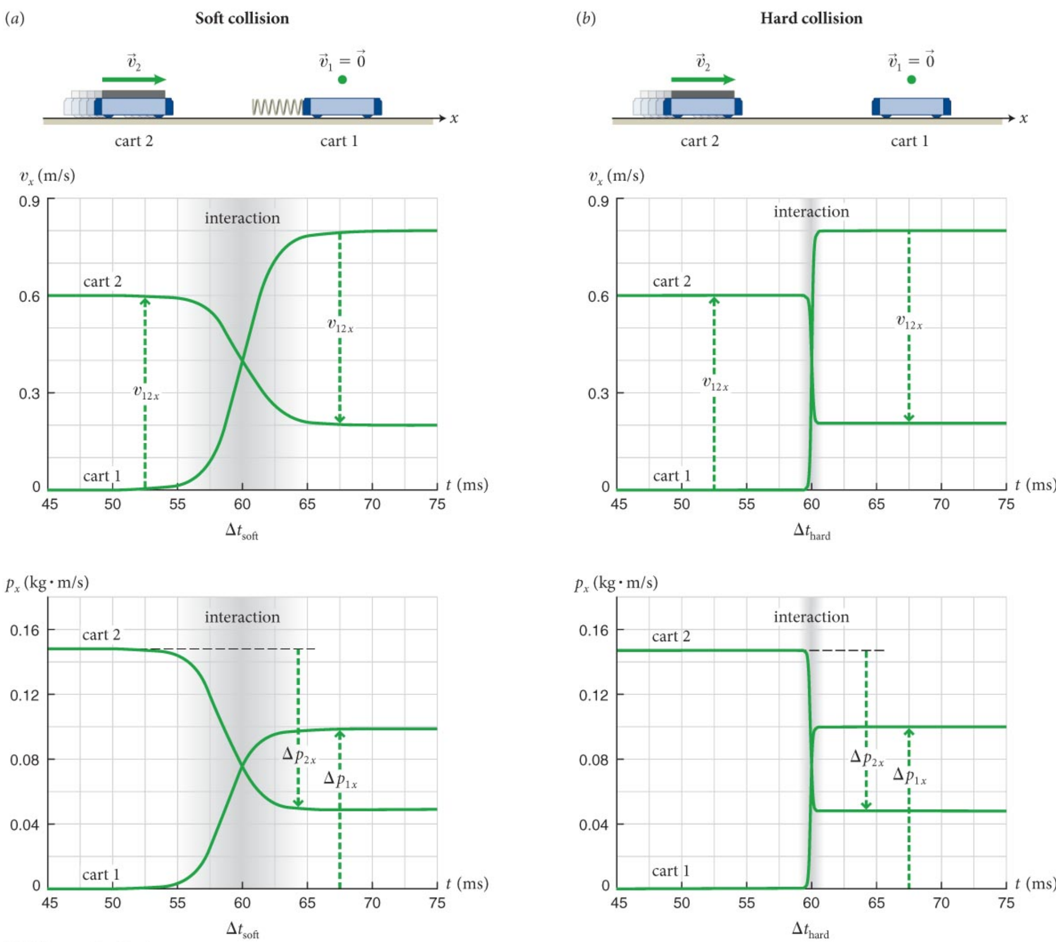
Section Goals

You will learn to

- Recognize that physical **interactions** always involve at least two objects so that **forces** always come in pairs.
- Describe the nature of **interactions pairs**, that is, the forces they exert on each other have **equal magnitudes** and **opposite directions**.

Section 8.2: The reciprocity of forces

- Because of the reciprocal nature of interactions, forces always come in pairs:
  - When two objects interact, each exerts a force on the other.
  - Such a pair of forces is called an **interacting pair**.
- To see this, consider the figure.



Section 8.2: The reciprocity of forces

- For the soft collision shown in the previous figure, the interaction time interval is about 10 ms. We can now compute the two forces exerted on each other:

$$(F_{\text{by 2 on 1}})_x = \Delta p_{1x}/\Delta t_{\text{soft}} = (+0.096\ \text{kg}\cdot\text{m/s})/(0.010\ \text{s}) = +9.6\ \text{kg}\cdot\text{m/s}^2$$

$$(F_{\text{by 1 on 2}})_x = \Delta p_{2x}/\Delta t_{\text{soft}} = (-0.096\ \text{kg}\cdot\text{m/s})/(0.010\ \text{s}) = -9.6\ \text{kg}\cdot\text{m/s}^2$$

- You can see the same relation between the two forces in the hard collision example shown in the previous figure.
- We can conclude that
  - **Whenever two objects interact, they exert on each other forces that are equal in magnitude and opposite in direction.**
  - **It doesn't matter if one is heavier.**

Checkpoint 8.4

8.4 Does the conclusion just stated apply to inelastic collisions?

Yes – the colliding objects can be considered isolated regardless of the type of collision.

This means the change in momentum for the two objects must be equal and opposite, $\Delta p_1 = -\Delta p_2$

Since the time interval is the same for both, they exert equal forces on each other in opposite directions, $F_{1 \text{ on } 2} = -F_{2 \text{ on } 1}$

Example 8.1 Splat!

A mosquito splatters onto the windshield of a moving bus and sticks to the glass.

(*a*) During the collision, is the magnitude of the change in the mosquito's momentum smaller than, larger than, or equal to the magnitude of the change in the bus's momentum?

(*b*) During the collision, is the magnitude of the average force exerted by the bus on the mosquito smaller than, larger than, or equal to the magnitude of the average force exerted by the mosquito on the bus?

Example 8.1 Splat! (cont.)

❶ GETTING STARTED Because the mosquito sticks to the windshield, the bus and mosquito move together after impact and so this event is a totally inelastic collision. I therefore choose the mosquito plus the bus as my system.

Example 8.1 Splat! (cont.)

❷ DEVISE PLAN During the collision, the system is isolated, and so its momentum doesn't change. I can use this fact to compare the magnitudes of the changes in momentum.

The average forces exerted on the mosquito and the bus are each equal to the corresponding momentum change divided by the time interval over which that momentum change takes place. I also know that this time interval is the same for the mosquito and the bus.

Example 8.1 Splat! (cont.)

❸ EXECUTE PLAN (*a*) Because the system's momentum is unchanged, I know that

$\Delta\vec{p}_{\text{mos}} = -\Delta\vec{p}_{\text{bus}}$ and so $\left|\Delta\vec{p}_{\text{mos}}\right| = \left|\Delta\vec{p}_{\text{bus}}\right|$. ✔

Example 8.1 Splat! (cont.)

❸ EXECUTE PLAN (*b*) Because the changes in momentum are equal in magnitude and because the time interval over which the change takes place is the same for both objects, the magnitudes of the average forces must also be equal: $\left|\vec{F}_{\text{by mos on bus, av}}\right| = \left|\vec{F}_{\text{by bus on mos, av}}\right|$. ✔

Example 8.1 Splat! (cont.)

❹ EVALUATE RESULT Although the answer to part *a* conforms to what I've seen before, the answer to part *b* is surprising.

Intuitively I expect the magnitude of the average force exerted by the bus to be much larger than the magnitude of the average force exerted by the mosquito because the mosquito is crushed while nothing happens to the bus.

Example 8.1 Splat! (cont.)

❹ EVALUATE RESULT However, I know that the force magnitude required to crush the mosquito is very small, and such a small force magnitude would have no effect on the bus. It therefore makes sense that the two forces are of equal magnitude.

Section 8.2
Question 2

If you drop a book from a certain height, it falls (accelerating all the while) because of the gravitational force exerted by Earth on it. Because forces always come in interaction pairs, the book must exert a force on Earth. How does the magnitude of $F_{\text{by book on Earth}}$ compare with the magnitude of $F_{\text{by Earth on book}}$?

1. $F_{\text{by book on Earth}} > F_{\text{by Earth on book}}$
2. $F_{\text{by book on Earth}} < F_{\text{by Earth on book}}$
3. $F_{\text{by book on Earth}} = F_{\text{by Earth on book}}$
4. Cannot be determined from the given information

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2. $F_{\text{by book on Earth}} < F_{\text{by Earth on book}}$
3. ✔ $F_{\text{by book on Earth}} = F_{\text{by Earth on book}}$
4. Cannot be determined from the given information

Section 8.3: Identifying forces

Section Goal

You will learn to

- Classify forces as due to **contact** or **"action at a distance."** Action at a distance interactions are also called **force fields.**

Section 8.3: Identifying forces

- To identify forces exerted on an object, it is convenient to distinguish between contact forces and field forces:
  - **Contact forces** - when objects physically touch each other:
    - These forces are due to pushing, pulling, and rubbing.
  - **Field forces** - associated with "action at a distance":
    - Interacting objects do not need to be physically touching.
    - For objects larger than atoms, gravitational and electromagnetic forces are the only ones that are in this category.

Exercise 8.2 What forces?

Identify all the forces exerted on the italicized object in each situation:

(a) A *book* is lying on top of a magazine on a table.

(b) A *ball* moves along a trajectory through the air.

(c) A person is sitting on a *chair* on the floor of a room.

(d) A *magnet* floats above another magnet that is lying on a table.

Exercise 8.2 What forces? (cont.)

SOLUTION

A *book* is lying on top of a magazine on a table.

Because the interaction with the air is usually very small, I'm ignoring this interaction in this problem.

(*a*) **Contact forces:** The book is in contact with the magazine only, and so the magazine exerts a contact force on the book. **Field forces:** Earth exerts a gravitational force on the book. ✔

Exercise 8.2 What forces? (cont.)

SOLUTION

A *ball* moves along a trajectory through the air.

(*b*) **Contact forces**: The ball is not in contact with anything during its flight, and so there are no contact forces. **Field forces**: There is a gravitational interaction between Earth and the ball, so Earth exerts a gravitational force on the ball. ✔

(if we did consider air resistance, that's a contact force)

Exercise 8.2 What forces? (cont.)

SOLUTION

A person is sitting on a *chair* on the floor of a room.

(*c*) **Contact forces**: The chair is in contact with the floor and the person, and so both the floor and the person exert a force on the chair. **Field forces**: Earth exerts a gravitational force on the chair. ✔

Exercise 8.2 What forces? (cont.)

SOLUTION

A *magnet* floats above another magnet that is lying on a table.

(*d*) **Contact forces**: The first magnet is floating, there are no contact forces on it. The second magnet is in contact with the table. **Field forces**: Earth exerts a gravitational force on both magnets. The magnets exert a magnetic force on each other. ✔

in principle, magnets interact with each other gravitationally, but this is negligibly small

Section 8.3
Question 3

Which of the following are characterized as contact force or field forces?

1. The force that causes a book sliding across a polished floor to eventually slow down
2. The force that causes a wine glass to fall downward after it has been knocked off a table
3. The force that causes one magnet to repel another
4. The force exerted by the wind on a sailboat
5. The force that causes an object attached to one end of a stretched rubber band to move toward the object attached to the other end

Section 8.3
Question 3

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Section 8.4: Translational equilibrium

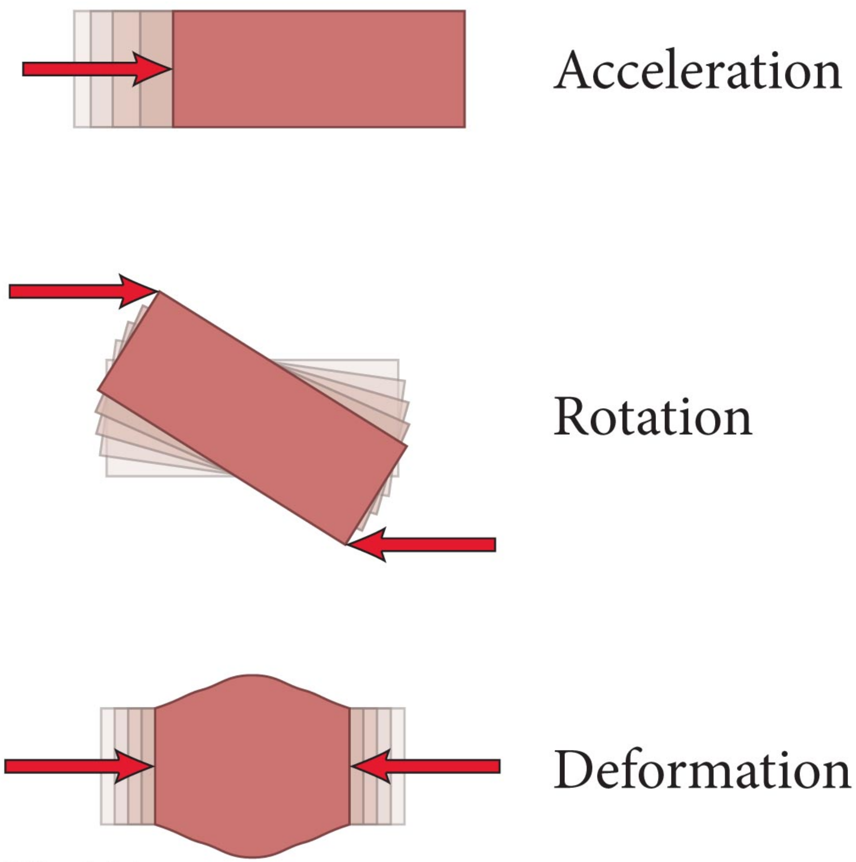
Section Goals

You will learn to

- Define the concept of **equilibrium** from both the motion and force perspectives.
- Establish the physical conditions in which translational equilibrium occur, that is, an object is either at **rest** or **moving with constant velocity.**

Section 8.4: Translational equilibrium

- A system whose motion or state is not changing is said to be in **equilibrium.**
- An object at rest or moving at constant velocity is said to be in **translational equilibrium.**
  - For an object in translational equilibrium, the force vectors add up to zero.
  - However, as seen in the figure, forces can cause an object to rotate or to deform in addition to causing it to accelerate

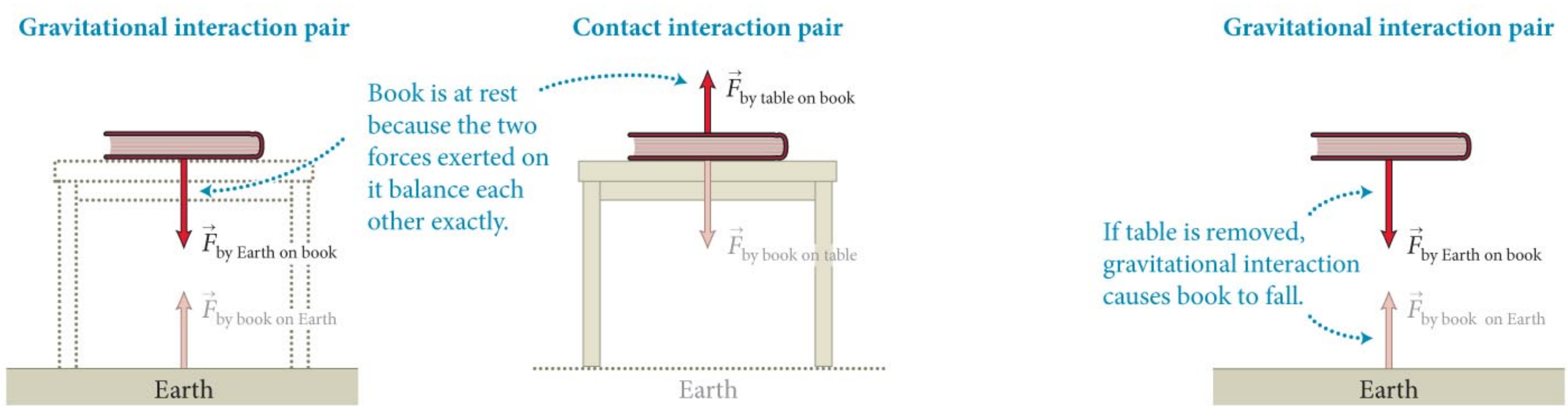


Section 8.4: Translational equilibrium

- For now we will only consider the acceleration caused by forces:
  - **Whenever an object is at rest or moving at constant velocity, the vector sum of the forces exerted on the object is zero, and the object is said to be in equilibrium.**
- An example of an object in translational equilibrium is shown below.

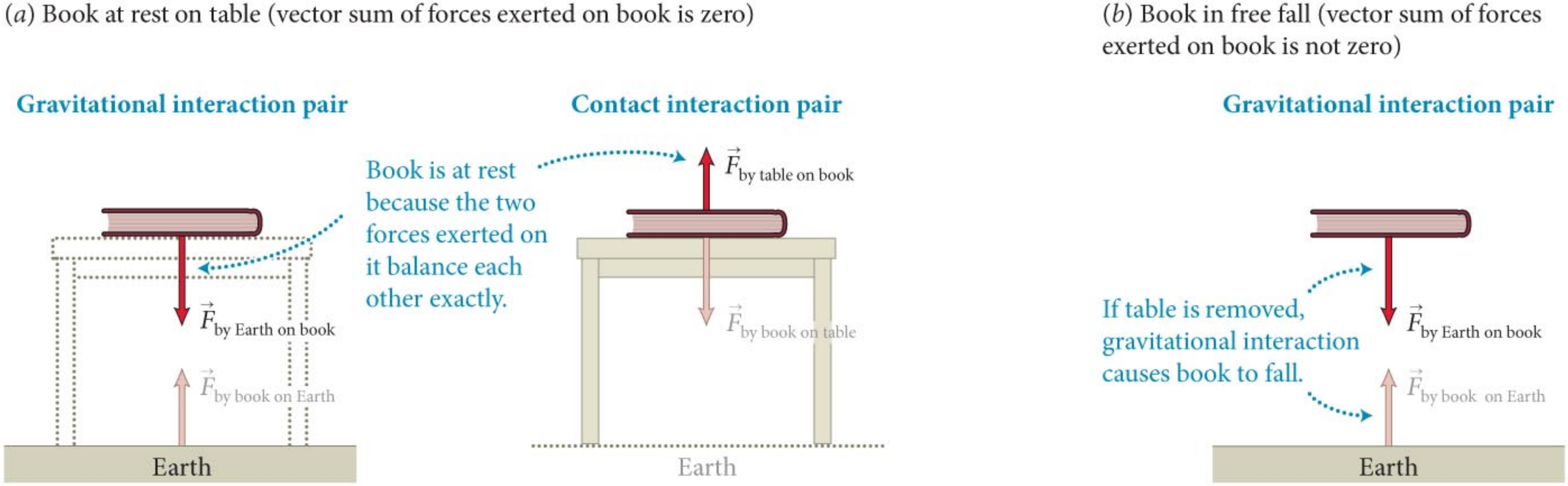
(*a*) Book at rest on table (vector sum of forces exerted on book is zero)

(*b*) Book in free fall (vector sum of forces exerted on book is not zero)



Checkpoint 8.8

8.8 In Figure 8.4, are the contact force exerted by the table on the book and the gravitational force exerted by Earth on the book an interaction pair?



Checkpoint 8.8

8.8 No – while they have the same magnitude and opposite direction, they result from different interactions.

The two forces in an interaction pair result from the same interaction and act on two different objects.

In this case, the two forces result from different interactions and act on the same object.

They are the same size due to the requirement of equilibrium (the book doesn't move)

Section 8.5: Free-body diagrams

Section Goals

You will learn to

- Develop **free-body diagrams** to assist in applying the **equations of motion.**
- Use free-body diagrams to calculate the **net force** on an object and its **time rate of change in momentum.**

Section 8.5: Free-body diagrams

- Knowing about forces gives you a powerful tool for analyzing physical situations:
  - **If the vector sum of the forces exerted on an object is known, the time rate of change in the object's momentum is known, and so the object's subsequent motion can be calculated.**
- To analyze the motion of an object you must first separate the object and forces that act on it from the environment.
- To facilitate this step we use what is called a **free-body diagram.**

Procedure: Free-body diagram

1. Draw a center-of-mass symbol (a circle with a cross) to indicate the object you wish to consider—this object is your system. Pretend the object is by itself in empty space (hence the name *free body*). If you need to consider more than one object in order to solve a problem, draw a separate free-body diagram for each.

