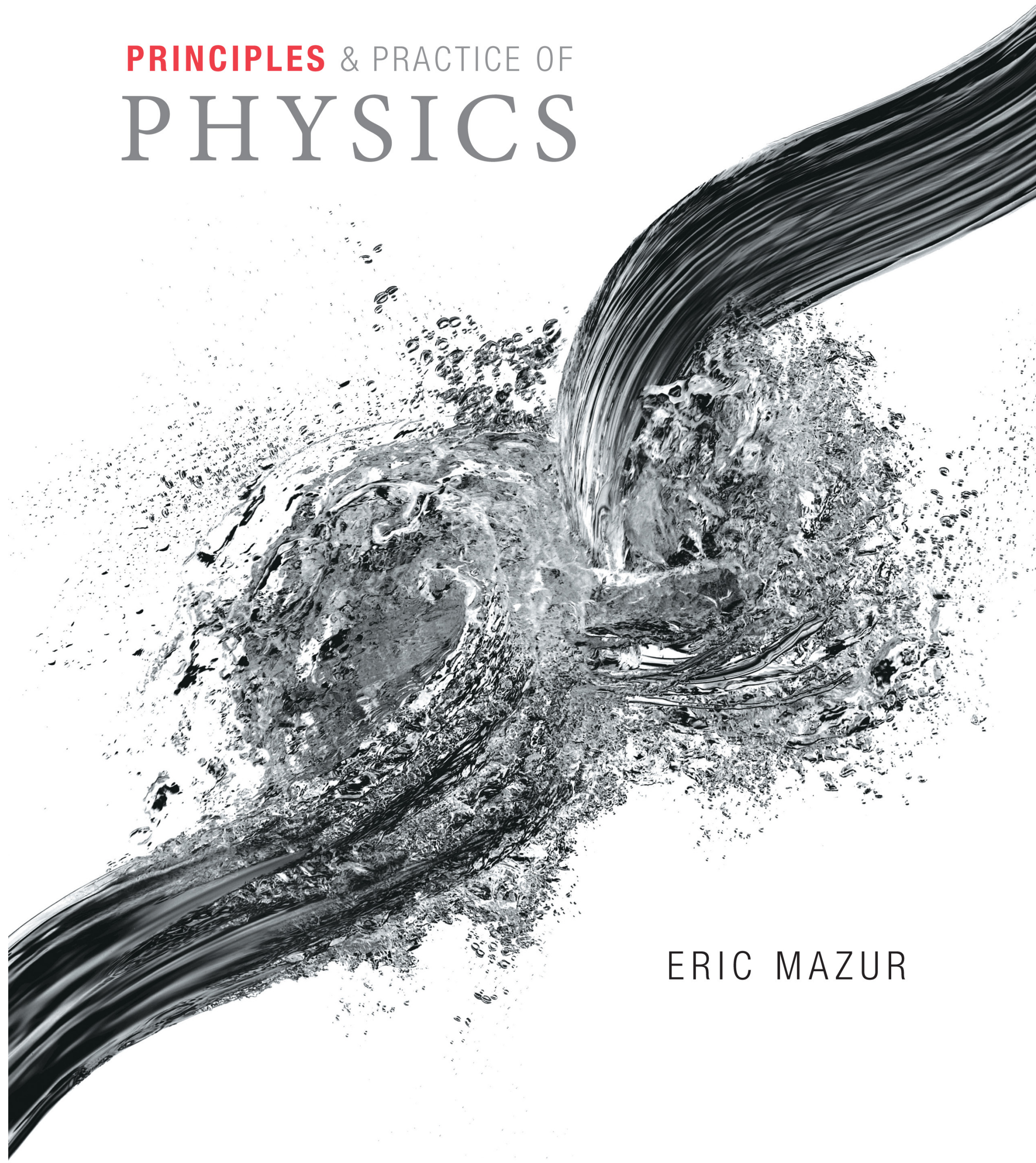
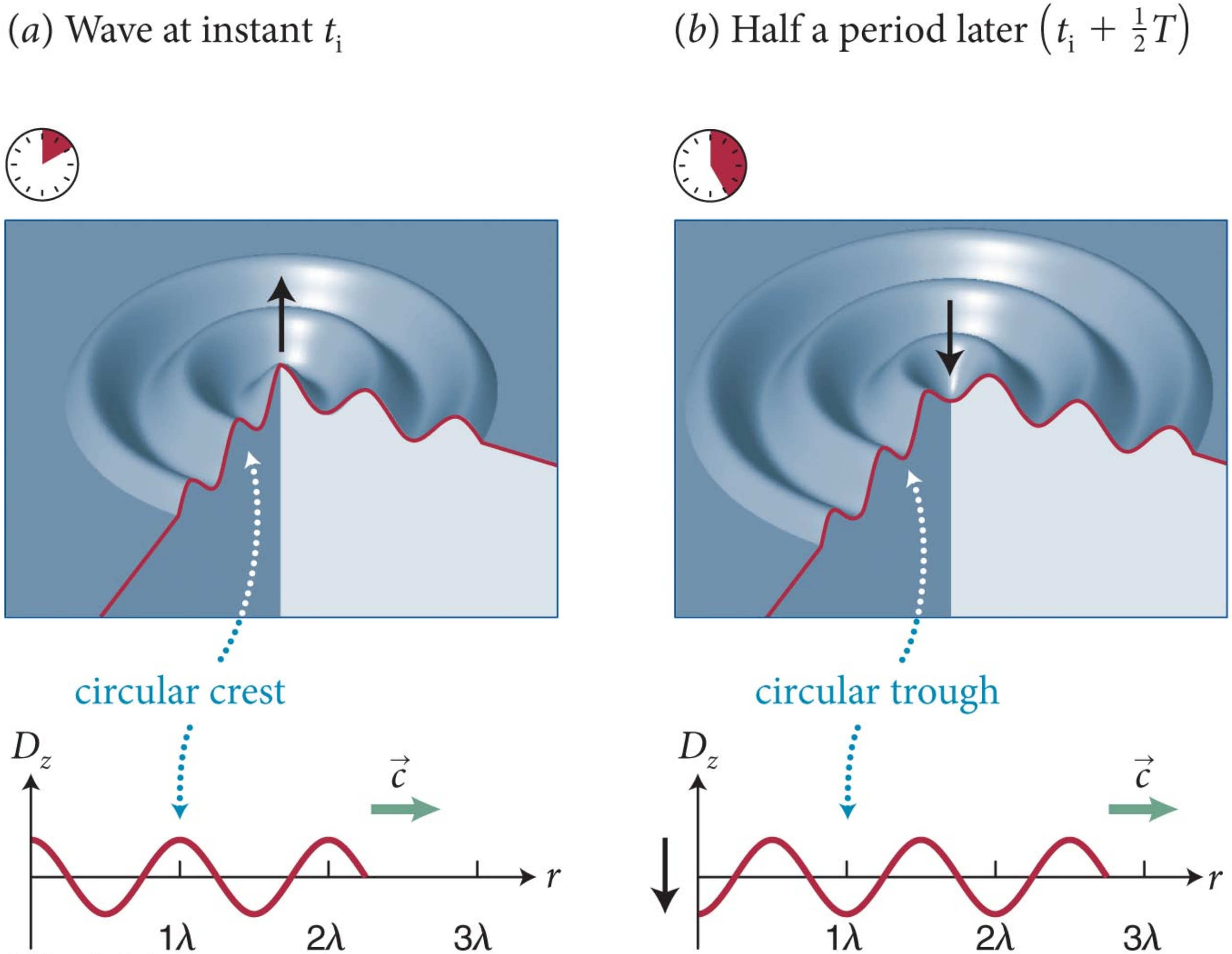
Chapter 17 Waves in Two and Three Dimensions



Concepts

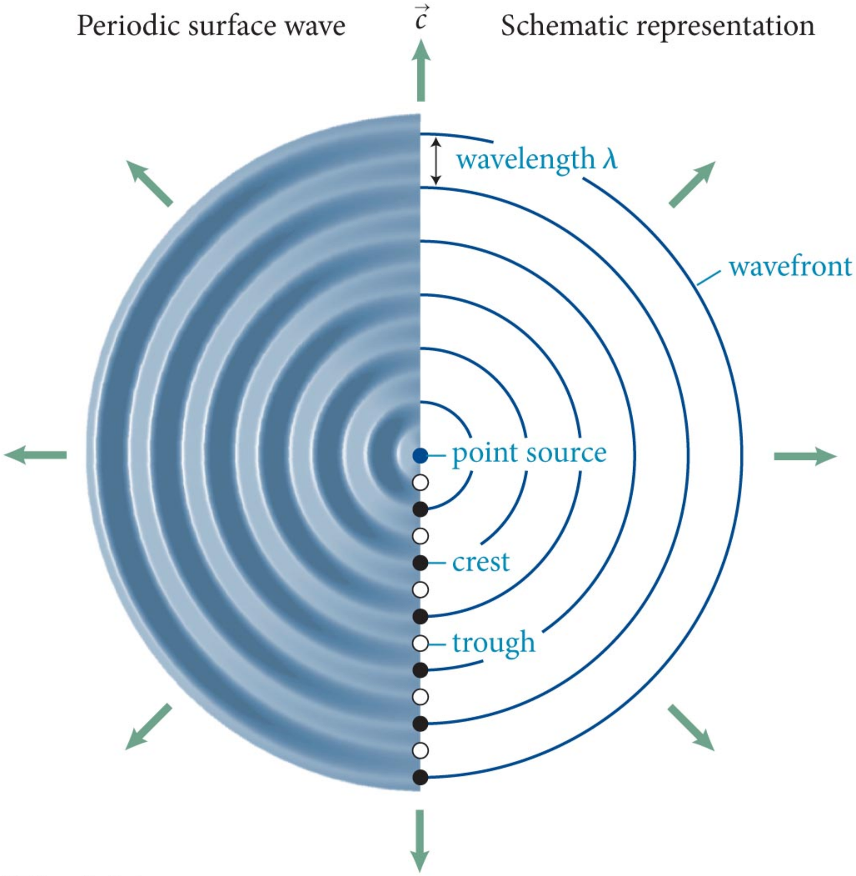
Section 17.1: Wavefronts

- The figure shows cutaway views of a periodic **surface wave** at two instants that are half a period apart.