Section 8.5: Free-body diagrams

Procedure: Free-body diagram (cont.)

2. List all the items in the object's environment that are in contact with the object. These are the items that exert *contact forces* on the object. *Do not add these items to your drawing*! If you do, you run the risk of confusing the forces exerted *on* the object with those exerted *by* the object.

Procedure: Free-body diagram (cont.)

3. Identify all the forces exerted *on* the object by objects in its environment. (For now, omit from consideration any force not exerted along the object's line of motion.) In general, you should consider (*a*) the *gravitational field force* exerted by Earth on the object and (*b*) the *contact force* exerted by each item listed in step 2.

Procedure: Free-body diagram (cont.)

4. Draw an arrow to represent each force identified in step 3. Point the arrow in the direction in which the force is exerted and place the tail at the center of mass. If possible, draw the lengths of the arrows so that they reflect the relative magnitudes of the forces.

Procedure: Free-body diagram (cont.)

4(cont.). Finally, label each arrow in the form

$$\vec{F}^{\text{type}}_{\text{by on}},$$

where "type" is a single letter identifying the origin of the force (c for contact force, *G* for gravitational force), "by" is a single letter identifying the object exerting the force, and "on" is a single letter identifying the object subjected to that force (this object is the one represented by the center of mass you drew in step 1).

Procedure: Free-body diagram (cont.)

5. Verify that all forces you have drawn are exerted **on** and not **by** the object under consideration. Because the first letter of the subscript on $\vec{F}$ represents the object exerting the force and the second letter represents the object on which the force is exerted, every force label in your free-body diagram should have the same second letter in its subscript.

Procedure: Free-body diagram (cont.)

6. Draw a vector representing the object's acceleration *next to* the center of mass that represents the object. Check that the vector sum of your force vectors points in the direction of the acceleration. If you cannot make your forces add up to give you an acceleration in the correct direction, verify that you drew the correct forces in step 4.

Procedure: Free-body diagram (cont.)

6(cont.). If the object is not accelerating (that is, if it is in translational equilibrium), write $\vec{a} = \vec{0}$ and make sure your force arrows add up to zero. If you do not know the direction of acceleration, choose a tentative direction for the acceleration.

(if you pick the wrong direction, your answer will have a negative sign)

Procedure: Free-body diagram (cont.)

7. Draw a reference axis. If the object is accelerating, it is often convenient to point the positive *x* axis in the direction of the object's acceleration.

Procedure: Free-body diagram (cont.)

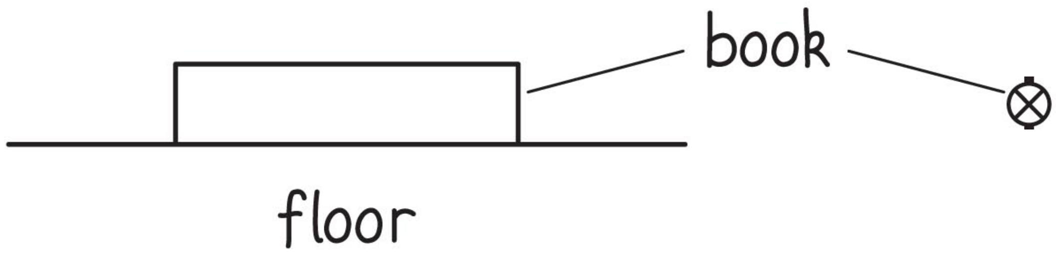
When your diagram is complete it should contain only the center-of-mass symbol, the forces exerted *on* the object (with their tails at the center of mass), an axis, and an indication of the acceleration of the object. Do not add anything else to your diagram.

Exercise 8.3 A book on the floor

Draw a free-body diagram for a book lying motionless on the floor.

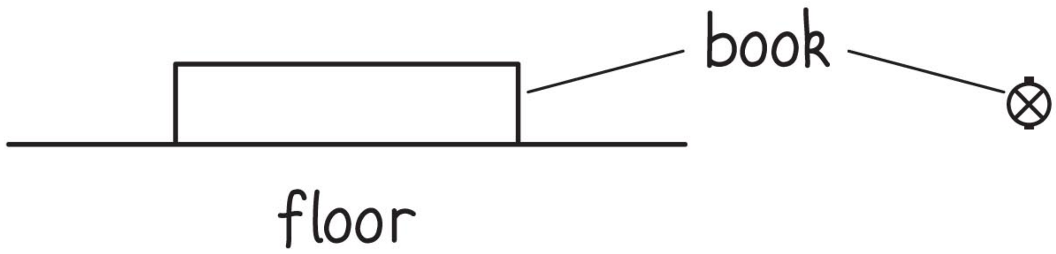
Exercise 8.3 A book on the floor (cont.)

SOLUTION I begin by drawing a small sketch of the situation. Next to it I draw a circle with a cross to represent the center of mass of the book—no line to represent the floor, just the center-of-mass symbol representing the book as if it were by itself in space (Figure 8.5).



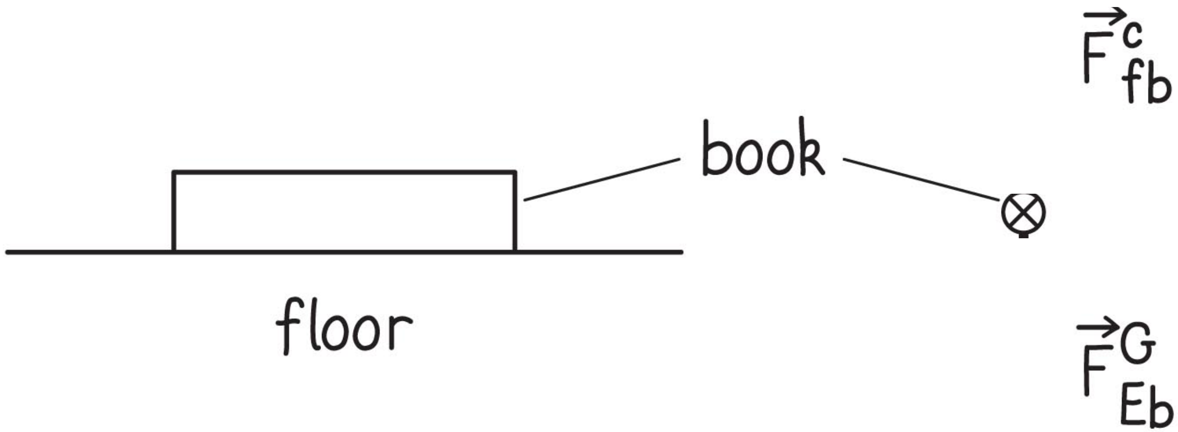
Exercise 8.3 A book on the floor (cont.)

SOLUTION Of the objects in the book's environment, the only one I need to consider is the floor because that is the only thing in contact with the book.



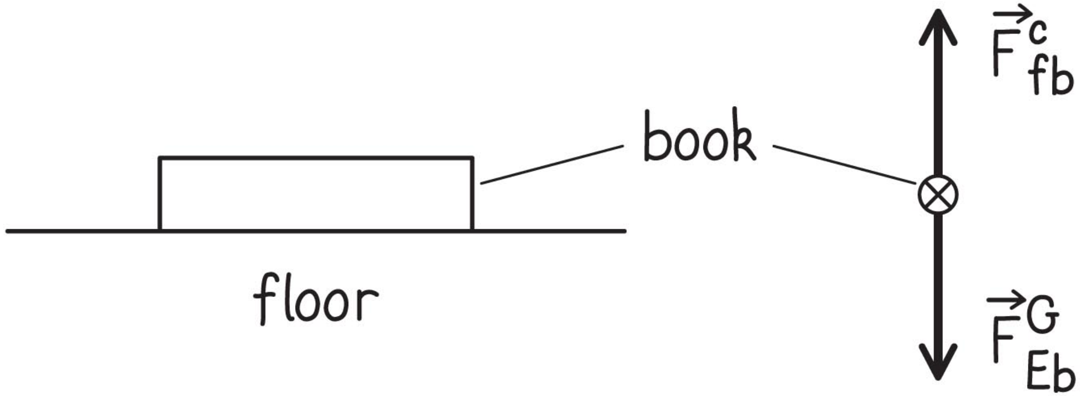
Exercise 8.3 A book on the floor (cont.)

SOLUTION Next I name the forces exerted on the book. First there is a downward gravitational force exerted by Earth on the book. In addition, there is the one contact force exerted by the floor on the book. This force, which prevents the book from falling down through the floor, is directed upward.



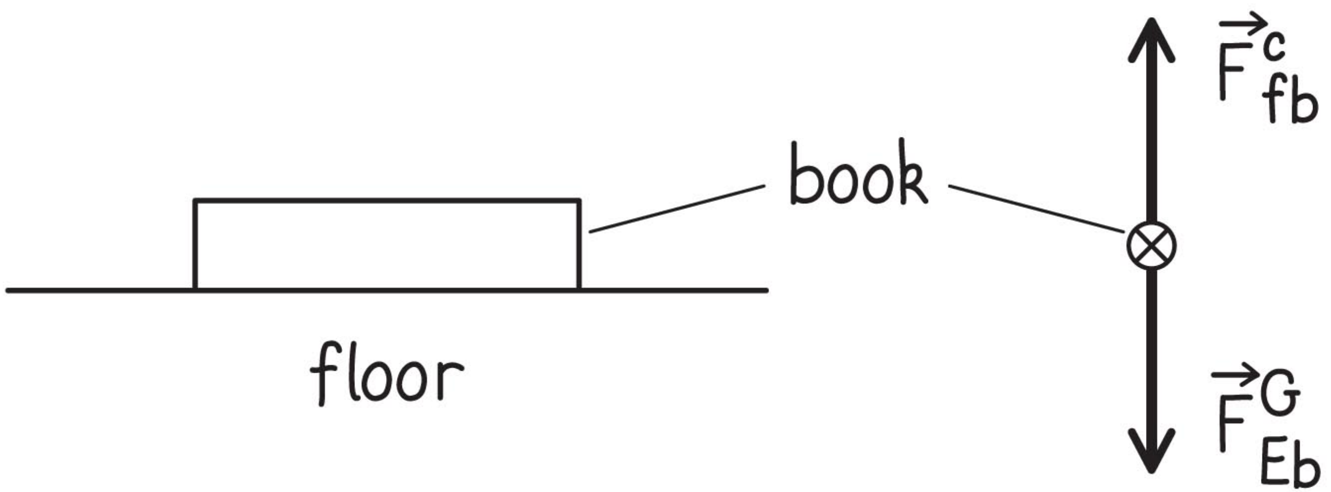
Exercise 8.3 A book on the floor (cont.)

SOLUTION Then I add arrows to the sketch to represent the forces I have identified. To represent the gravitational force, I draw an arrow pointing vertically downward with its tail at the center of mass. I label this force with a superscript *G* (for *gravity*) and the subscript Eb to indicate that it is a force exerted *by* Earth (E) *on* the book (b).



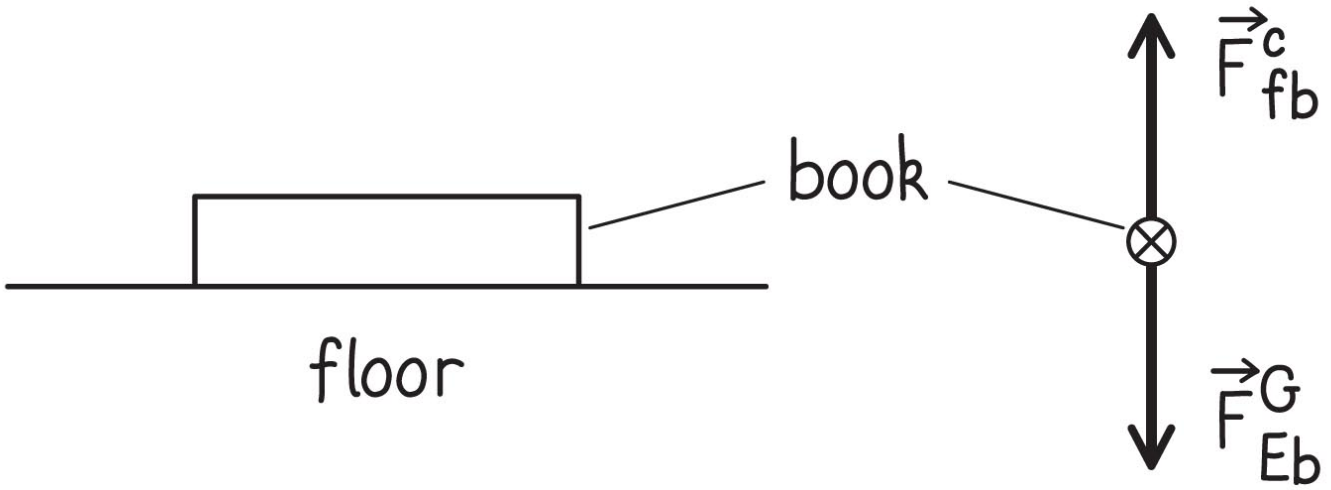
Exercise 8.3 A book on the floor (cont.)

SOLUTION For the contact force, I draw an upward-pointing arrow with its tail at the center of mass. I label this force with a superscript c (for *contact*) and the subscript fb to indicate that it is a force exerted *by* the floor (f) *on* the book (b).



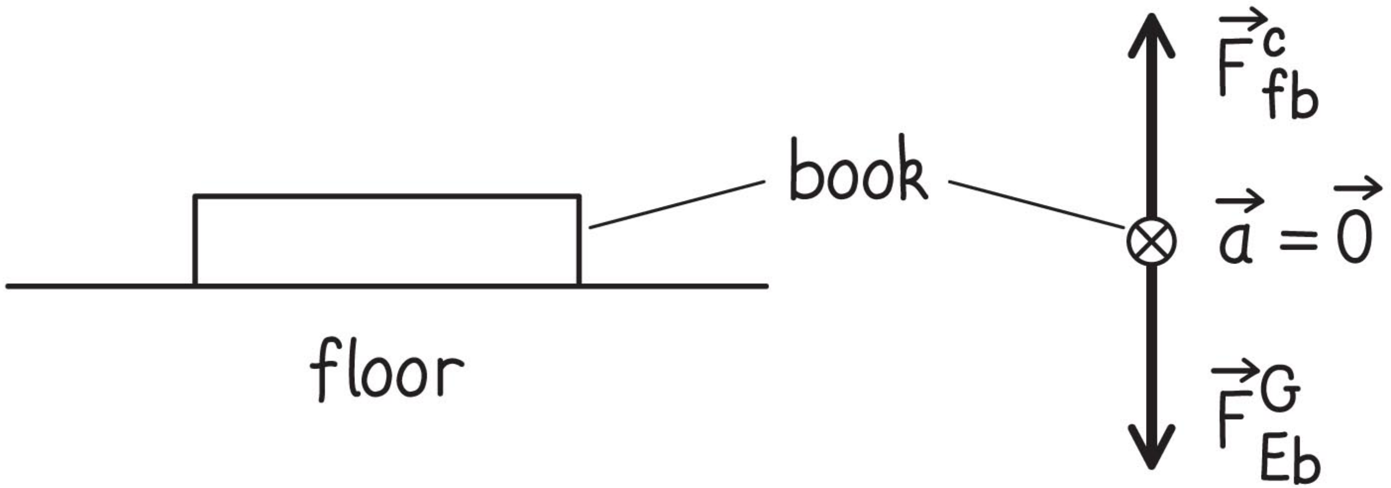
Exercise 8.3 A book on the floor (cont.)

SOLUTION Finally, I verify that all forces in the diagram have subscripts ending in b because my diagram should contain only forces exerted *on* the book.



Exercise 8.3 A book on the floor (cont.)

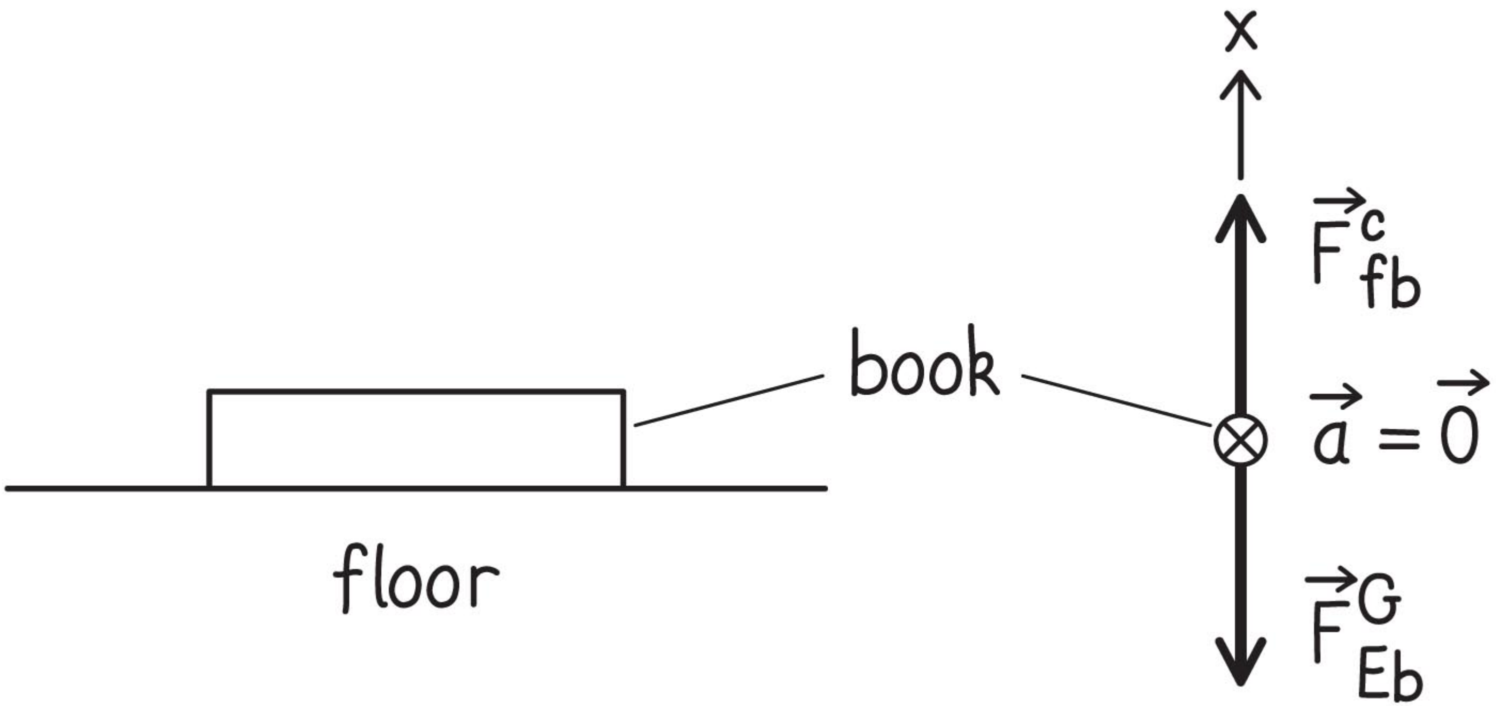
SOLUTION Because the book is at rest, its acceleration is zero, and therefore I write $\vec{a} = \vec{0}$ near the center of mass. From this information, I conclude that the vector sum of the forces exerted on the book (the sum of the two force vectors I have drawn) must be zero.