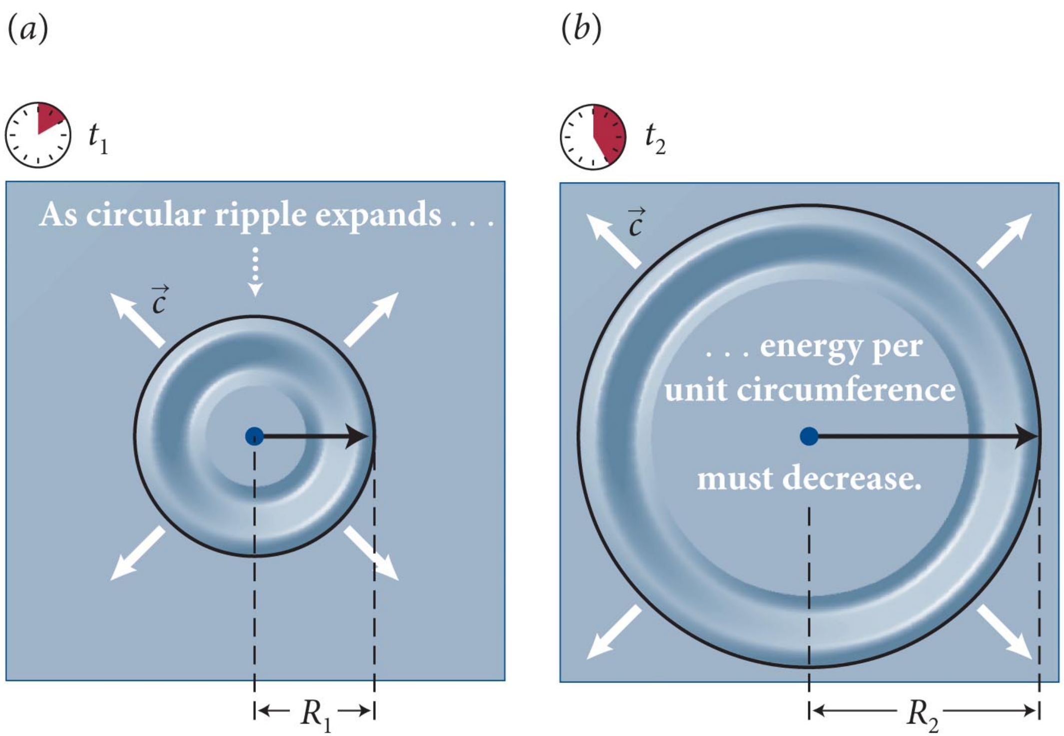
Section 17.1: Wavefronts

- When the source of the wavefront can be localized to a single point, the source is said to be a **point source**.
- The figure shows a periodic surface wave spreading out from a point source.
- The curves (or surfaces) in the medium on which all points have the same phase is called a **wavefront**.



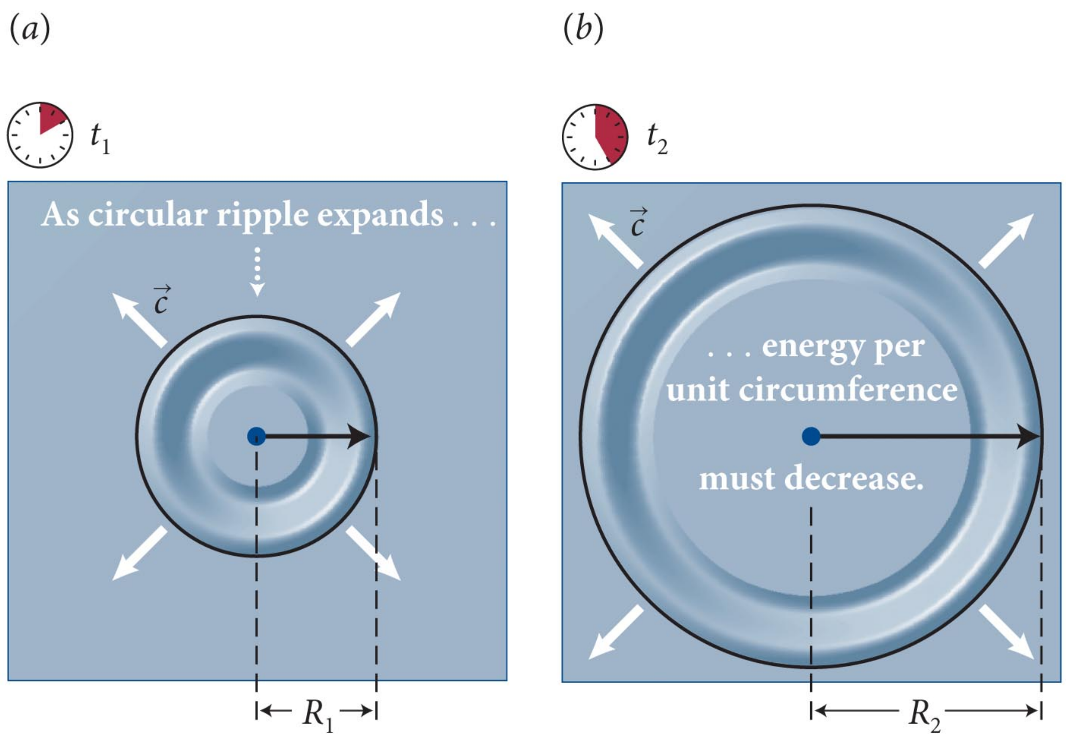
Section 17.1: Wavefronts

- Consider the figure.
- If we assume that there is no energy dissipation, then there is no loss of energy as the wave moves outward.
- As the wavefront spreads, the circumference increases, and hence the energy per unit length decreases.



Checkpoint 17.1

17.1 Let $t_2 = 2t_1$ in Figure 17.3. (*a*) How does R_1 compare with R_2? (*b*) If the energy in the wave is E and there is no dissipation of energy, what is the energy per unit length along the circumference at R_1? At R_2? (*c*) How does the energy per unit length along a wavefront vary with radial distance r?



Checkpoint 17.1

17.1 (a) The wave speed c is constant, so in twice the time it covers twice the distance, $R_2 = 2R_1$

(b) Energy per unit length?

At 1: $E_1 = E/2\pi R_1$.

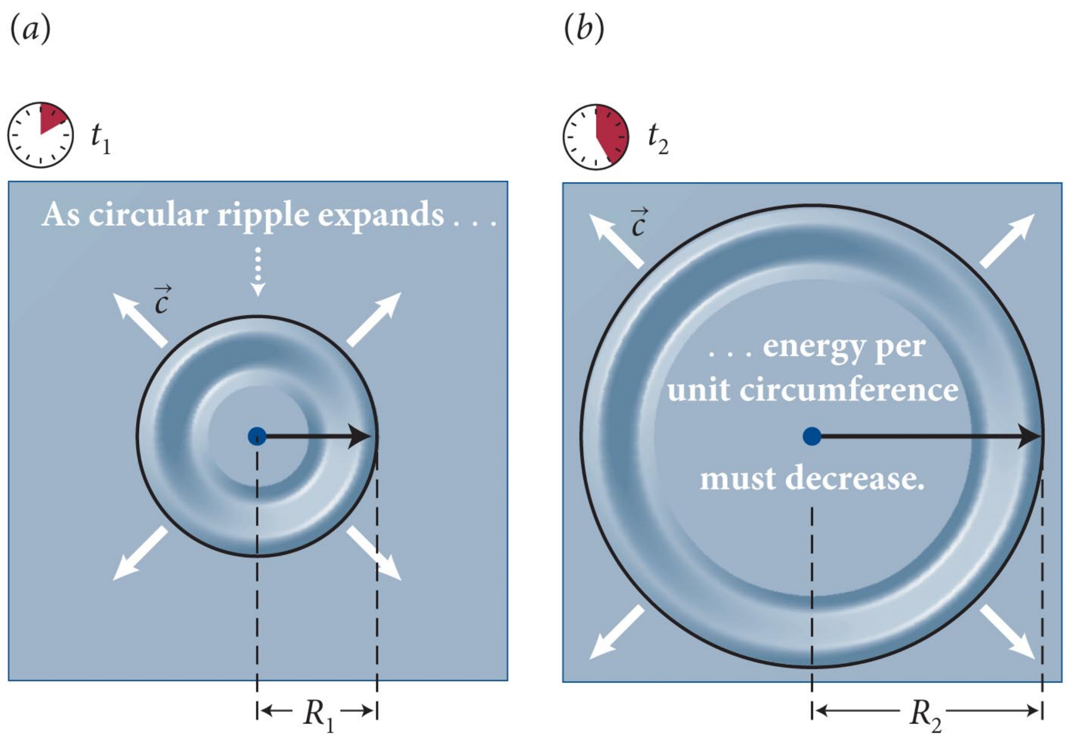
At 2: If the radius doubles, so does the circumference. Now at point 2, same energy but double the circumference, so

$E_2 = E/2\pi R_2 = E/4\pi R_1 = E_1/2$

(c) Energy per unit length goes as $1/r$ since circumference increases with r

Section 17.1: Wavefronts

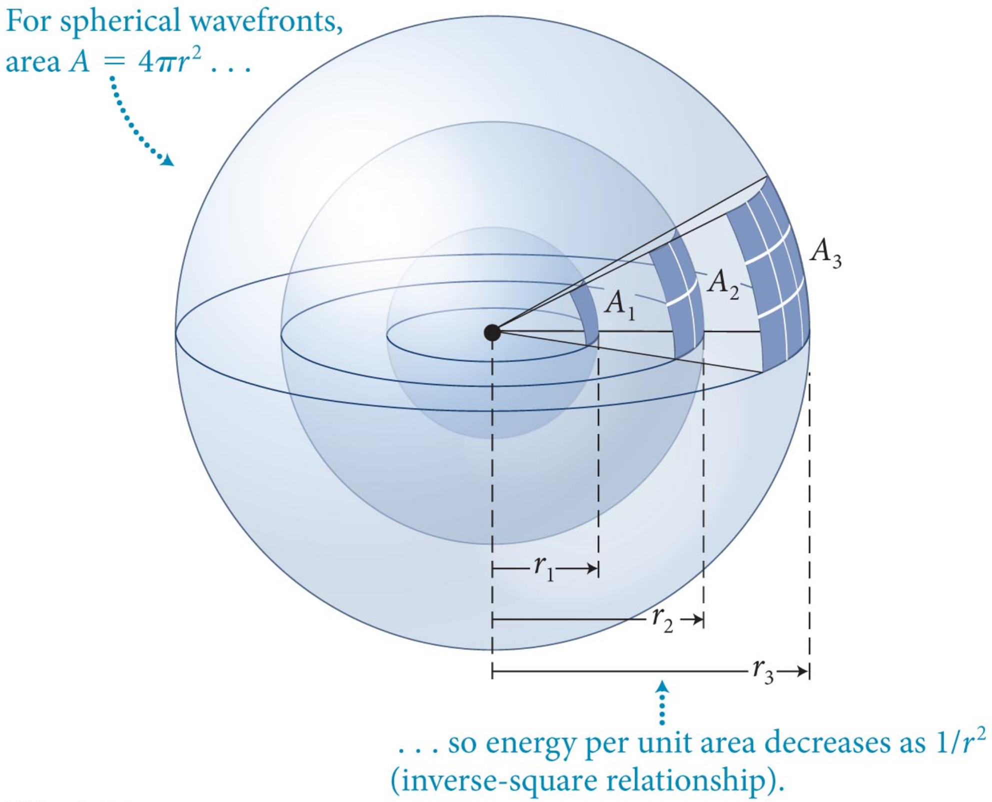
- The expansion of the circular wavefronts causes the energy per unit length along the wavefront to decrease as $1/r$.
- In Chapter 16 we saw $E_\lambda = \frac{1}{2}(\mu\lambda)\omega^2 A^2$ (Eq. 16.41),
- Therefore, it follows that for waves in two dimensions $A \sim 1/\sqrt{r}$.
  - *e.g., water waves,* $y(r,t) \sim \sin(\omega t)/\sqrt{r}$