Section 8.5: Free-body diagrams

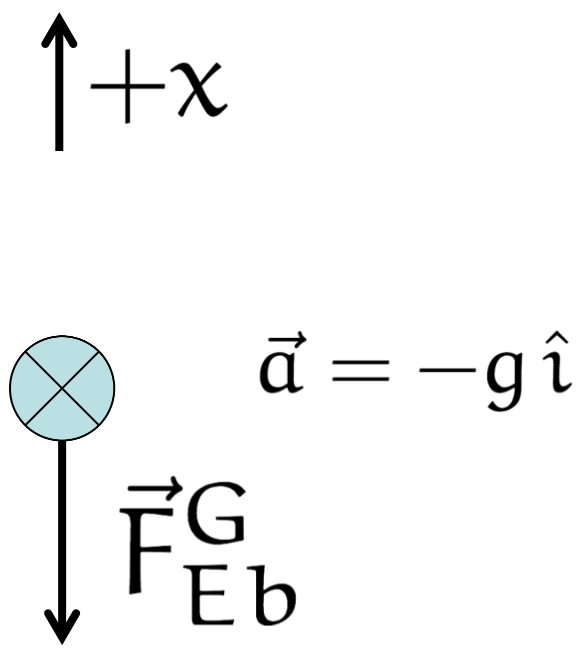
Exercise 8.3 A book on the floor (cont.)

SOLUTION I therefore adjust the lengths of the two arrows in the diagram so that they are of equal length, as shown in Figure 8.5. I complete the diagram by drawing an upward-pointing *x* axis. ✔



Checkpoint 8.9

8.9 If Exercise 8.3 had asked about a book in free fall rather than one on the floor, what would the free-body diagram look like?

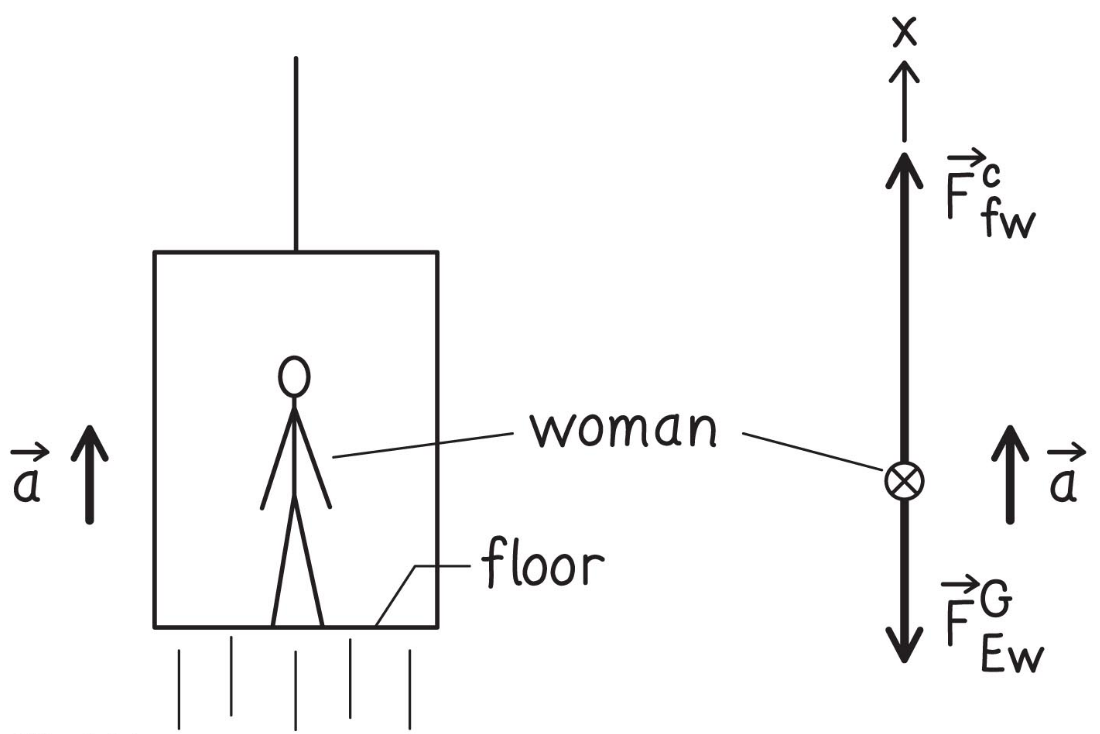


Exercise 8.5 Elevator woman

A woman stands in an elevator that is accelerating upward. Draw a free-body diagram for her.

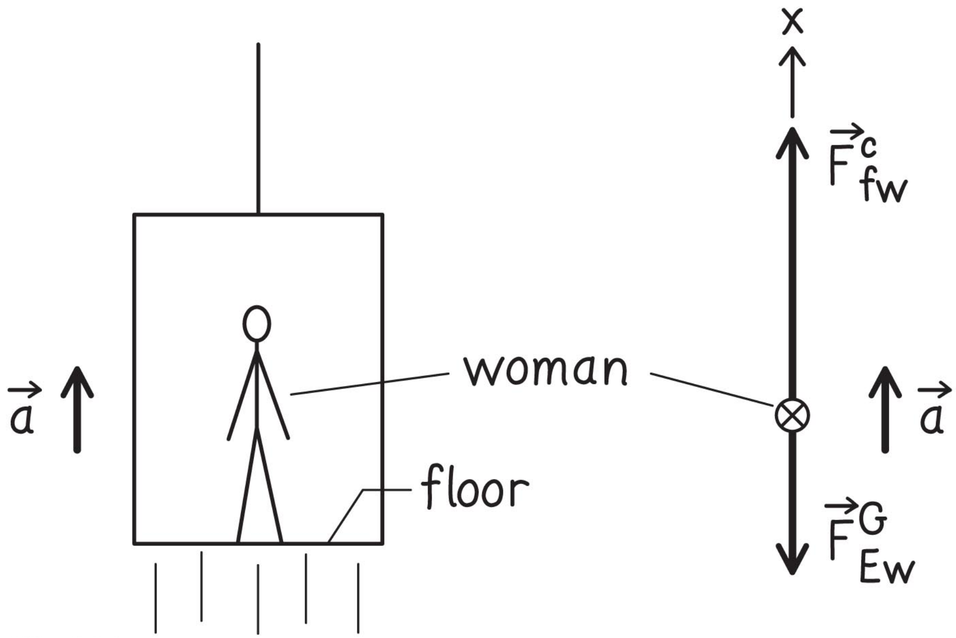
Exercise 8.5 Elevator woman (cont.)

SOLUTION After making a situation sketch, I draw a center-of-mass symbol to represent the woman. The only thing she is in contact with is the elevator floor—it is the floor that pushes her upward.



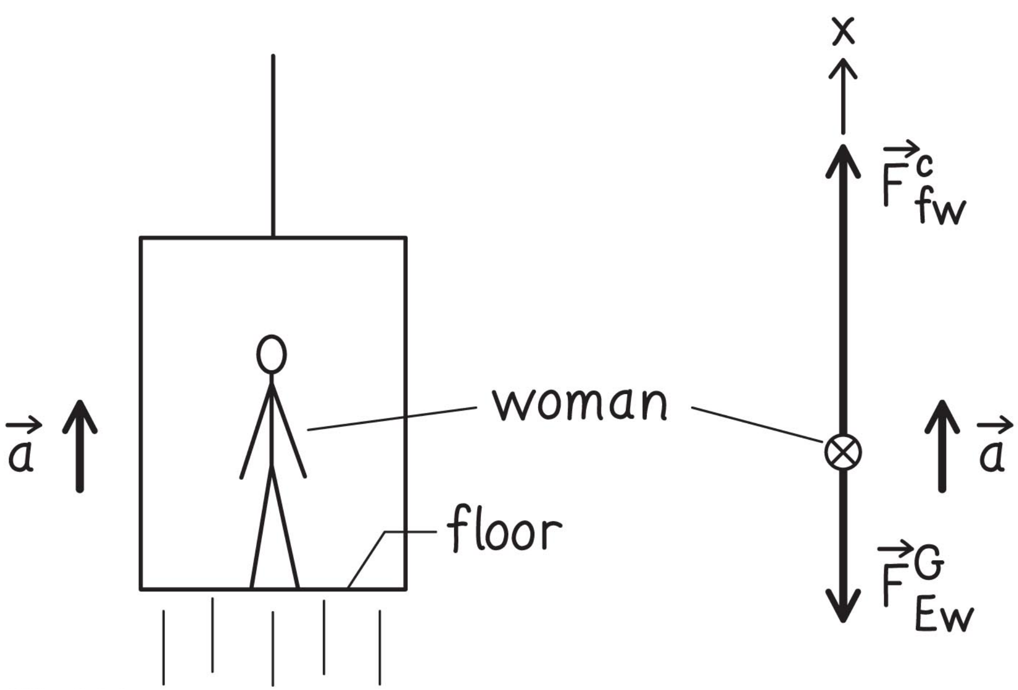
Exercise 8.5 Elevator woman (cont.)

SOLUTION The forces exerted on the woman are a **downward gravitational force** exerted by Earth and an **upward contact force** exerted by the floor. I add arrows to the diagram to represent these forces, label them, and verify that they all have subscripts ending in w for woman.



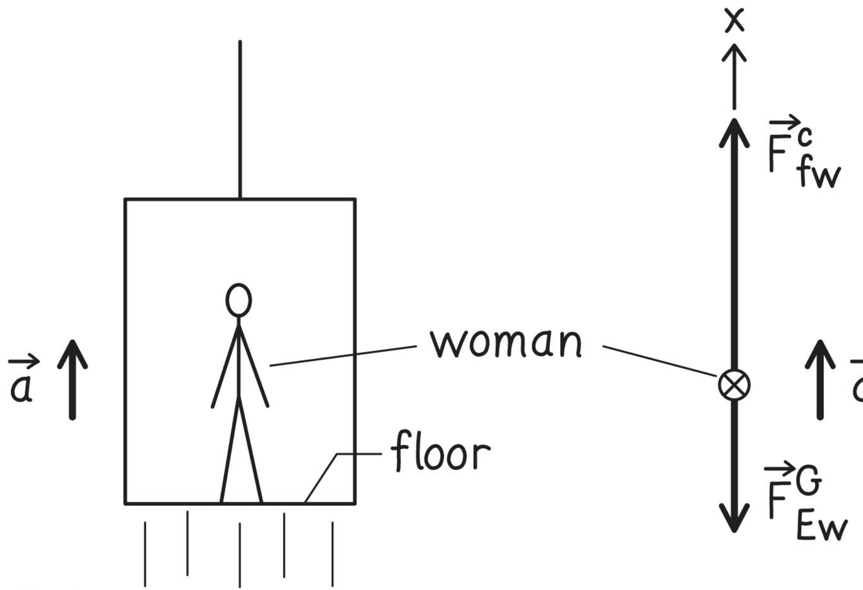
Exercise 8.5 Elevator woman (cont.)

SOLUTION What is the woman's acceleration? Even though she is at rest in the elevator, her acceleration is not zero because the elevator and she are both accelerating upward. I indicate this acceleration with an arrow pointing upward.



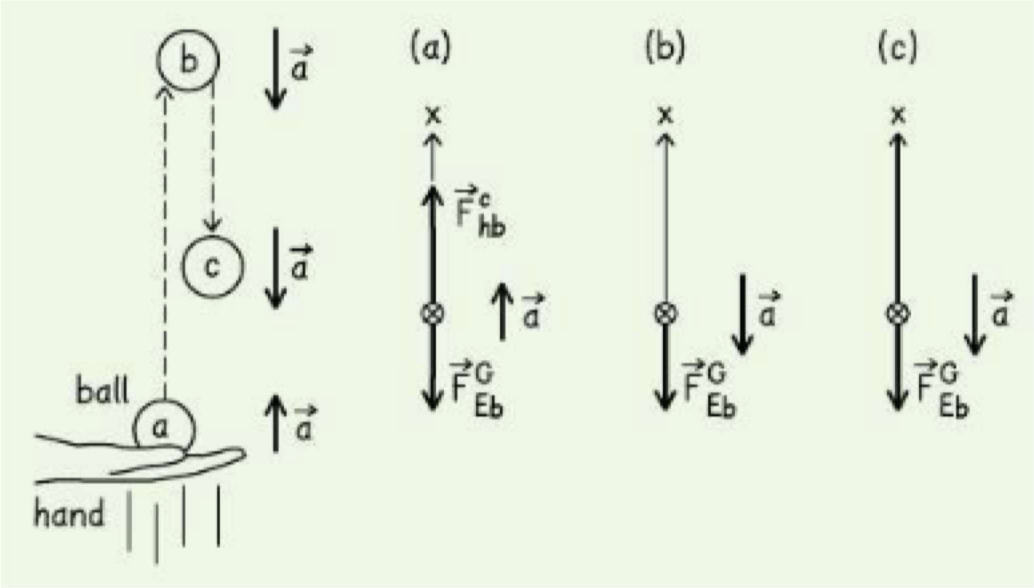
Exercise 8.5 Elevator woman (cont.)

SOLUTION The upward acceleration means that the vector sum of the forces exerted on the woman is nonzero and directed upward. This can happen only if the upward force exerted by the elevator floor is larger than the gravitational force exerted by Earth. Therefore I adjust the lengths of the arrows as shown. Because the elevator has an upward acceleration, I add an upward pointing *x* axis to my drawing.✔



Checkpoint 8.11

8.11 You throw a ball straight up. Draw a free-body diagram for the ball (*a*) while it is still touching your hand and is accelerating upward, (*b*) at its highest point, and (*c*) on the way back down.



Section 8.6: Springs and tension

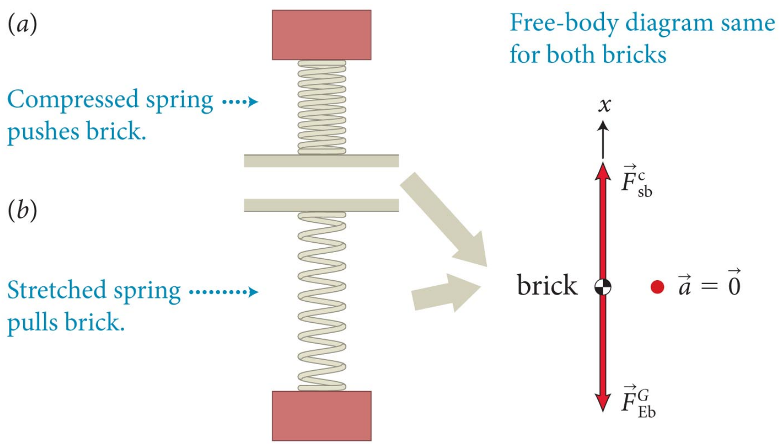
Section Goals

You will learn to

- Understand the forces of **elasticity and tension** and enumerate some of their physical properties.
- Represent spring forces and tensions on free-body diagrams.

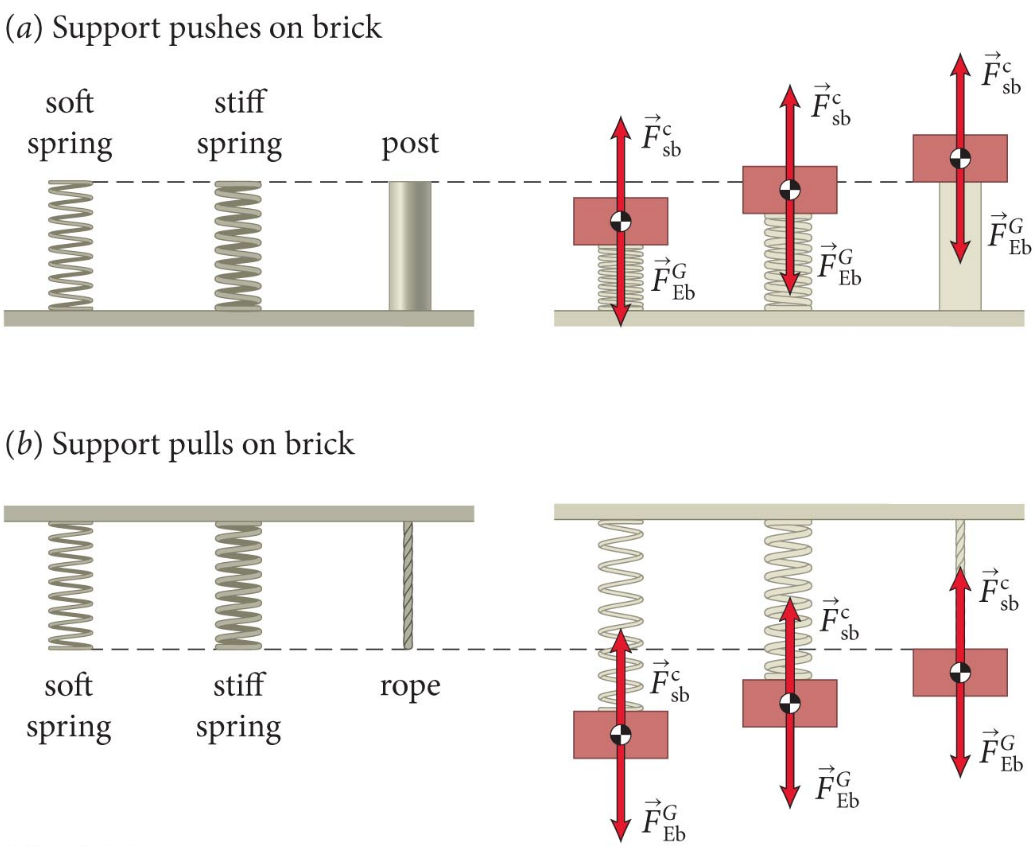
Section 8.6: Springs and tension

- To better understand the behavior of contact forces let us examine the behavior of springs.
- The figure shows
  - (*a*) A spring being compressed by a brick lying on top of it.
  - (*b*) A spring being stretched by a hanging brick.
- In both cases the force exerted by the spring always tends to return the spring to its *relaxed length*.
- The spring force counteracts the gravitational force, and because the spring is at rest the vector sum of the two forces is zero.



Section 8.6: Springs and tension

- The amount of stretching or compression of a spring depends on
  - The force exerted on the spring
  - The stiffness of the spring.
- As the figure below shows, soft and stiff springs exert the same support force on the load.
  - The soft spring stretches and compresses more.