Section 17.1: Wavefronts

- The waves that spread out in 3 dimensions are called **spherical waves**.
- The energy carried by a spherical wavefront is spread out over a spherical area of $4\pi r^2$.
- So, for waves in three dimensions, $E \sim 1/r^2$, and therefore $A \sim 1/r$.
- *e.g. sound waves*

$$y(r,t) \sim \sin(\omega t)/r$$



Example 17.1 Ripple amplitude

The amplitude of a surface wave for which $\lambda = 0.050$ m is 5.0 mm at a distance of 1.0 m from a point source. What is the amplitude of the wave

(a) 10 m from the source and

(b) 100 m from the source

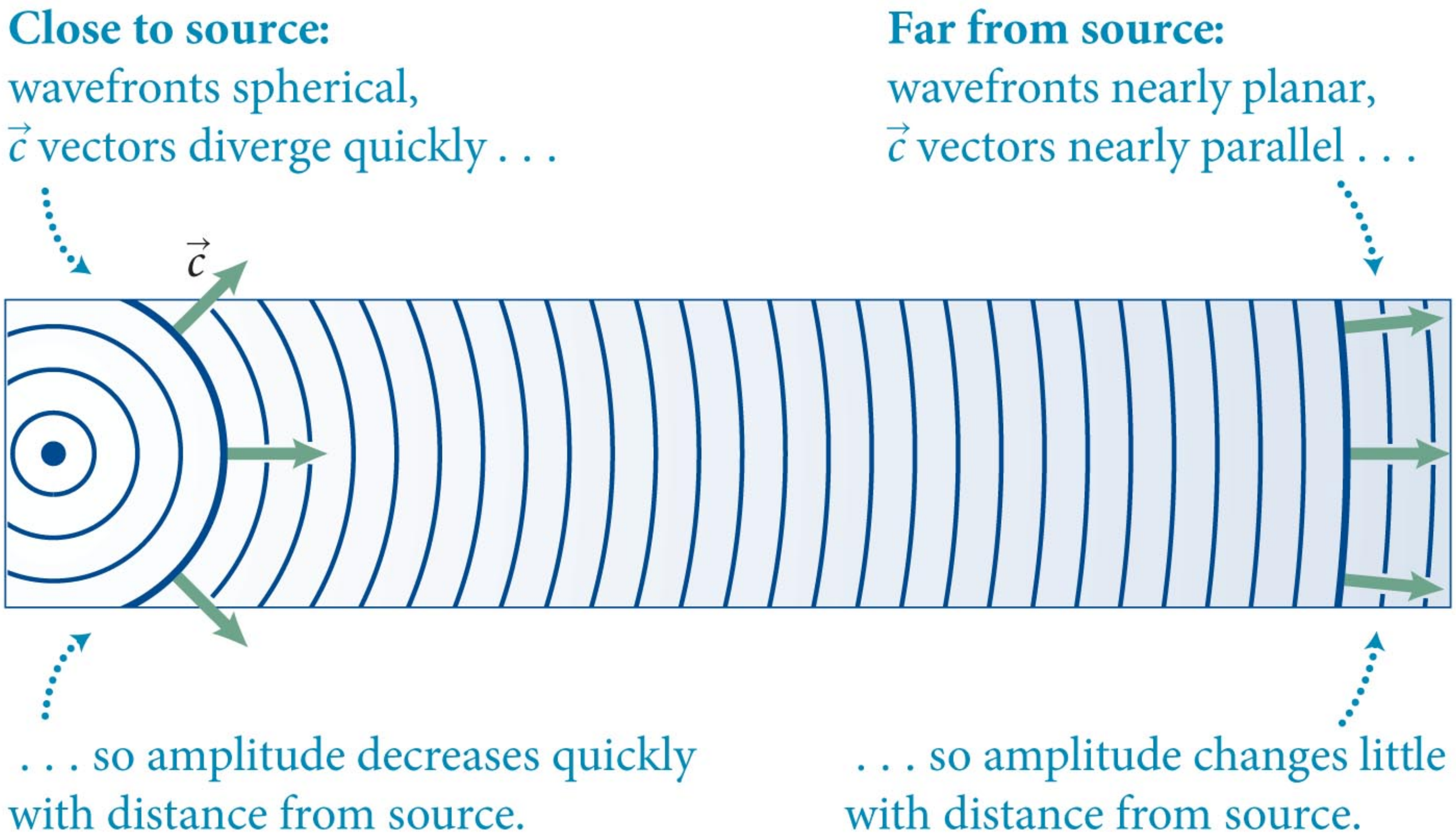
From 1 m to 10 m, factor 3 decrease

goes as $\sqrt{(r_1/r_2)} \sim \sqrt{10} \sim 3$.

F*rom 10 m to 100 m also a factor of three*, even though distance is 10 times larger.