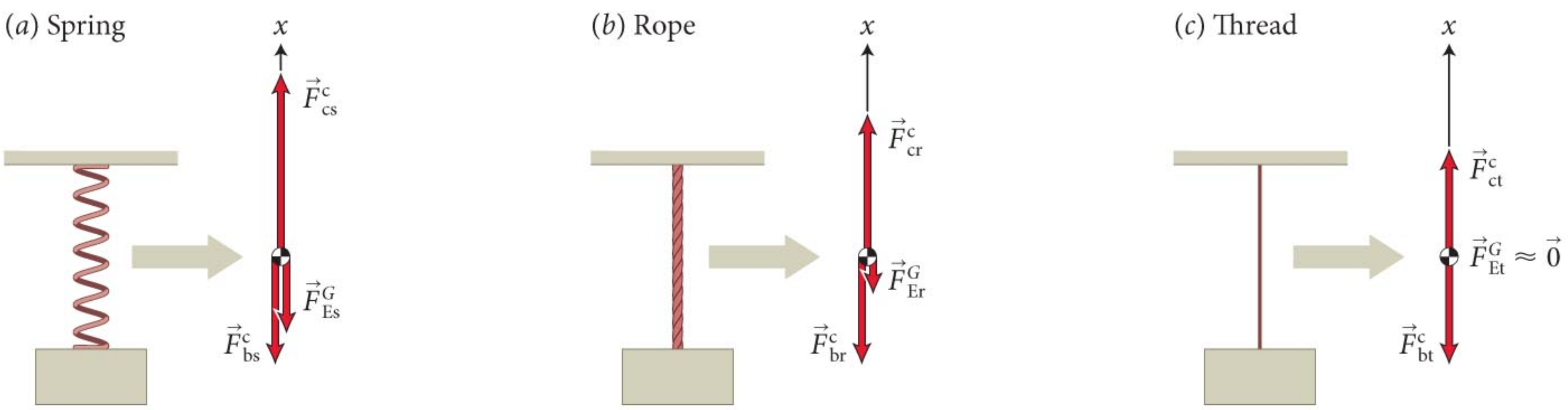


Section 8.6: Springs and tension

- Over a certain range, called the *elastic range*, the deformation of the spring is reversible.
- In this range, the forces exerted by a compressed or stretched material is called an **elastic force.**

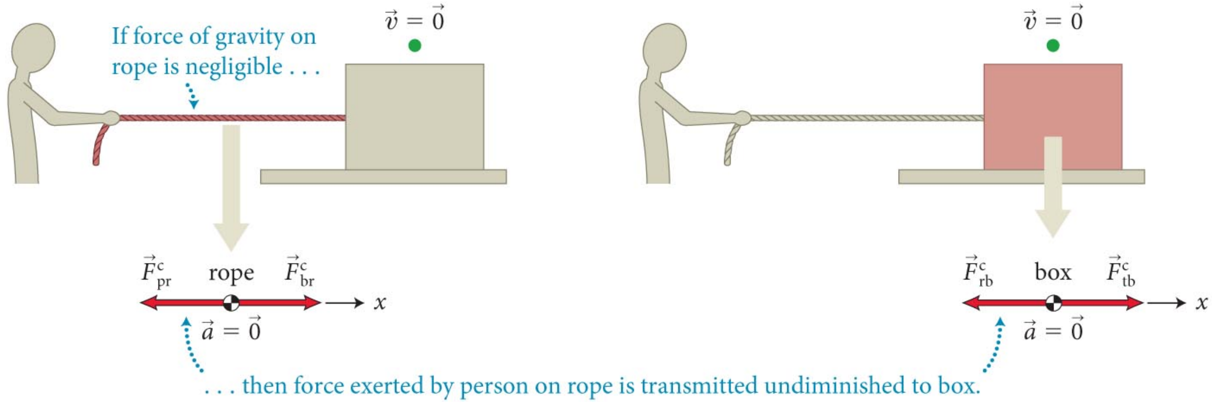
Section 8.6: Springs and tension

- Consider the figure below, where identical bricks are suspended by (*a*) a spring, (*b*) a rope, (*c*) a thread.
- For objects with lower inertia, the gravitational force approaches zero; the magnitude of the two contact forces (exerted by the ceiling and the brick) become equal.
  - As seen in the case of the thread, due to reciprocity of forces, the force exerted *by the thread on the ceiling* becomes equal to the magnitude of the force exerted by the brick on the thread.



Section 8.6: Springs and tension

- Usually the force of gravity exerted on a rope, spring, or thread is much smaller than the forces that cause the stretching, and we can ignore the force of gravity (see figure).
- Therefore, **the force exerted on one end of a rope, spring, or thread is transmitted undiminished to the other end, provided the force of gravity on the rope, spring, or thread is much smaller than the force that causes the stretching.**
- The stress caused by the pair of forces on each end of the rope is called the **tension,** represented by $\mathcal{T}$.



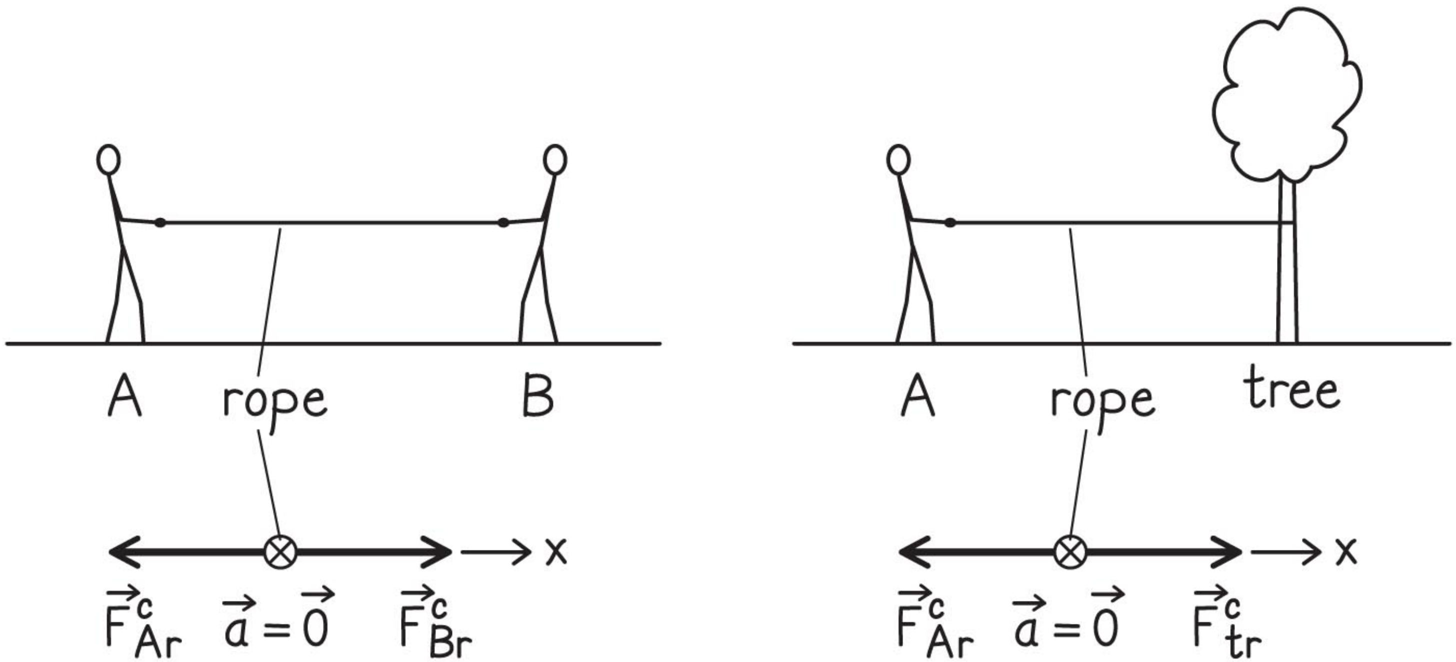
Example 8.6 Tug of war

If two people, A and B, pull on opposite ends of a rope that is at rest, each exerting a horizontal tensile force of magnitude F, the tension in the rope is $\mathcal{T} = F$.

Suppose instead that one end of the rope is tied to a tree and A pulls on the other end by himself with the same force magnitude F. Is the tension in the rope larger than, equal to, or smaller than the tension when A and B pull on opposite ends?

Example 8.6 Tug of war (cont.)

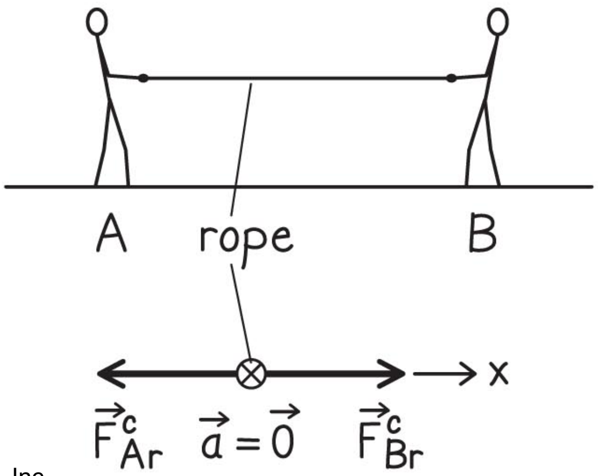
❶ GETTING STARTED To determine the tension in each case, I need to know the magnitude of the tensile force exerted on each end of the rope. Start with a sketch

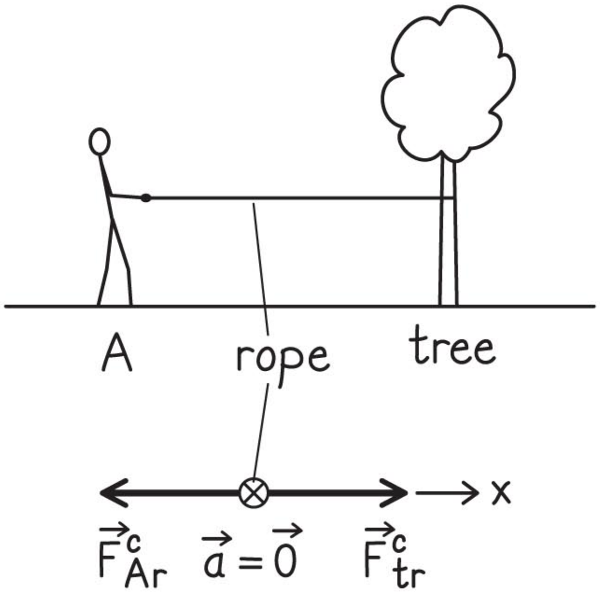


Example 8.6 Tug of war (cont.)

❷ DEVISE PLAN Begin with a free-body diagram for the rope.

In a tug of war only the horizontal forces matter, so I ignore the effect of the vertical gravitational force exerted by Earth and consider only the horizontal forces.

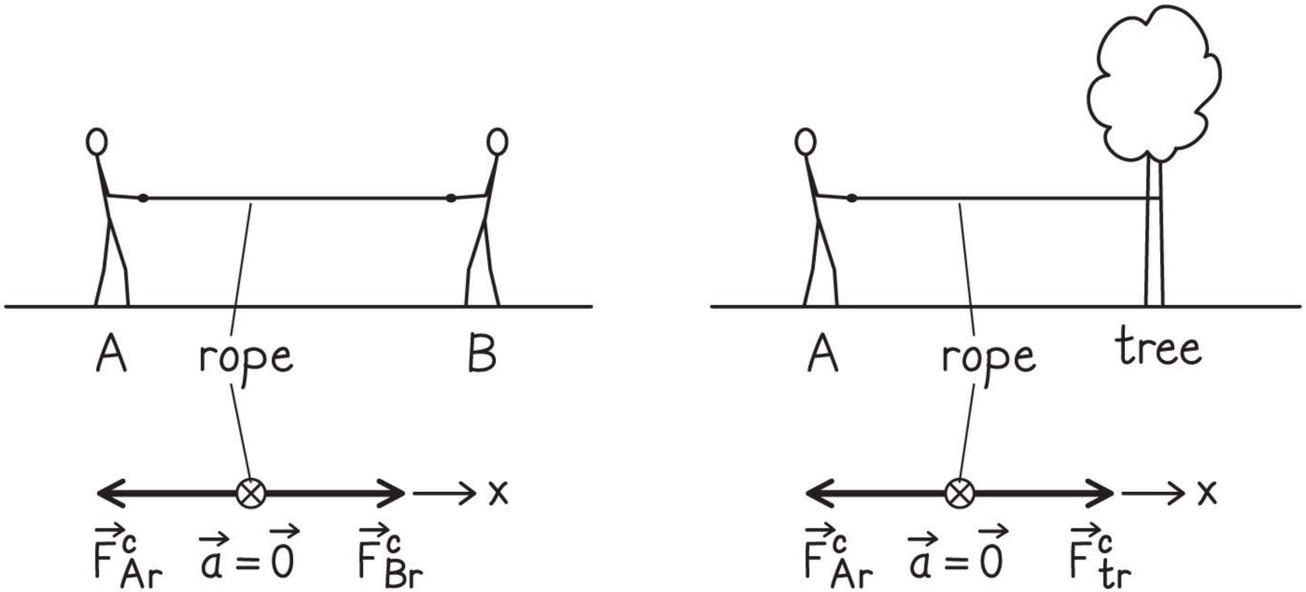




Example 8.6 Tug of war (cont.)

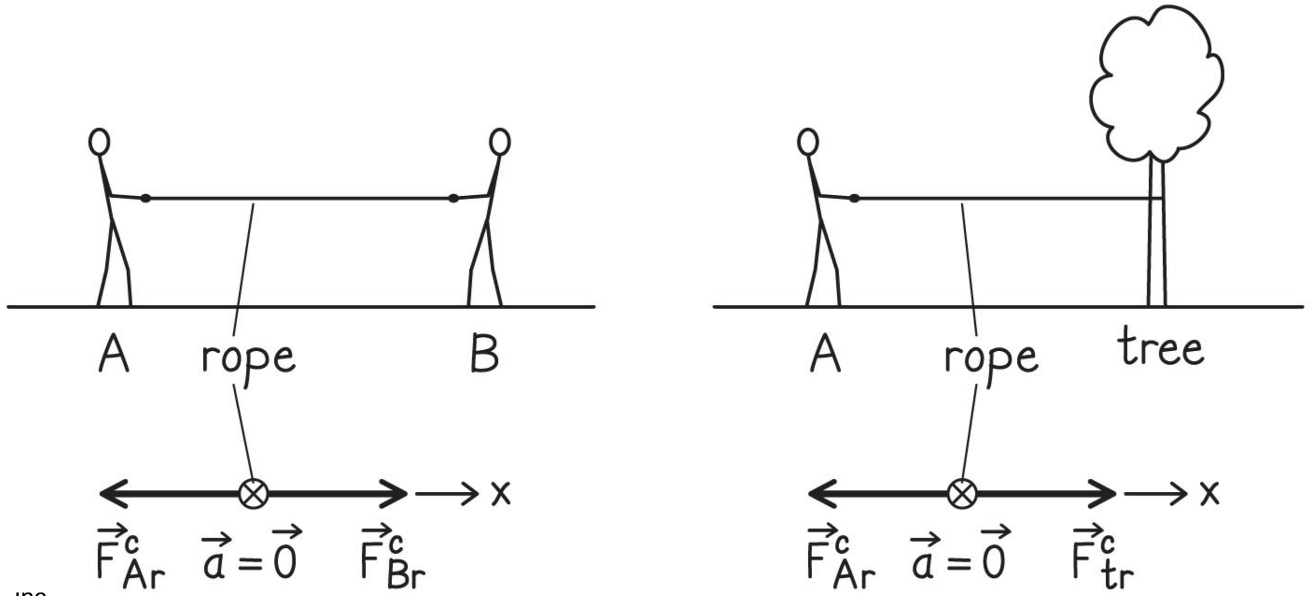
❷ DEVISE PLAN The rope is always at rest, I know that the vector sum of the forces exerted on the rope must be zero.

The magnitudes of the forces exerted on either end are therefore equal, and the tension in the rope is equal to the magnitude of the force exerted on either end.



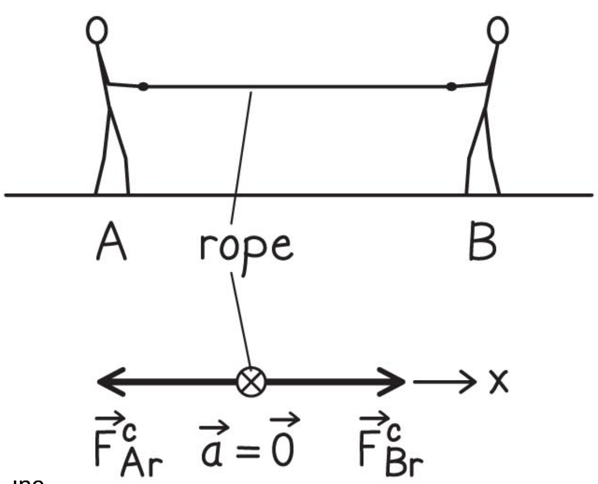
Example 8.6 Tug of war (cont.)

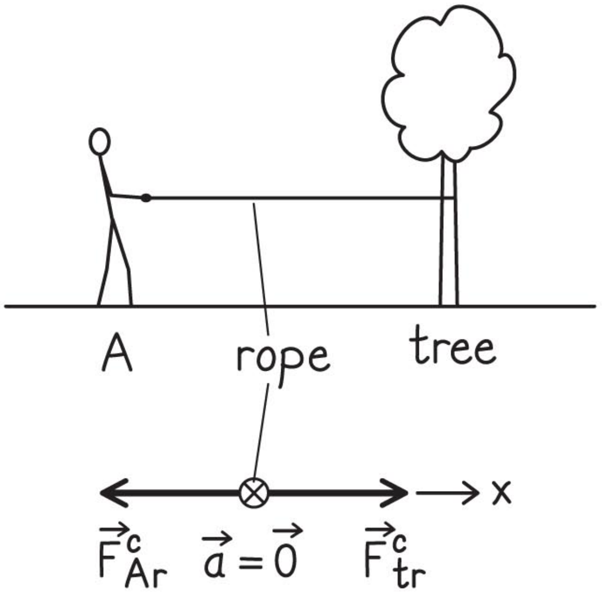
❸ EXECUTE PLAN The rope tied to the tree is subject to two contact forces, one exerted by A and the other exerted by the tree. Because the vector sum of the forces exerted on the rope must be zero, I make the two force arrows of equal length.



Example 8.6 Tug of war (cont.)

❸ EXECUTE PLAN Because A pulls on the rope with a force of magnitude F, the tree must pull with a force of magnitude F in the opposite direction, and so the tension in the rope is $\mathcal{T} = F$. which is the same as when A and B pull on opposite ends. ✔





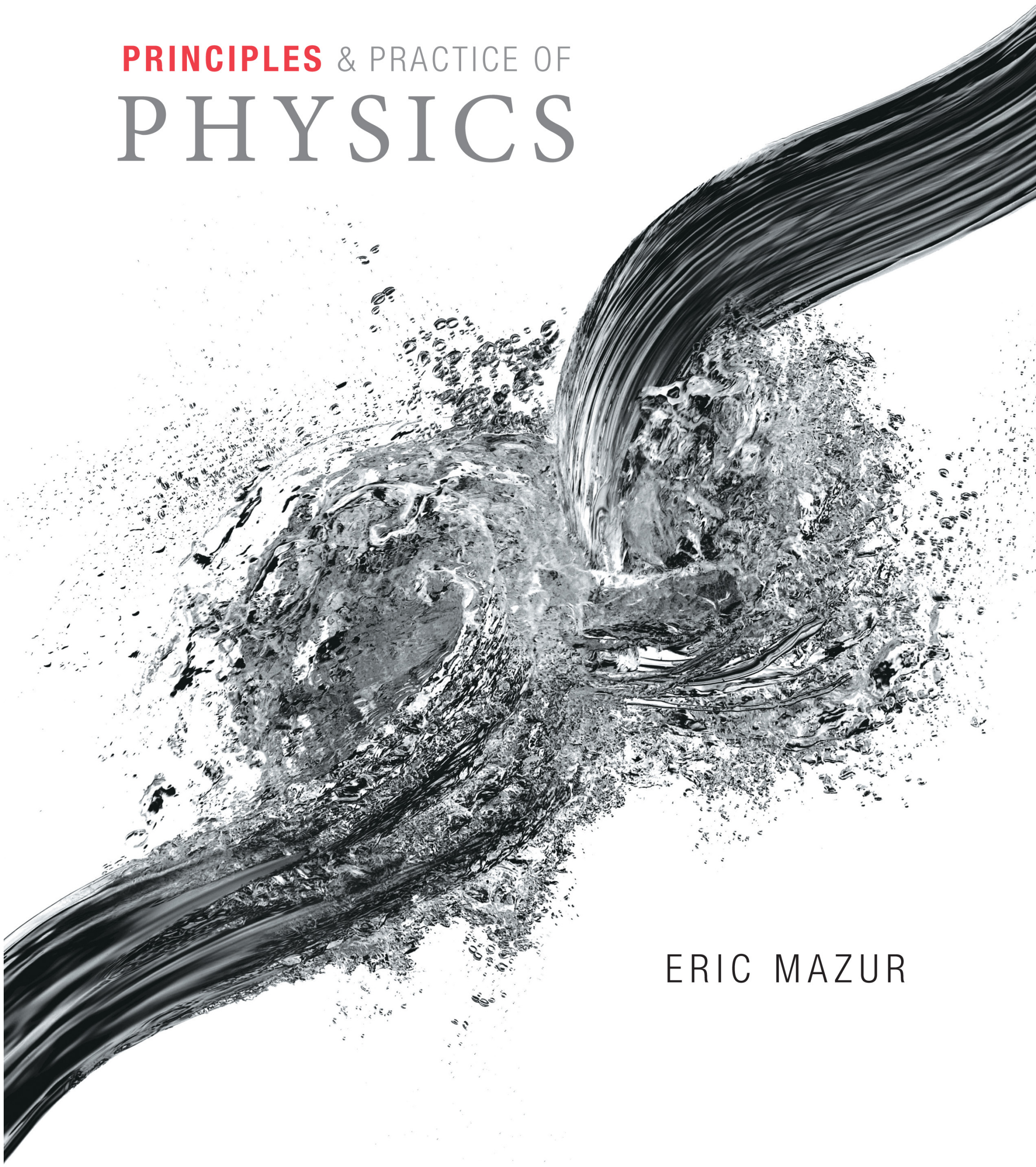
Example 8.6 Tug of war (cont.)