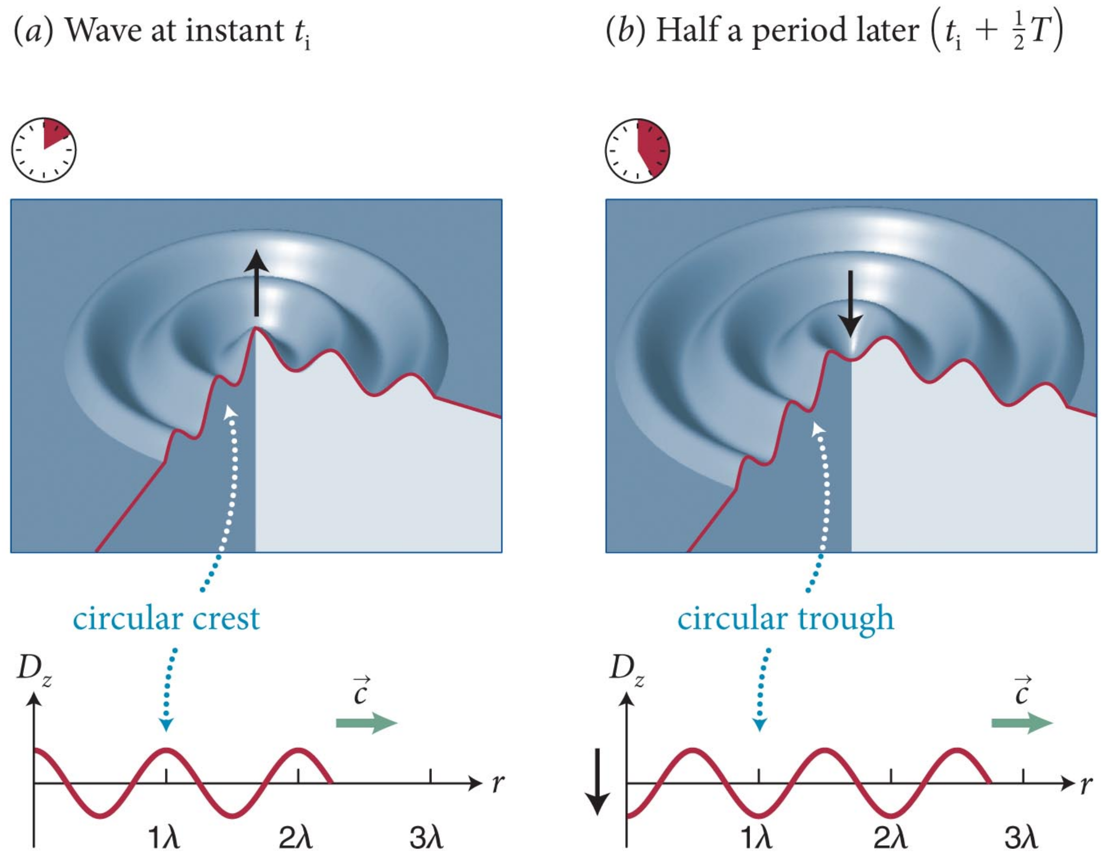
Section 17.1: Wavefronts

- Far from a point source, the spherical wavefronts essentially become a two-dimensional flat wavefront called a **planar wavefront**.



Checkpoint 17.2

17.2 Notice that in the views of the surface wave in Figure 17.1 the amplitude does not decrease with increasing radial distance *r*. How could such waves be generated?



Checkpoint 17.2

17.2 Would work to *decrease* the source amplitude as a function of time.

First wave out is diminished when the second one is created, so make the second one smaller to compensate.

By the time the third one comes out, both the first and second are smaller (but still equal), so make the third one even smaller …

Makes it uniform over space, but not in time – uniformly decreases over entire wave pattern.

Section 17.1
Question 1

Which of the following factors plays a role in how much a wave's amplitude decreases as the wave travels away from its source? Answer all that apply.

1. Dissipation of the wave's energy
2. Dimensionality of the wave
3. Destructive interference by waves created by other sources

Section 17.1
Question 1

Which of the following factors plays a role in how much a wave's amplitude decreases as the wave travels away from its source? Answer all that apply.

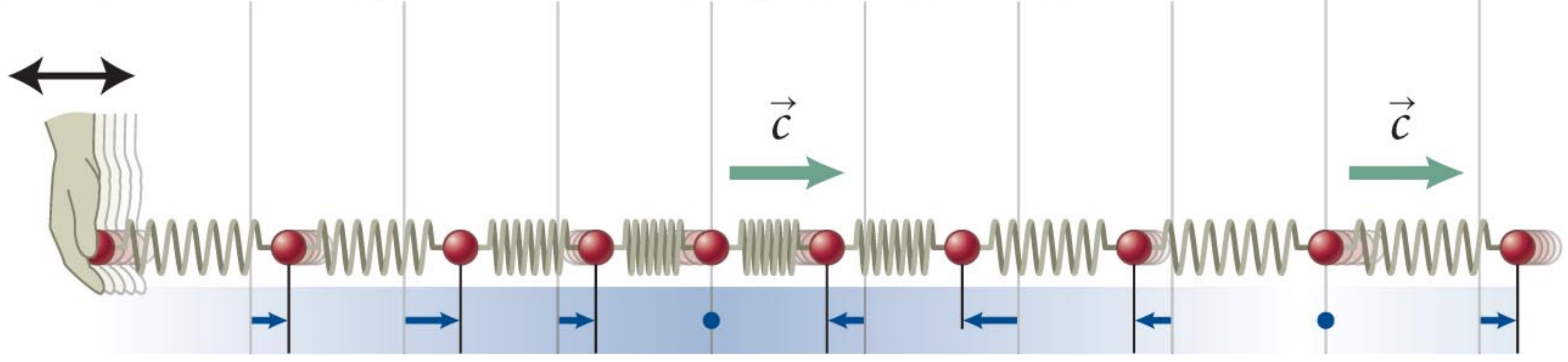
✔ 1. Dissipation of the wave's energy
✔ 2. Dimensionality of the wave
3. Destructive interference by waves created by other sources (don't lose any energy/amplitude this way!)