❹ EVALUATE RESULT The magnitude of the force exerted by A is F in both cases.

Because the rope has zero acceleration in both cases, the magnitude of the force exerted by the tree must be the same as the magnitude of the force exerted by B when A and B pull.

Therefore it makes sense that the tension in the rope is the same in the two cases.

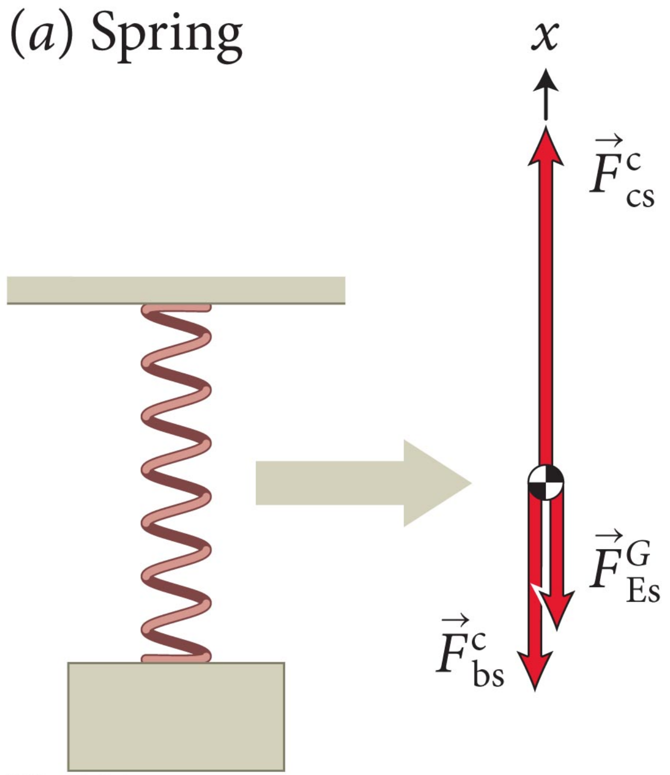
PRINCIPLES & PRACTICE OF PHYSICS



ERIC MAZUR

Checkpoint 8.12

8.12 In Figure 8.10*a*, how does the magnitude of the downward force exerted *by the spring on the ceiling* compare with the magnitudes of the downward gravitational forces exerted by Earth on the spring and on the brick?



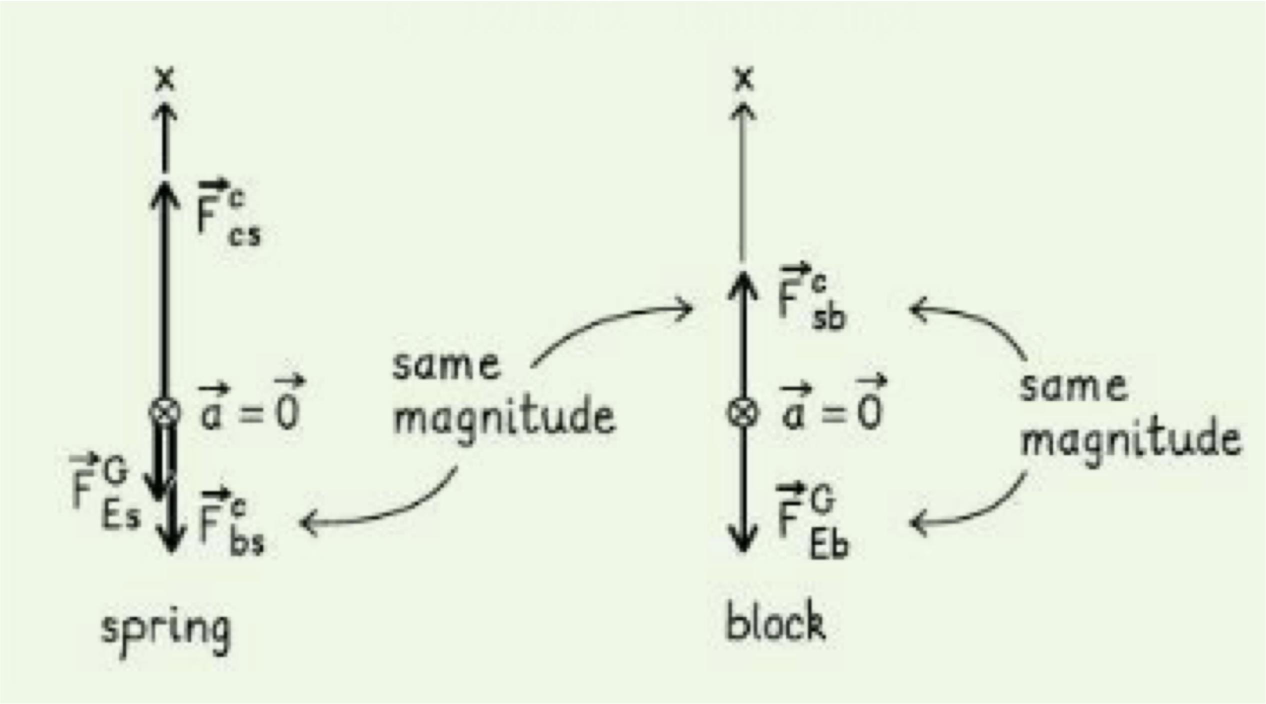
Ceiling supports both block and spring

Combining the two free body diagrams, noting F_{sb}=-F_{bs}

(1) $F_{cs} - F_{Es} - F_{bs} = 0$

(2) $F_{sb} - F_{Eb} = -F_{bs} - F_{Eb} = 0$ or $F_{bs} = F_{Eb}$

$\Rightarrow F_{cs} = F_{Es} + F_{Eb}$



Quantitative Tools

Section 8.7: Equations of motion

- We know that the time rate of change of momentum of an object is equal to the vector sum of forces acting on it.

$$\lim_{\Delta t \to 0} \frac{\Delta \vec{p}_{\text{object}}}{\Delta t} = \frac{d\vec{p}_{\text{object}}}{dt} \equiv \sum \vec{F}_{\text{object}}$$

The vector sum of the forces acting on the object is represented by

$$\sum \vec{F}_{\text{object}} = \vec{F}_{1\text{ object}} + \vec{F}_{2\text{ object}} + \vec{F}_{3\text{ object}} + \cdots$$

- Dropping the subscripts in the previous equation we get

$$\sum \vec{F} \equiv \frac{d\vec{p}}{dt}$$

- Substituting $p=mv$ we can write

$$\frac{d\vec{p}}{dt} \equiv \frac{d(m\vec{\upsilon})}{dt} = m\frac{d\vec{\upsilon}}{dt} \qquad \text{(rigid objects)}$$

Section 8.7: Equations of motion

- Substituting $d\vec{\upsilon}/dt = \vec{a}$ in the previous two equations, we get

$$\sum \vec{F} = m\vec{a}$$

or

$$\vec{a} = \frac{\sum \vec{F}}{m}$$

- In component form these equations become

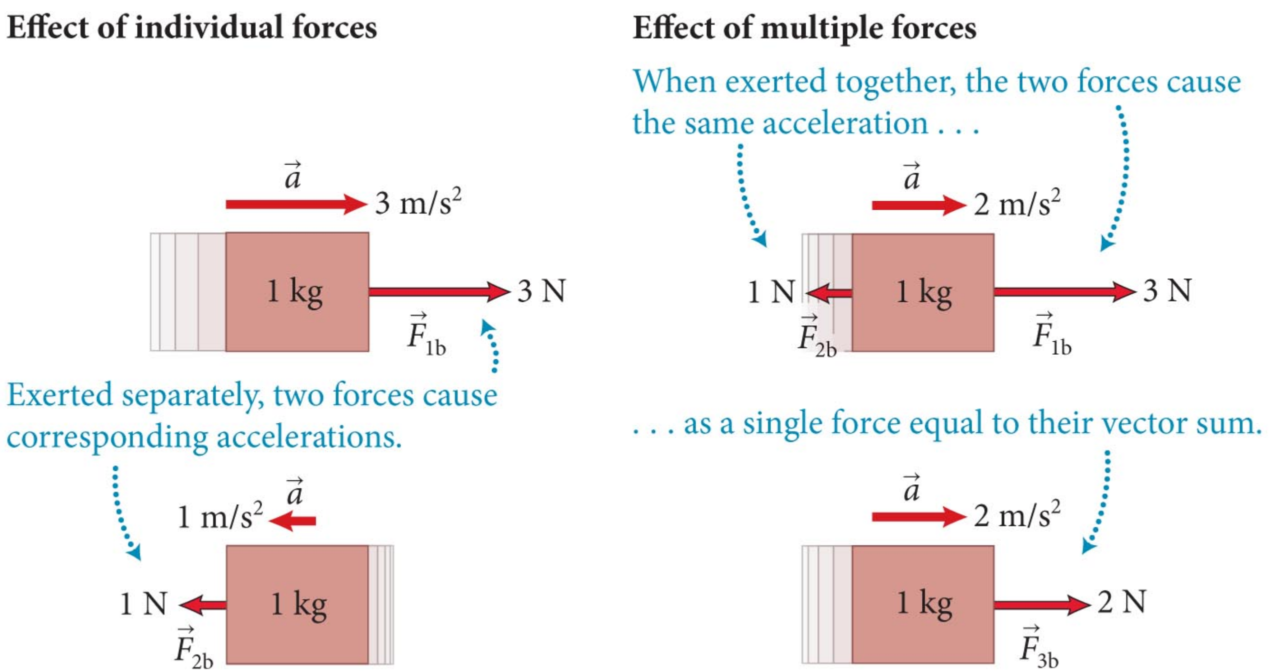
$$\sum F_x = ma_x \quad \text{and} \quad a_x = \frac{\sum F_x}{m}$$

- The second equation is called the **equation of motion** of the object.
- The SI unit of force is called a **newton** (N):

$$1\ \text{N} \equiv 1\ \text{kg} \cdot \text{m/s}^2$$

Section 8.7: Equations of motion

- The property of adding forces vectorially is called the *superposition of forces*
- We can use the equation of motion to determine the acceleration.



Example 8.7 Elevator stool

A person is sitting on a stool in an elevator. The forces exerted on the stool are a downward force of magnitude 60 N exerted by Earth, a downward force of magnitude 780 N exerted by the person, and an upward force of magnitude 850 N exerted by the elevator floor.

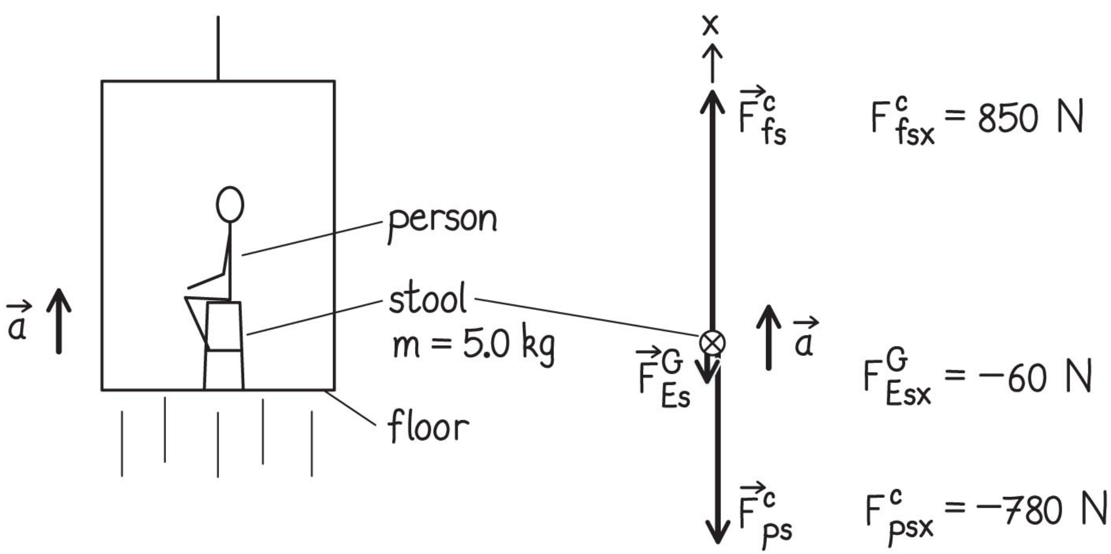
If the inertia of the stool is 5.0 kg, what is the acceleration of the elevator?

Example 8.7 Elevator stool (cont.)

❶ GETTING STARTED I know all the forces that are exerted on the stool, and I must determine its acceleration.

Begin by drawing a free-body diagram for the stool that includes the three forces mentioned.

Choose an *x* axis pointing upward.



Section 8.7: Equations of motion

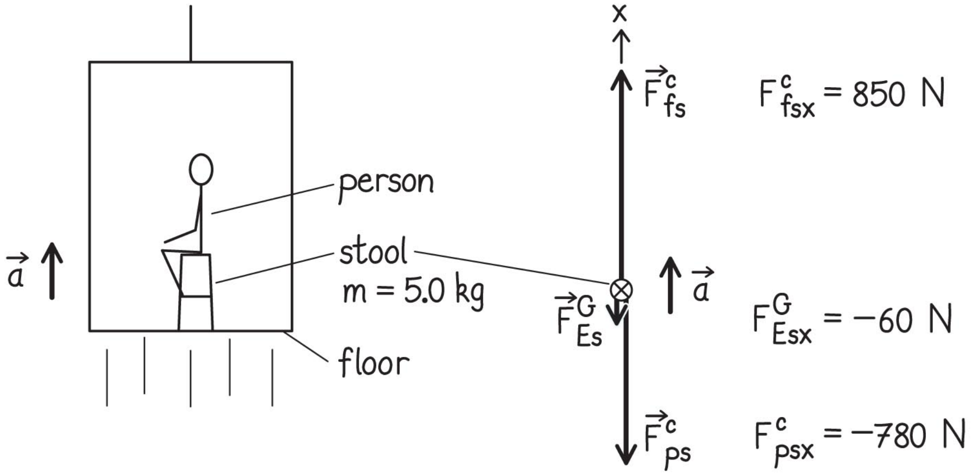
Example 8.7 Elevator stool (cont.)

❸ EXECUTE PLAN From Eq. 8.8, I have

$$a_x = \frac{\sum F_x}{m} = \frac{1}{m}\left(F^c_{\text{fs}\,x} + F^c_{\text{ps}\,x} + F^G_{\text{Es}\,x}\right)$$

$$= \frac{1}{5.0\ \text{kg}}\left[(+850\ \text{N}) + (-780\ \text{N}) + (-60\ \text{N})\right]$$

$$= +2.0\ \text{m/s}^2. \checkmark$$



Example 8.7 Elevator stool (cont.)

❹ EVALUATE RESULT The acceleration I obtained is positive, which means that the elevator has an upward acceleration, which is what I expect because the force exerted by the floor on the stool is upward.

The magnitude of the acceleration is about one-fifth the acceleration due to gravity, which strikes me as a reasonable value.

Checkpoint 8.14

8.14

(a) You exert a constant force of 200 N on a friend on roller skates. If she starts from rest, estimate how far she moves in 2.0 s.

(*b*) When a person jumps off a wall, what is the magnitude of his acceleration during the fall?

(*c*) Estimate the magnitude of the force exerted by Earth on the person during the jump in part *b*.

(a) You exert a constant force of 200 N on a friend on roller skates. If she starts from rest, estimate how far she moves in 2.0 s.

Need to estimate mass. Say m=60kg (about 132 lbs)

$$\text{Then } a = F/m = (200\text{N})/(60\text{kg}) = 3.3\text{m/s}^2$$

$$\Delta x = \tfrac{1}{2}a_x(\Delta t)^2 = 6.7\text{m}$$

(*b*) When a person jumps off a wall, what is the magnitude of his acceleration?

free fall, so $a = 9.8 m/s^2$

(*c*) Estimate the magnitude of the force exerted by Earth on the person during the jump in part *b*.

Constant acceleration, so constant force

$$F_{EP}^{G} = ma = (60\text{kg})(9.8\text{m/s}^2) = 590\text{N}$$

Section 8.7: Equations of motion

- For an isolated system of two colliding carts, we can write (for an infinitesimal time interval)

$$d\vec{p}_1 + d\vec{p}_2 = \vec{0}$$

- Because *dt* is the same for both carts, we can write

$$\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = \vec{0}$$

- Since the rate of change of momentum is equal to the vector sum of forces, we get

$$\sum \vec{F}_1 + \sum \vec{F}_2 = \vec{0}$$

Section 8.7: Equations of motion

- For the isolated system of two carts colliding,

$$\sum \vec{F}_1 = \vec{F}_{21} \quad \text{and} \quad \sum \vec{F}_2 = \vec{F}_{12}$$

- Thus we get $\vec{F}_{12} = -\vec{F}_{21}$.
- The forces $\vec{F}_{12}$ and $\vec{F}_{21}$ form an interacting pair.
- Both forces in an interacting pair can never be part of the same free-body diagram – they act on different objects
- The second equation above is called **Newton's third law of motion**.

Checkpoint 8.15

8.15 If forces always come in interaction pairs and the forces in such a pair are equal in magnitude and opposite in direction (Eq. 8.15), how can the vector sum of the forces exerted on an object ever be nonzero?

You have to consider forces from other objects – the force of **you on the crate** is the same as the **force of the crate on you** …

… but the former might still be larger than the force of friction

For nonzero net force, you're considering multiple interaction pairs

Newton's laws of motion

In the modern view, the conservation laws are the heart and soul of physics—they best represent the generally held belief that the universe has an underlying simplicity.

In the 17th century, however, 200 years before the concept of energy and the conservation laws were formulated, Newton published a work describing his ideas concerning the fundamental principles that govern forces and their effects on the motion of objects.

Newton’s laws of motion

Newton’s three laws of motion, as they are known, have been the backbone of physics for more than two centuries.

We have already discussed each of these three laws—albeit under different, and in the current context more appropriate, names.

Section 8.7: Equations of motion

Newton's laws of motion

Newton's first law of motion, first formulated by Galileo Galilei, is what we called the *law of inertia* in Chapter 6:

In an inertial reference frame (non-accelerating)**, any isolated object that is at rest remains at rest, and any isolated object that is in motion keeps moving at a constant velocity.**

Section 8.7: Equations of motion

Newton's laws of motion

Newton's second law of motion corresponds to the *definition of force* given in Eq. 8.4:

The vector sum of the forces exerted on an object is equal to the time rate of change in the momentum of that object.

Equation 8.6, $\Sigma\vec{F} = m\vec{a}$, is frequently referred to as Newton's second law even though it holds only when m is constant.

Newton's laws of motion

Newton's third law of motion expresses the law of conservation of momentum in terms of forces (Eq. 8.15):

Whenever two objects interact, they exert on each other forces that are equal in magnitude but opposite in direction.