Section Goals

You will learn to

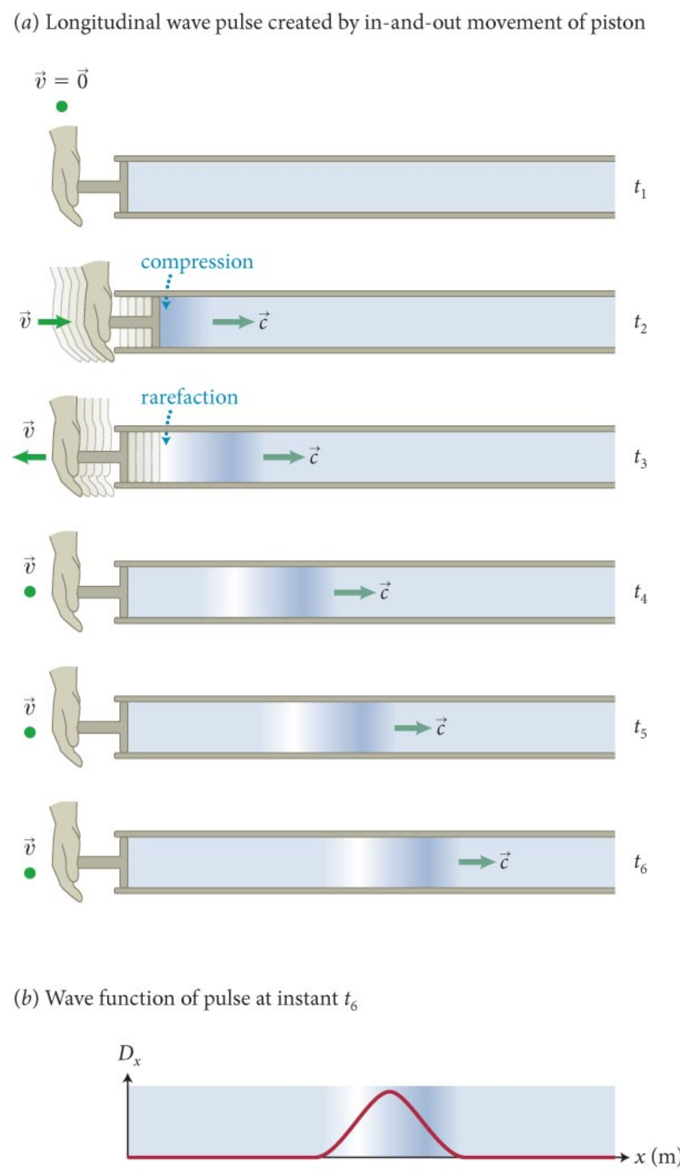
- Define the **physical characteristics** of sound.
- Represent sound graphically.

(*b*) Snapshot of longitudinal wave propagating along chain



Section 17.2: Sound

- Longitudinal waves propagating through any kind of material is what we call **sound**.
- The human ear can detect longitudinal waves at frequencies from 20 Hz to 20 kHz.
- Sound waves consist of an alternating series of *compressions* and *rarefactions*.
- For dry air at 20C, the speed of sound is ~343 m/s.



Exercise 17.2 Wavelength of audible sound

Given that the speed of sound waves in dry air is 343 m/s, determine the wavelengths at the lower and upper ends of the audible frequency range (20 Hz–20 kHz).

Exercise 17.2 Wavelength of audible sound (cont.)

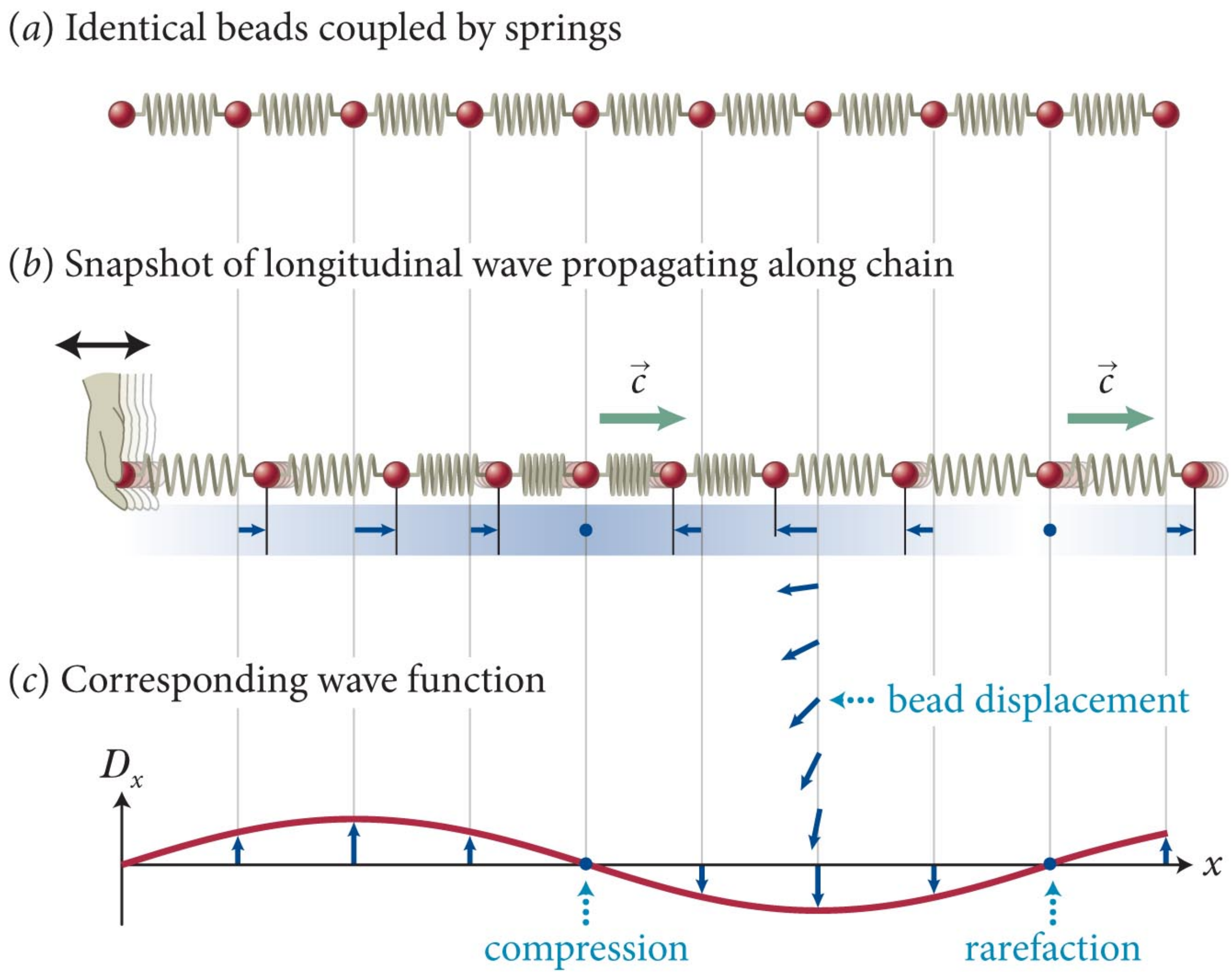
SOLUTION The wavelength is equal to the distance traveled in one period. At 20 Hz, the period is 1/(20 Hz) = 1/(20 s^{-1}) = 0.050 s, so the wavelength is (343 m/s)(0.050 s) = 17 m. ✔