Section 8.8: Force of gravity

- All objects in free fall near Earth's surface have a downward acceleration of $a_x = -g$. ($+x$ is upwards)
- The left hand side of the equation of motion for a free-falling object is

$$\sum F_x = F^G_{\mathrm{Eo}\,x}$$

- So, the x component of the gravitational force exerted by Earth on an object is

$$F^G_{\mathrm{Eo}\,x} = -mg \text{ (near Earth's surface, } x \text{ axis vertically upward)}$$

- This is the object's weight

Example 8.8 Tennis ball launch

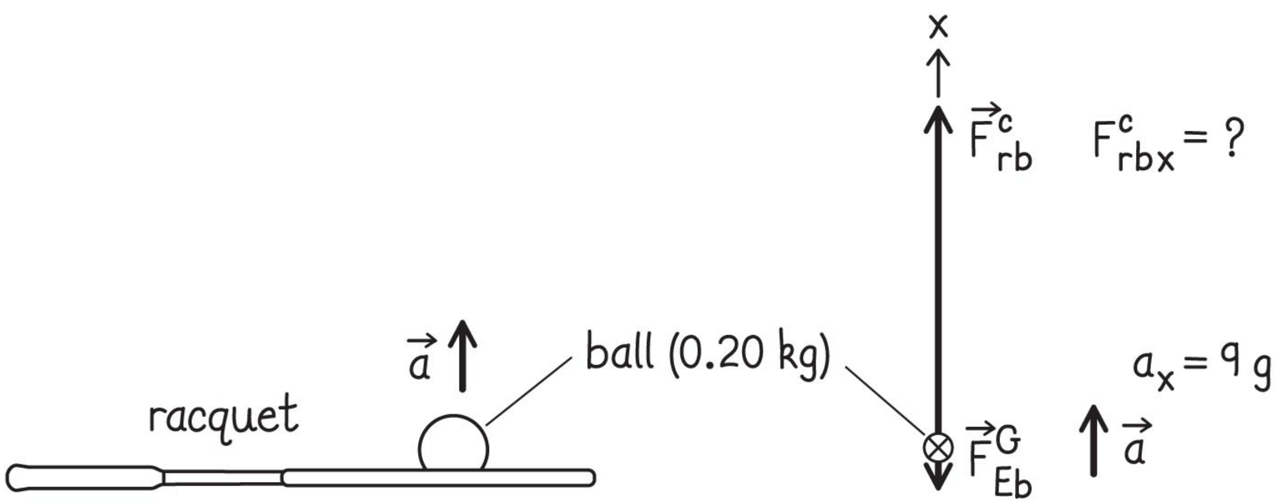
A tennis ball of inertia 0.20 kg is launched straight up in the air by hitting it with a racquet. If the magnitude of the acceleration of the ball while it is in contact with the racquet is $9g$, what are the magnitude and direction of the force exerted by the racquet on the ball?

Example 8.8 Tennis ball launch (cont.)

❶ GETTING STARTED I am given the inertia and acceleration magnitude for a tennis ball hit by a racquet.

I should calculate the force responsible.

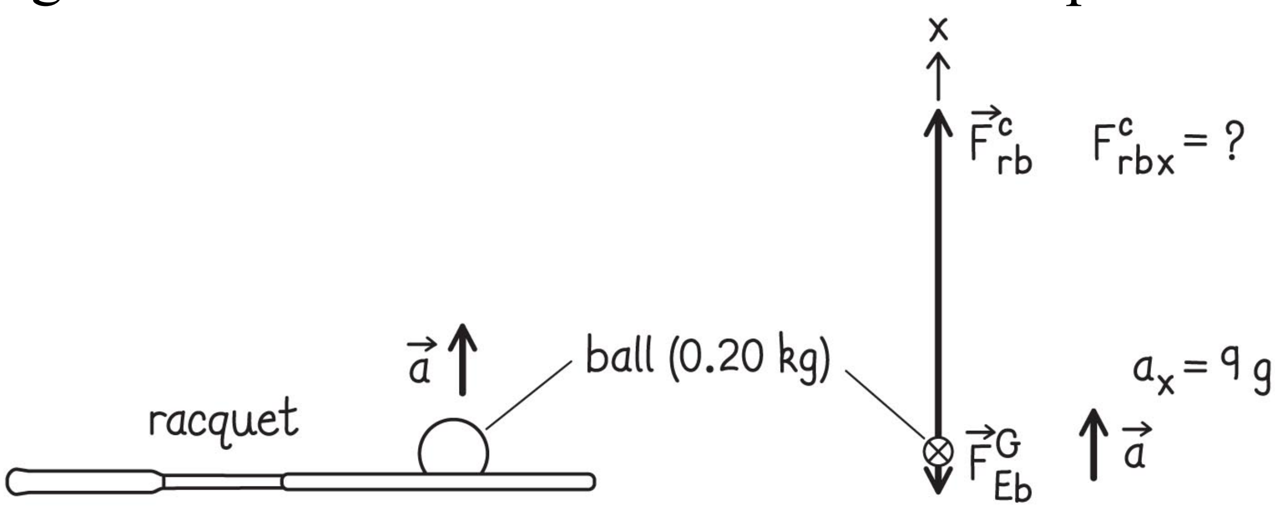
Start with a free-body diagram.



Example 8.8 Tennis ball launch (cont.)

❶ GETTING STARTED While it is being hit, the ball is in contact with the racquet and so I include an upward contact force exerted by the racquet as well as a downward force of gravity exerted by Earth.

Because the ball accelerates upward, I make the upward force vector longer than the downward force vector and point the positive .



Example 8.8 Tennis ball launch (cont.)

❷ DEVISE PLAN I can relate the ball’s acceleration to the vector sum of the two forces exerted on it.

I know one of the forces. Since the acceleration and mass are known, I can find the second force.

Example 8.8 Tennis ball launch (cont.)

❸ EXECUTE PLAN Equation 8.8 yields

$$\sum F_x = F^G_{\text{Eb}x} + F^c_{\text{rb}x} = ma_x.$$

Because I've defined upward as the positive x direction, $a_x = +9g$. Substituting this value and Eq. 8.17 for the gravitational force exerted by Earth on the object, I get

$$(-mg) + F^c_{\text{rb}x} = +9mg$$

$$F^c_{\text{rb}x} = +10mg = +10(0.20\ \text{kg})(9.8\ \text{m/s}^2)$$

$$= +20\ \text{N}.$$

Example 8.8 Tennis ball launch (cont.)

❹ EVALUATE RESULT The plus sign indicates that this force is directed upward, as expected. The magnitude of the force is ten times larger than the gravitational force exerted by Earth, which also makes sense if the ball is to be accelerated upward during the very short time interval that the racquet is in contact with the ball.

Section 8.8
Question 6

The magnitude of the gravitational force exerted by Earth on an object of inertia m_1 is $m_1 g$. What is the magnitude of the force exerted by the object on Earth (inertia m_E)?

1. $m_1 g$
2. $m_E g$
3. 0
4. Cannot be determined from the given information

Section 8.8
Question 6

The magnitude of the gravitational force exerted by Earth on an object of inertia m_1 is $m_1 g$. What is the magnitude of the force exerted by the object on Earth (inertia m_E)?

1. ✔ $m_1 g$
2. $m_E g$
3. 0
4. Cannot be determined from the given information

Section 8.8
Question 7

The magnitude of the gravitational force exerted by Earth on an object of inertia m_1 is m_1g. The inertia of the Earth is inertia m_E. What is the acceleration of Earth due to its gravitational interaction with the object?

1. 0
2. g
3. m_1g/m_E
4. m_Eg/m_1
5. Cannot be determined from the given information

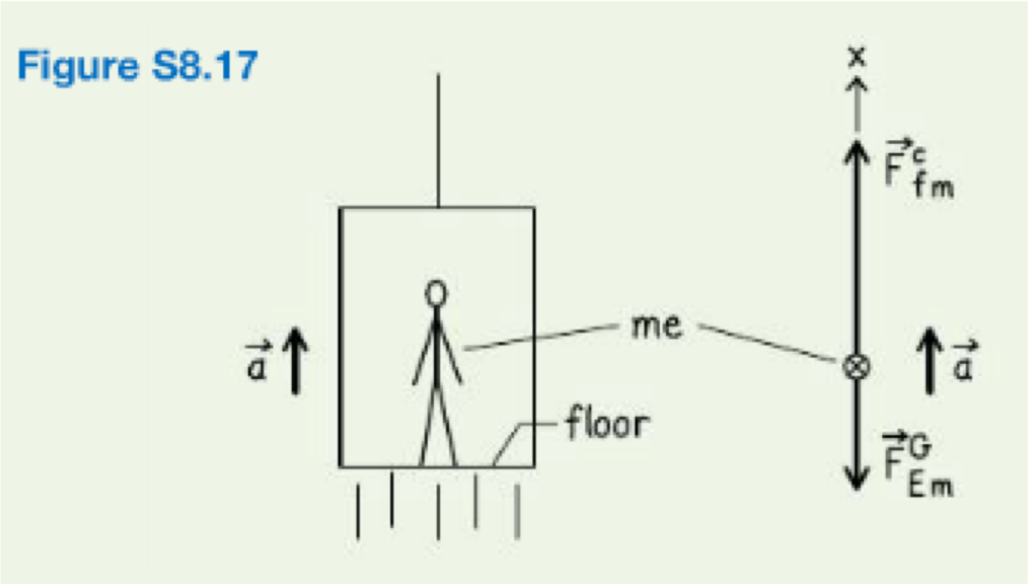
Question 7

The magnitude of the gravitational force exerted by Earth on an object of inertia m_1 is $m_1 g$. The inertia of the Earth is inertia m_E. What is the acceleration of Earth due to its gravitational interaction with the object?

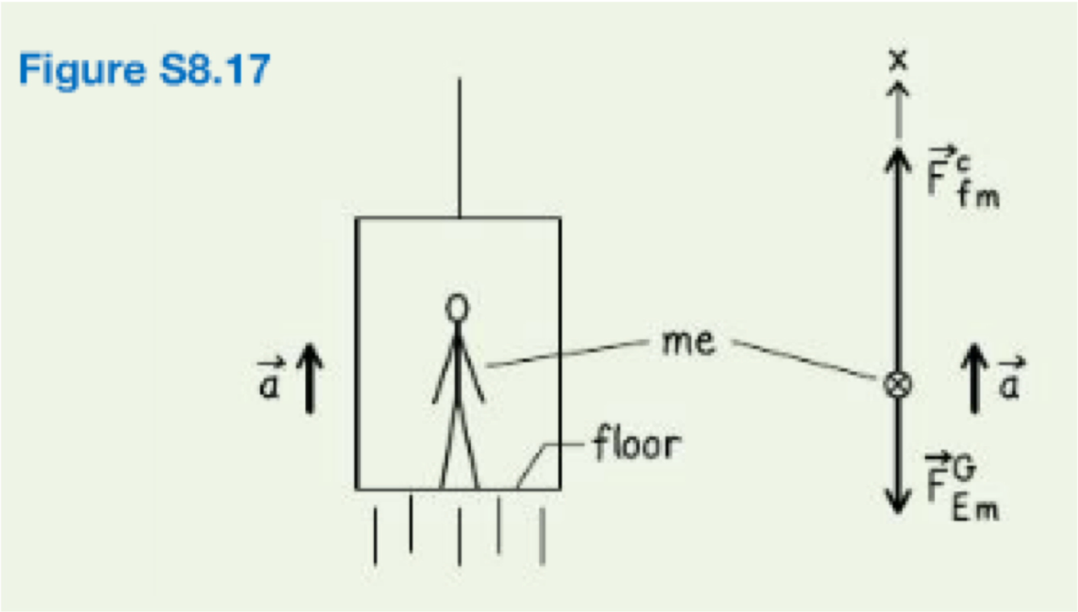
1. 0
2. g
3. ✔ $m_1 g/m_E = F/m_E$
4. $m_E g/m_1$
5. Cannot be determined from the given information

Checkpoint 8.17

8.17 Suppose you are in an elevator that is accelerating upward at 1 m/s^2. (*a*) Draw a free-body diagram for your body. (*b*) Determine the magnitude of the force exerted by the elevator floor on you.



Checkpoint 8.17



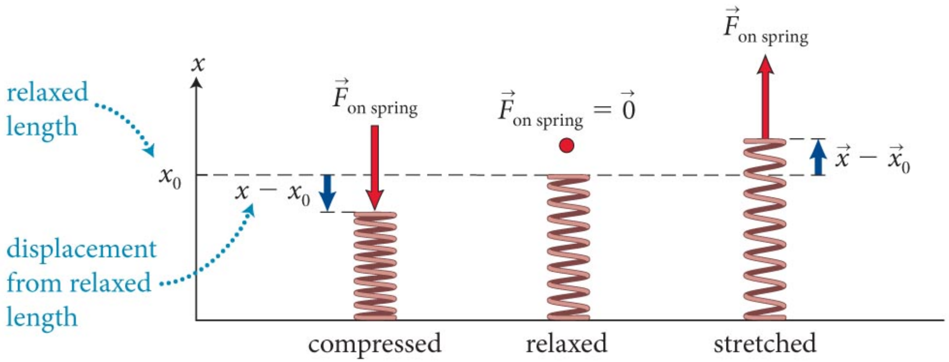
$$\sum F_x = ma_x = F^c_{fm,x} + F^G_{Em,x}$$

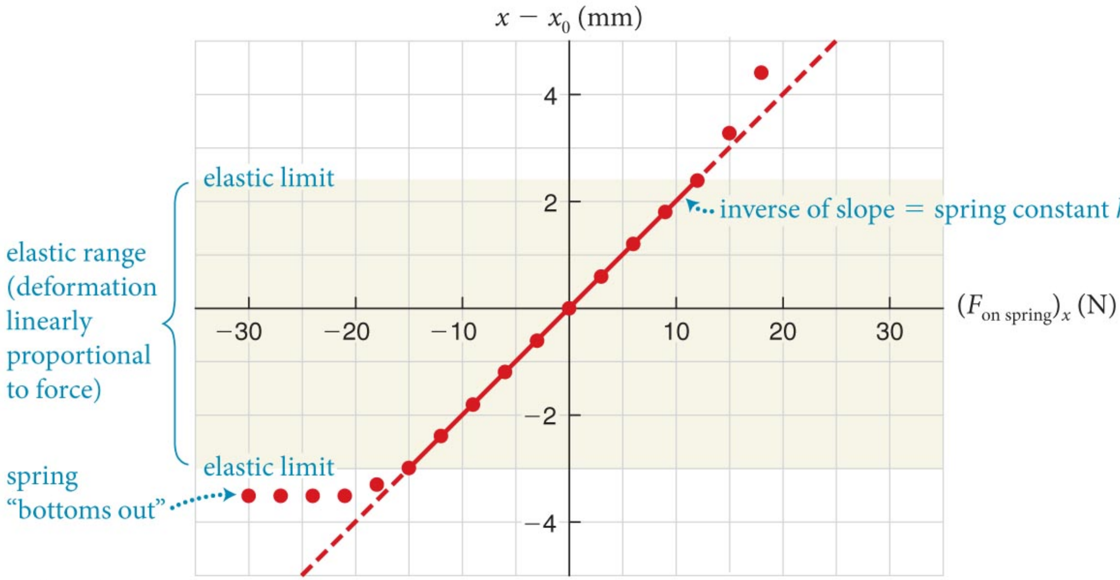
$$F^c_{fm,x} = ma_x - F^G_{Em,x} = ma_x - (-mg) = m(a_x + g)$$

For an inertia of $m = 73\text{kg}$, this gives a force of 790N

Section 8.9: Hooke's law

- The graph below shows the x component of the displacement of the spring from its relaxed position (x_0) versus the x component of the force exerted on it.





Section 8.9: Hooke's law

- In the graph shown on the previous slide, the relationship between the displacement and force can be quantitatively expressed as

$$(F_{\text{by load on spring}})_x = k(x - x_0) \text{ (small displacement)}$$

where k is called the **spring constant**.

- We know from Newton's third law that

$$\vec{F}_{\text{by spring on load}} = -\vec{F}_{\text{by load on spring}}$$

- So, the force exerted by the spring is given by

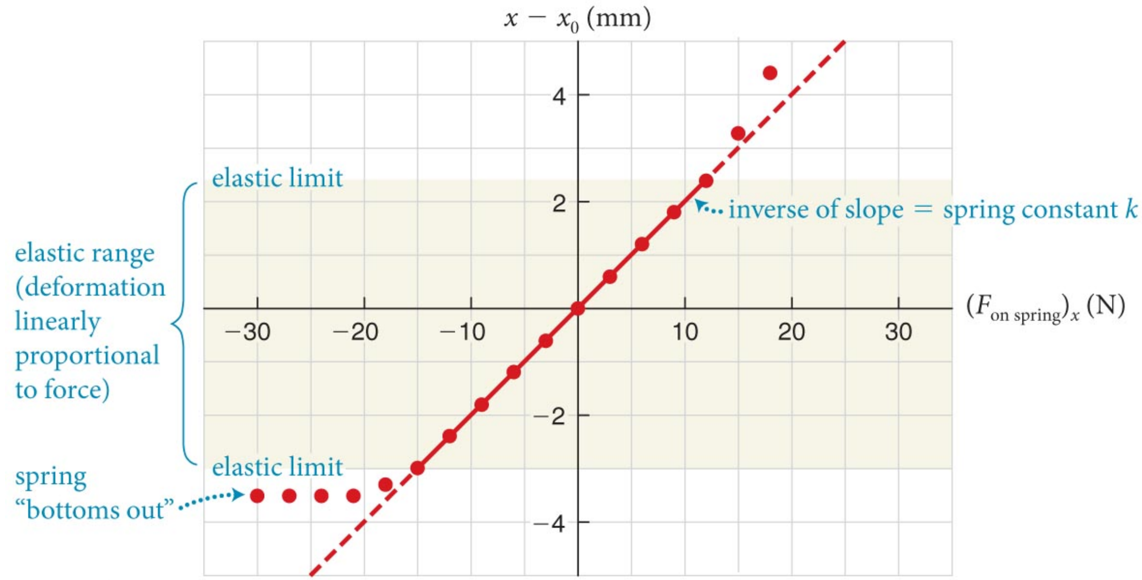
$$(F_{\text{by spring on load}})_x = -k(x - x_0) \text{ (small displacement)}$$

- This equation is called **Hooke's law**.

Section 8.9: Hooke's law

Example 8.9 Spring compression

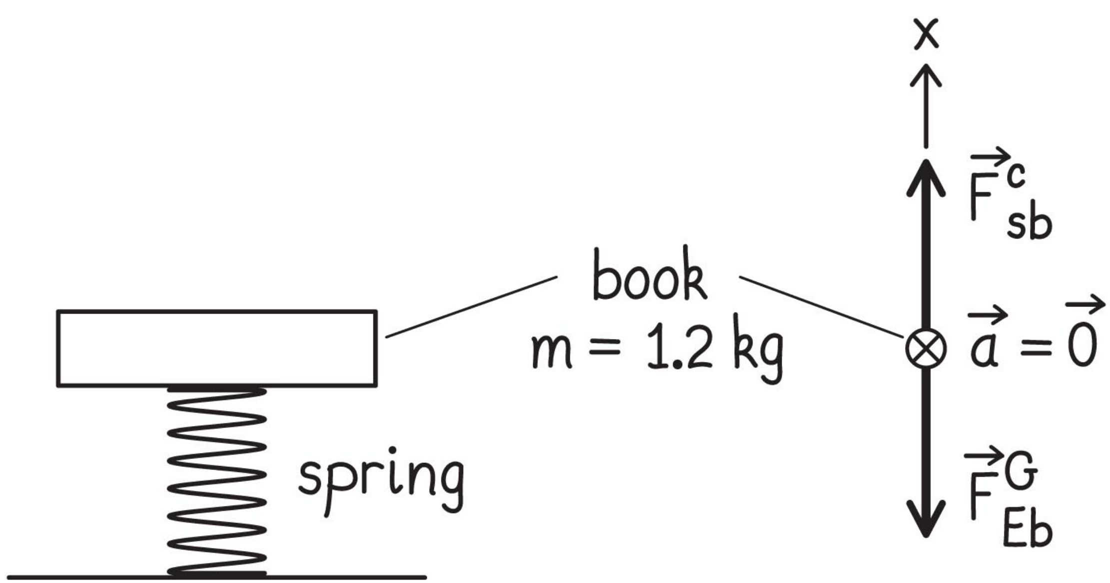
- A book of inertia 1.2 kg is placed on top of the spring from the previous figure.
- What is the displacement of the top end of the spring from the relaxed position when the book is at rest on top of the spring?



Example 8.9 Spring compression (cont.)

❶ GETTING STARTED My task is to determine how much the top end of a spring is displaced when a 1.2-kg book is placed on the spring.

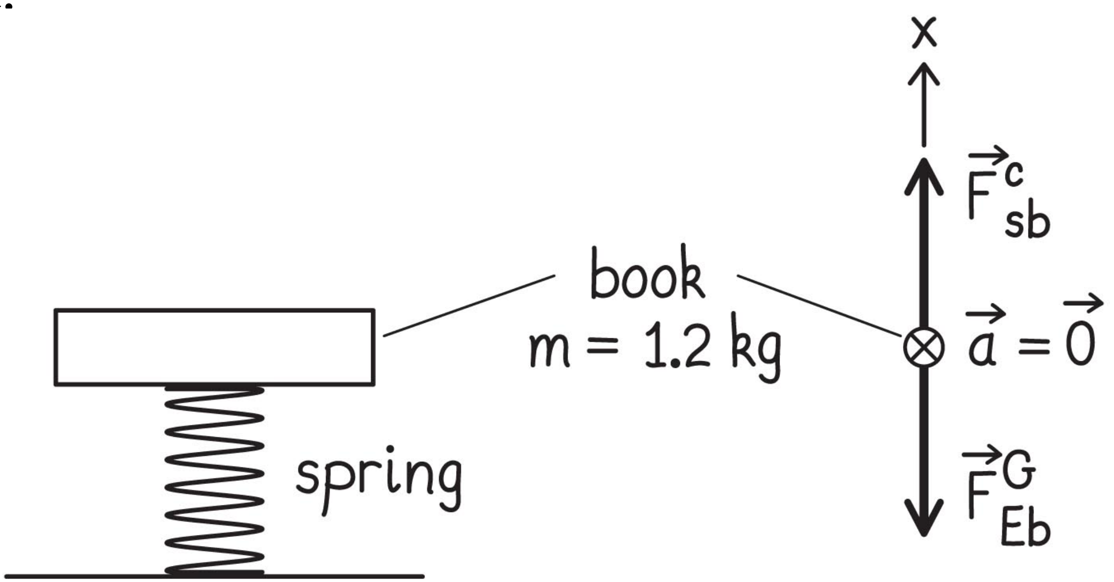
Displacement is proportional to the force exerted by the spring on the book. First sketch + free-body diagram.



Example 8.9 Spring compression (cont.)

❶ GETTING STARTED The book is subject to a downward gravitational force exerted by Earth and an upward contact force exerted by the spring.

The book is in translational equilibrium (acceleration is zero), and so the two forces are equal in magnitude. I arbitrarily point the *x* axis upward.



Example 8.9 Spring compression (cont.)

❷ DEVISE PLAN I need to know the spring constant k and the x component F^{c}_{sbx} of the force exerted by the spring on the book.

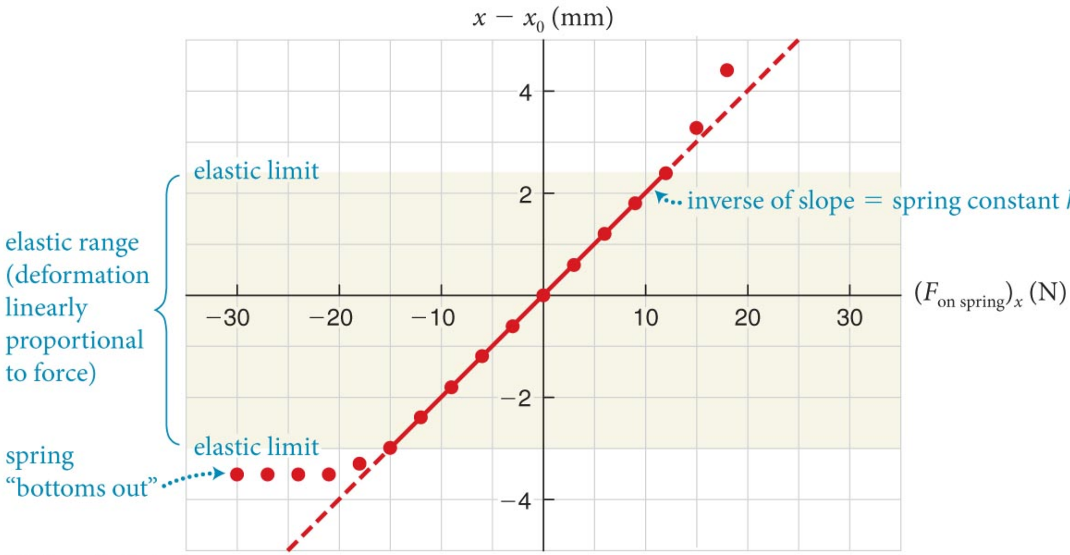
The spring constant is given by the slope of the curve in Figure 8.18.

From my free-body diagram I know that $\vec{F}^{c}_{sb}$ is equal to the negative of the force of gravity $\vec{F}^{G}_{Eb}$ exerted by Earth on the book.

Example 8.9 Spring compression (cont.)

❸ EXECUTE PLAN From the graph in Figure 8.18, I arbitrarily choose 10 N as my force value and see that it causes a displacement of 2.0 mm. The spring constant is:

$$k = \frac{(F_{\text{by load on spring}})}{x - x_o} = \frac{+10\ \text{N}}{+5.0 \times 10^{-3}\ \text{m}} = 5 \times 10^{3}\ \text{N/m}$$



Example 8.9 Spring compression (cont.)

❸ EXECUTE PLAN Because the book is at rest, I know that $\vec{F}^{c}_{sb} + \vec{F}^{G}_{Eb} = \vec{0}$, and so $\vec{F}^{G}_{Eb} = -\vec{F}^{c}_{sb}$.

The force of gravity is given by -*mg*, and so I get for the force exerted by the spring on the book

$$F^{c}_{sbx} = -F^{G}_{Ebb} = -(-mg) = (1.2\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = +12\,\mathrm{N}$$

Example 8.9 Spring compression (cont.)

❸ EXECUTE PLAN Substituting this value and the spring constant into Eq. 8.20, I obtain for the displacement:

$$x - x_0 = \frac{-F^{c}_{sbx}}{k}$$

$$= \frac{-12\text{ N}}{5.0 \times 10^{3}\text{ N/m}}$$

$$= -2.4 \times 10^{-3}\text{ m.}$$

Example 8.9 Spring compression (cont.)

❹ EVALUATE RESULT It makes sense that the force exerted by the book on the spring is equal to the force exerted by Earth on the book ($\vec{F}^{G}_{\text{Eb}} = \vec{F}^{\text{c}}_{\text{bs}}$) because the spring has to keep the book from being pulled downward into the ground.

I can thus also obtain my answer by reading off the displacement from Figure 8.18 for a force of magnitude 12 N (the magnitude of the force of gravity exerted on the book). This procedure yields a value that is very close to the result I obtained. The negative sign in my answer indicates that the top end of the spring is displaced downward, as I expect.

Checkpoint 8.18

8.18 (*a*) Is a spring that has a large spring constant k stiffer or softer than a spring that has a small spring constant? (*b*) Which has a larger spring constant: steel or foam rubber?

(a) stiffer – larger k means the same displacement requires larger force

(b) steel – compresses less for the same load force, $k_{steel} > k_{foam}$

Section 8.10: Impulse

Section Goals

You will learn to

- Derive the **impulse equation** that relates change in momentum and force.
- Represent the impulse equation on **force-versus-time graphs**.

Section 8.10: Impulse

- In Chapter 4 we defined the change in momentum of a system as the impulse delivered to it.
- We will now determine the relation between impulse and force.
- First let us consider the case of a single object subject to a constant force:
  - Constant force will give the object a constant acceleration:

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

  - Multiplying both sides with m, we get

$$m\vec{a} = m\frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta \vec{p}}{\Delta t}$$

or

$$\Delta \vec{p} = m\vec{a}\ \Delta t$$

Section 8.10: Impulse

- Using $\Sigma\vec{F} = m\vec{a}$ in previous equation, we get

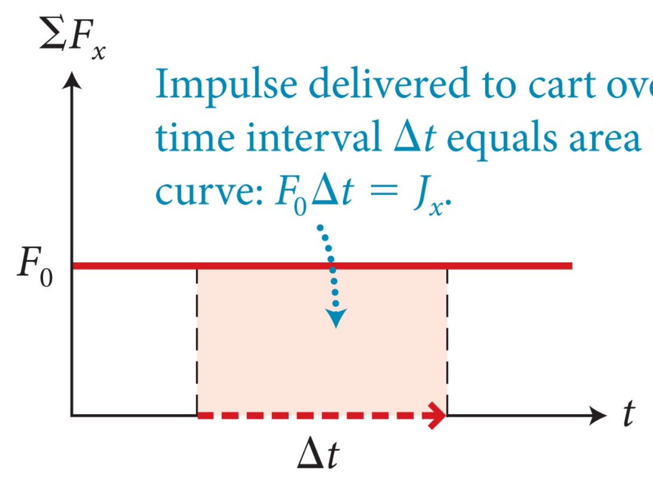
$$\Delta\vec{p} = \left(\sum\vec{F}\right)\Delta t \text{ (constant force)}$$

- Comparing this to the momentum law ($\Delta\vec{p} = \vec{J}$), we get the **impulse equation**:

$$\vec{J} = \left(\sum\vec{F}\right)\Delta t \text{ (constant force)}$$

- Impulse delivered by a constant force during a time interval Δt is shown in the area of the shaded rectangle.

(*a*) Impulse delivered by constant force exerted on object



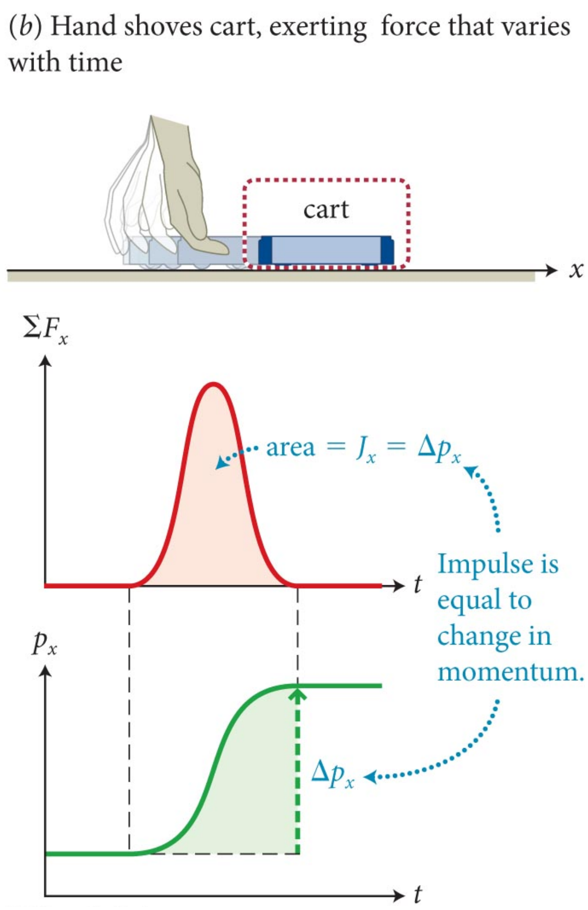
Section 8.10: Impulse

- The relationship between impulse and area under the $F_x(t)$ curve holds for a time-varying force.
- Thus, the impulse delivered by a time-varying force is

$$\Delta\vec{p} = \vec{J} = \int_{t_i}^{t_f} \sum \vec{F}(t)\, dt \text{ (time-varying force)}$$

- If the object is in translational equilibrium, then $\sum F=0$ and we get

$$\Delta\vec{p} = \vec{0} \text{ (translational equilibrium)}$$

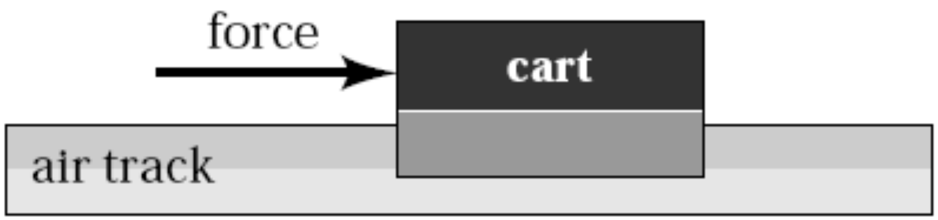


Section 8.10
Question 8

A constant force is exerted on a cart that is initially at rest on an air track. Friction between the cart and the track is negligible. The force acts for a short time interval and gives the cart a certain final speed.

To reach the same final speed with a force that is only half as big, the force must be exerted on the cart for a time interval

1. four times as long as
2. twice as long as
3. equal to
4. half as long as
5. a quarter of that for the stronger force.



Section 8.10
Question 8

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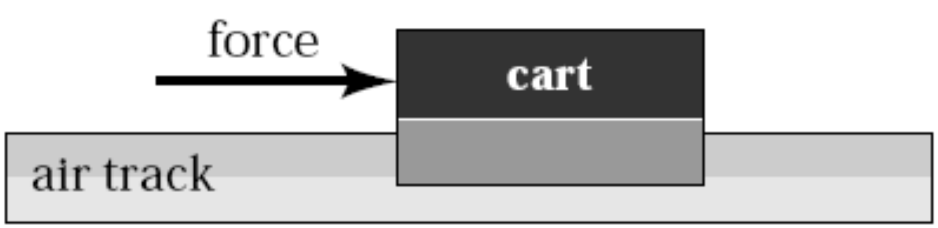
To reach the same final speed with a force that is only half as big, the force must be exerted on the cart for a time interval

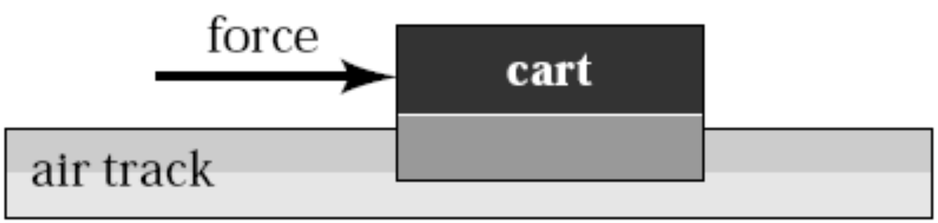
1. four times as long as
2. twice as long – same impulse with half the force = double time ✔
3. equal to
4. half as long as
5. a quarter of that for the stronger force.



Section 8.10
Question 9

A constant force is exerted for a short time interval on a cart that is initially at rest on an air track. This force gives the cart a certain final speed. The same force is exerted for the same length of time on another cart, also initially at rest, that has twice the mass of the first one. The final speed of the heavier cart is

1. one-fourth
2. four times
3. half
4. double
5. the same as
   that of the lighter cart.



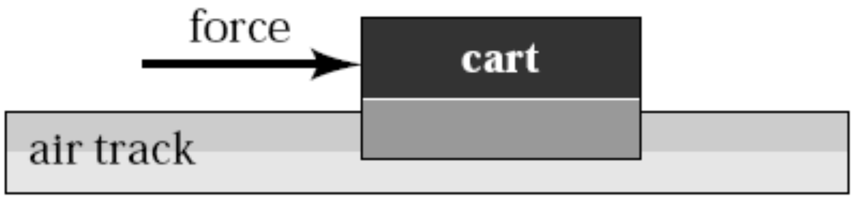
Section 8.10
Clicker Question 9

A constant force is exerted for a short time interval on a cart that is initially at rest on an air track. This force gives the cart a certain final speed. The same force is exerted for the same length of time on another cart, also initially at rest, that has twice the mass of the first one. The final speed of the heavier cart is

1. one-fourth
2. four times
3. ✔ half - same impulse but twice the mass, so half Δv
4. double
5. the same as
   that of the lighter cart.

Section 8.10
Question 10

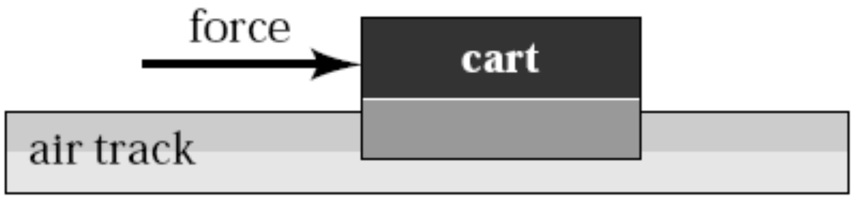
A constant force is exerted for a short time interval on a cart that is initially at rest on an air track. This force gives the cart a certain final speed. Suppose we repeat the experiment but, instead of starting from rest, the cart is already moving with constant speed in the direction of the force at the moment we begin to apply the force. After we exert the same constant force for the same short time interval, the increase in the cart's speed



1. is equal to two times its initial speed.
2. is equal to the square of its initial speed.
3. is equal to four times its initial speed.
4. is the same as when it started from rest.
5. cannot be determined from the information provided.

Section 8.10
Question 10

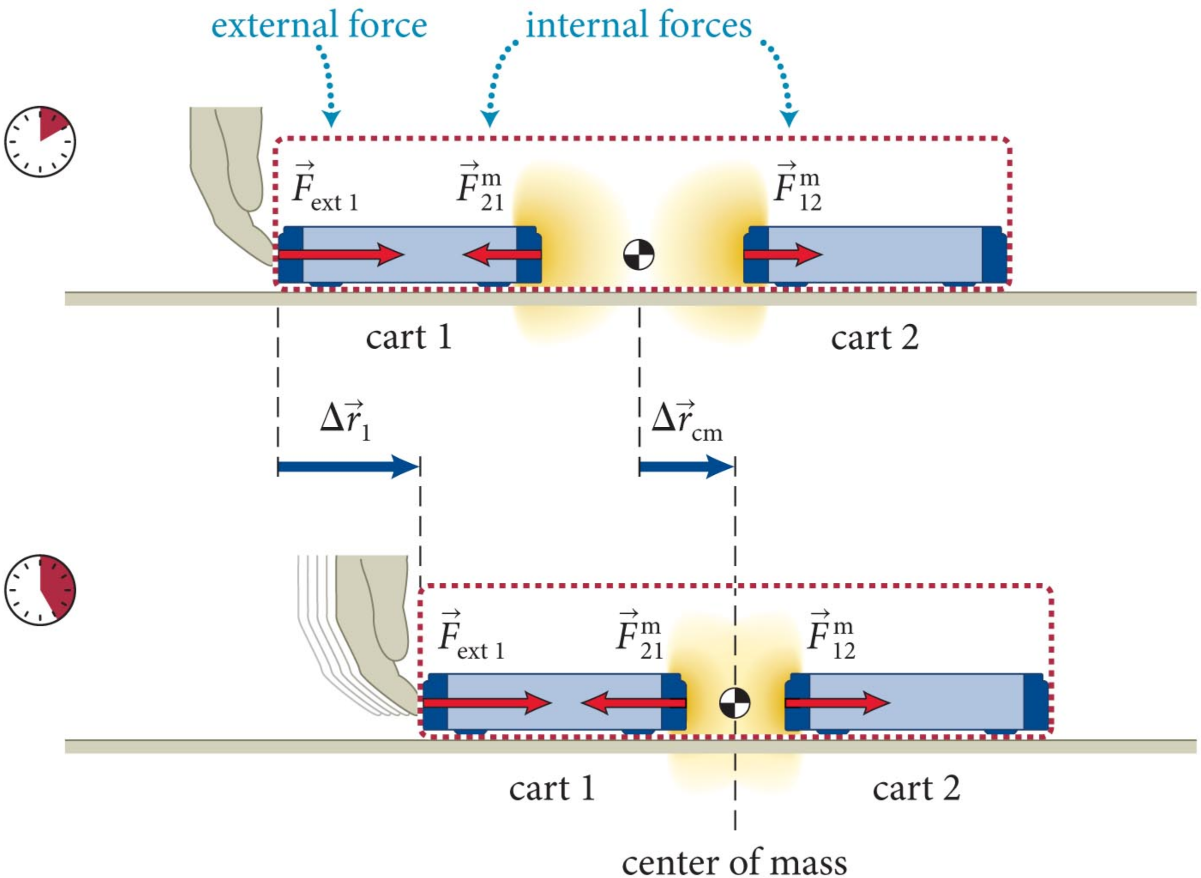
A constant force is exerted for a short time interval on a cart that is initially at rest on an air track. This force gives the cart a certain final speed. Suppose we repeat the experiment but, instead of starting from rest, the cart is already moving with constant speed in the direction of the force at the moment we begin to apply the force. After we exert the same constant force for the same short time interval, the increase in the cart's speed



1. is equal to two times its initial speed.
2. is equal to the square of its initial speed.
3. is equal to four times its initial speed.
4. ✔ is the same as when it started from rest – same impulse given
5. cannot be determined from the information provided.

Section 8.11: Systems of two interacting objects

- Let us now look at a system of two interacting objects.
  - In the figure below, two carts equipped with repelling magnets move on a track while cart 1 gets a push.



Section 8.11: Systems of two interacting objects

- The momentum of the two carts is $\vec{p} = \vec{p}_1 + \vec{p}_2$.
- Differentiating with respect to time gives

$$\frac{d\vec{p}}{dt} = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = \sum \vec{F}_1 + \sum \vec{F}_2$$

- The vector sum of forces exerted on cart 1 is

$$\sum \vec{F}_1 = \vec{F}_{\text{ext 1}} + \vec{F}_{21}^{\text{m}}$$

- The vector sum of forces on cart 2 is

$$\sum \vec{F}_2 = \vec{F}_{12}^{\text{m}}$$

Because $\vec{F}_{12}^{\text{m}}$ and $\vec{F}_{21}^{\text{m}}$ form an interacting pair, $\vec{F}_{12}^{\text{m}} = -\vec{F}_{21}^{\text{m}}$, and so

$$\frac{d\vec{p}}{dt} = \vec{F}_{\text{ext 1}}$$

Section 8.11: Systems of two interacting objects

- Using the definition of center-of-mass velocity, for the system of both carts together, $\vec{p} = m\vec{\upsilon}_{\text{cm}}$.
- We can therefore write

$$\frac{d\vec{p}}{dt} = \frac{d(m\vec{\upsilon}_{\text{cm}})}{dt} = m\frac{d\vec{\upsilon}_{\text{cm}}}{dt} \equiv m\vec{a}_{\text{cm}}$$

- Hence we can write Equation 8.33 as simply

$$\vec{F}_{\text{ext 1}} = m\vec{a}_{\text{cm}}$$

or

$$\vec{a}_{\text{cm}} = \frac{\vec{F}_{\text{ext 1}}}{m}$$

- **The center of mass of a two-object system accelerates as though both objects were located at the center of mass and the external force were exerted at that point.**

Section 8.12: Systems of many interacting objects

- Let us now look at a system with more than two objects.
- As in the case of the two-object system we can write

$$\frac{d\vec{p}}{dt} = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} + \cdots = \sum \vec{F}_1 + \sum \vec{F}_2 + \cdots$$

- The right-hand side of this equation consists of a sum of external and internal forces:

$$\sum \vec{F}_1 = \sum \vec{F}_{\text{ext 1}} + \sum \vec{F}_{\text{int 1}}$$

- All the internal forces in the previous equation cancel out because they come in pairs ($F_{ij} = -F_{ji}$), and we get

$$\frac{d\vec{p}}{dt} = \sum (\vec{F}_{\text{ext 1}} + \vec{F}_{\text{ext 2}} + \cdots) \equiv \sum \vec{F}_{\text{ext}}$$

Cancellation of internal forces

There is another way to see that internal forces always add to zero. We consider here a system consisting of three particles, but the proof can be extended to any number of particles.

Section 8.12: Systems of many interacting objects

Cancellation of internal forces

Each particle is subject to forces from the other two particles and to an external force:

$$\Sigma\vec{F}_1 = \Sigma\vec{F}_{\text{ext 1}} + \vec{F}_{21} + \vec{F}_{31}$$

$$\Sigma\vec{F}_2 = \Sigma\vec{F}_{\text{ext 2}} + \vec{F}_{12} + \vec{F}_{32}$$

$$\Sigma\vec{F}_3 = \Sigma\vec{F}_{\text{ext 3}} + \vec{F}_{13} + \vec{F}_{23}.$$

Section 8.12: Systems of many interacting objects

Cancellation of internal forces

Summing up all the forces, we get

$$\Sigma\vec{F}_1 + \Sigma\vec{F}_2 + \Sigma\vec{F}_3$$

$$= \Sigma\vec{F}_{\text{ext }1} + \Sigma\vec{F}_{\text{ext }2} + \Sigma\vec{F}_{\text{ext }3} + \left(\vec{F}_{12} + \vec{F}_{21}\right)$$

$$+\left(\vec{F}_{13} + \vec{F}_{31}\right) + \left(\vec{F}_{32} + \vec{F}_{23}\right). \qquad (1)$$

Section 8.12: Systems of many interacting objects

Cancellation of internal forces

Remember, however, that two interacting particles exert on each other forces that are equal in magnitude but opposite in direction (Eq. 8.15). This means that the following equalities must hold:

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\vec{F}_{13} = -\vec{F}_{31}$$

$$\vec{F}_{32} = -\vec{F}_{23}.$$

Section 8.12: Systems of many interacting objects

Cancellation of internal forces

So each term in parentheses in Eq. 1 vanishes, and only the external forces remain:

$$\Sigma\vec{F}_1 + \Sigma\vec{F}_2 + \Sigma\vec{F}_3$$
$$= \Sigma\vec{F}_{\text{ext}\,1} + \Sigma\vec{F}_{\text{ext}\,2} + \Sigma\vec{F}_{\text{ext}\,3}.$$

Section 8.12: Systems of many interacting objects

- Using the definition of center-of-mass velocity, we get

$$\frac{d\vec{p}}{dt} = \frac{d(m\vec{\upsilon}_{\rm cm})}{dt} = m\frac{d\vec{\upsilon}_{\rm cm}}{dt} \equiv m\vec{a}_{\rm cm}$$

- Combining the two previous equations, we obtain

$$\Sigma\vec{F}_{\rm ext} = m\vec{a}_{\rm cm}$$

- The equation of motion for the center of mass of a nonisolated system is

$$\vec{a}_{\rm cm} = \frac{\Sigma\vec{F}_{\rm ext}}{m}$$

Chapter 8: Self-Quiz #1

(*a*) Two forces, one twice the magnitude of the other, are exerted on identical objects during the same time interval. Which force causes the greater change in momentum?

(*b*) Two identical forces are exerted on identical objects, one during a time interval twice as long as the other. Which force causes the greater change in momentum?

Chapter 8: Self-Quiz #1

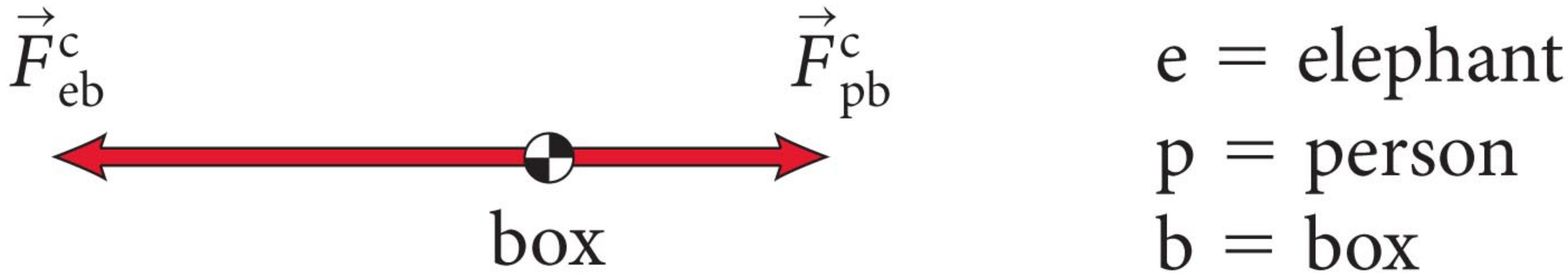
Answer

(*a*) The larger force causes the greater change in momentum because force and change in momentum are proportional to each other over a given time interval.

(*b*) The force exerted over the longer time interval causes the greater change in momentum because time interval and change in momentum are proportional to each other for a constant force.

Chapter 8: Self-Quiz #2

The free-body diagram of a box subjected to forces is shown in the figure. Because we are concerned about only the horizontal motion, the forces in the vertical direction have been omitted. (*a*) What is the direction of the change in the momentum of the box? (*b*) If the box is at rest before the forces shown in the diagram are exerted on it, what is the direction of its momentum once the forces are exerted?



Chapter 8: Self-Quiz #2

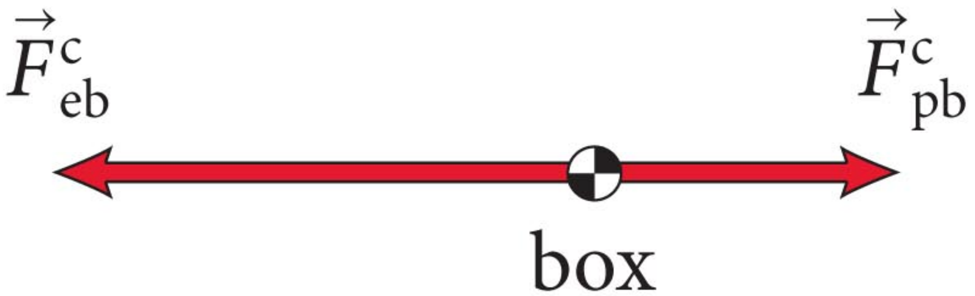
Answer

(*a*) The vector sum of the forces exerted on the box points to the left because the force exerted on the box by the elephant is larger than the force exerted on the box by the person. The change in momentum is to the left, in the same direction as the vector sum of the forces.

(*b*) The change in momentum is equal to the final momentum minus the initial momentum of zero. Therefore, the final momentum of the box must be to the left.

Chapter 8: Self-Quiz #3

For the free-body diagram in the previous question, list the objects in the environment of the box.



e = elephant
p = person
b = box

Answer

The objects in the environment of the box are an elephant and a person. If you also consider the vertical direction, you need to include the ground and Earth as being in the environment of the box.

Chapter 8: Self-Quiz #4

A block of wood rests on a shelf, and a second block of wood rests on top of the first.

(*a*) List the objects in the environment of the top block.

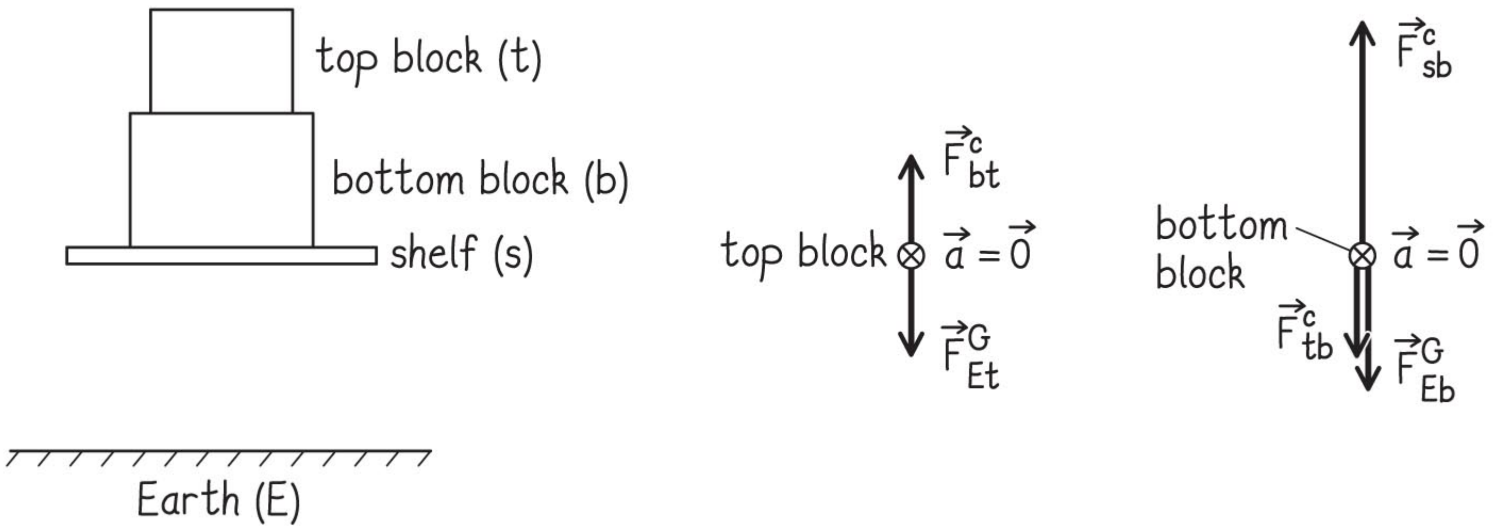
(*b*) List the objects in the environment of the bottom block.

(*c*) Draw a free-body diagram for each block.

Chapter 8: Self-Quiz #4

Answer

(*a*) The objects in the environment of the top block are the bottom block and Earth. The shelf is not in contact with the top block and so is not part of the top block's environment. (*b*) The objects in the environment of the bottom block are the shelf, the top block, and Earth. (*c*) See the figure. Earth pulls downward on each block. The surface below each block pushes upward. If the bottom block pushes upward on the top block, the top block must push down on the bottom block.



Chapter 8: Self-Quiz #5

Complete the following sentences, applying the reciprocity of forces. (Example: If I push down on the seat of my chair, *the seat of my chair pushes up on me*.)

If I push to the right on the wall, . . .

If I pull to the right on a spring, . . .

If the floor pushes up on my feet, . . .

If Earth pulls down on me, . . .

Chapter 8: Self-Quiz #5

Answer

For each condition, swap the nouns and reverse the given direction.

If I push to the right on the wall, *the wall pushes to the left on me*.

If I pull to the right on a spring, *the spring pulls to the left on me*.

If the floor pushes up on my feet, *my feet push down on the floor*.

If Earth pulls down on me, *I pull up on Earth*.*

*Many people find this last statement difficult to accept and are inclined to answer: If Earth pulls down on me, the ground pushes up on me. This statement, however, includes three objects (ground, Earth, me), not just the two required by the reciprocity of forces.

Chapter 8: Summary

Concepts: Characteristics of forces

- When an object participates in one interaction only, the **force** exerted on the object is given by the time rate of change in the object's momentum.

- A **contact force** is a force that one object exerts on another object only when the two objects are in physical contact.

Chapter 8: Summary

Concepts: Characteristics of forces

- A **field force** is a force (such as gravity) that one object exerts on another object without the requirement that the two objects be in physical contact.
- When two objects interact, the forces they exert on each other form an **interaction pair,** and these forces have equal magnitudes but opposite directions.

Quantitative Tools: Characteristics of forces

- The SI unit of force is the **newton** (N):

$$1\ \text{N} \equiv 1\ \text{kg} \cdot \text{m/s}^2$$

- For an interaction pair of forces,

$$\vec{F}_{12} = -\vec{F}_{21}$$

Concepts: Some important forces

- A taut, flexible object (such as a spring, rope, or thread), when subjected to equal and opposite *tensile forces* applied at either end, experiences along its length a *stress* called **tension**. If the object is very light, the tension is the same everywhere in it.

- **Hooke's law** relates the force that a spring exerts on a load to the distance the spring is stretched (or compressed) from its relaxed position.

Quantitative Tools: Some important forces

- The x component of the force of gravity exerted on an object of inertia m near Earth's surface is

$$F^{G}_{\text{Eo}\,x} = -mg$$

where the minus sign indicates that the force is directed downward.

- **Hooke's law:** If a spring is stretched (or compressed) by a *small* distance $x - x_0$ from its unstretched length x_0, the x component of the force it exerts on the load is

$$(F_{\text{by spring on load}})_x = -k(x - x_0)$$

where k is called the **spring constant** of the spring.

Chapter 8: Summary

Concepts: Effects of force

- The vector sum of the forces exerted on an object is equal to the time rate of change of the momentum of the object.
- The **equation of motion** for an object relates the object's acceleration to the vector sum of the forces exerted on it.
- **Newton's laws of motion** describe the effects forces have on the motion of objects.
- If an object is at rest or moving with constant velocity, it is in **translational equilibrium.** In this case, the vector sum of the forces exerted on it is equal to zero.
- A **free-body diagram** for an object is a sketch representing the object by a dot and showing all the forces exerted *on* it.

Chapter 8: Summary

Concepts: Effects of force

- The vector sum of the forces exerted on an object is equal to the time rate of change of the momentum of the object.
- The **equation of motion** for an object relates the object’s acceleration to the vector sum of the forces exerted on it.

Chapter 8: Summary

Concepts: Effects of force

- **Newton's laws of motion** describe the effects forces have on the motion of objects.
- If an object is at rest or moving with constant velocity, it is in **translational equilibrium**. In this case, the vector sum of the forces exerted on it is equal to zero.
- A **free-body diagram** for an object is a sketch representing the object by a dot and showing all the forces exerted *on* it.

Chapter 8: Summary

Quantitative Tools: Effects of forces

- Vector sum of forces exerted on an object:

$$\Sigma\vec{F} \equiv \frac{d\vec{p}}{dt}$$

- Equation of motion:

$$\vec{a} = \frac{\Sigma\vec{F}}{m}$$

- If the inertia m of an object is constant, Newton's second law is usually written as

$$\Sigma\vec{F} = m\vec{a}$$

- For two interacting objects, 1 and 2, Newton's third law of motion states that

$$\vec{F}_{12} = -\vec{F}_{21}$$

Chapter 8: Summary

Concepts: Impulse

- The **impulse** produced by forces exerted on an object is the product of the vector sum of the forces and the time interval over which the forces are exerted. The impulse delivered to the object is also equal to the change in its momentum.

Chapter 8: Summary

Quantitative Tools: Impulse

- For a constant force,

$$\Delta\vec{p} = \vec{J} = \left(\Sigma\vec{F}\right)\Delta t$$

- For a time-varying force,

$$\Delta\vec{p} = \vec{J} = \int_{t_1}^{t_f} \Sigma\vec{F}(t)\,dt$$

Concepts: System of interacting objects

- The center of mass of a system of objects accelerates as though all the objects were located at the center of mass and the external force were applied at that location.

Chapter 8: Summary

Quantitative Tools: System of interacting objects

- Acceleration of a system of objects:

$$\vec{a}_{\text{cm}} = \frac{\Sigma \vec{F}_{\text{ext}}}{m}$$

- Internal forces between interacting objects cancel out!
- ***Net*** motion is only due to external forces