The period of a wave of 20 kHz is 1/(20,000 Hz) = 5 × 10^{-5} s, so the wavelength is (343 m/s)(5.0 × 10^{-5} s) = 17 mm. ✔

Conveniently, the size of everyday objects …

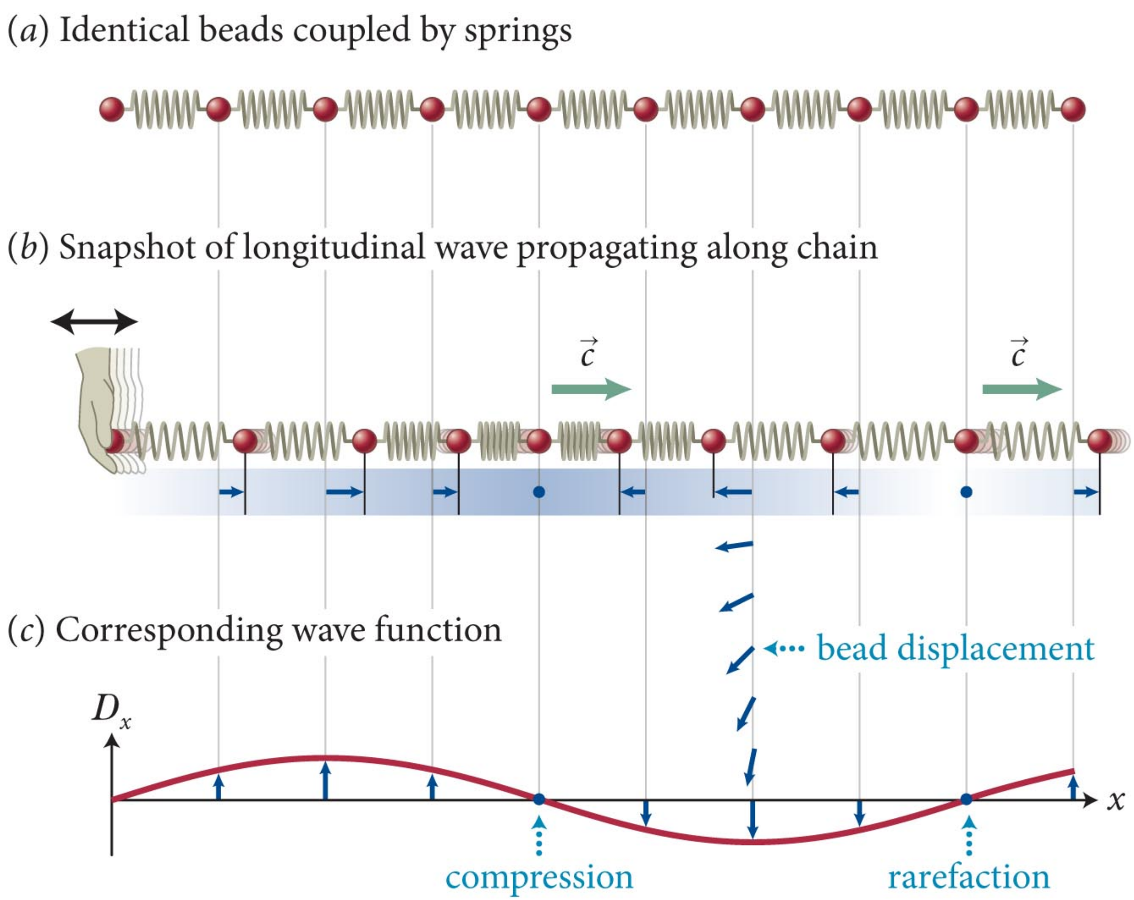
Section 17.2: Sound

- The figure illustrates a mechanical model for a longitudinal waves.



Checkpoint 17.3

17.3 Does the wave speed along the chain shown in Figure 17.9 increase or decrease when (*a*) the spring constant of the springs is increased and (*b*) the mass of the beads is increased?



Checkpoint 17.3

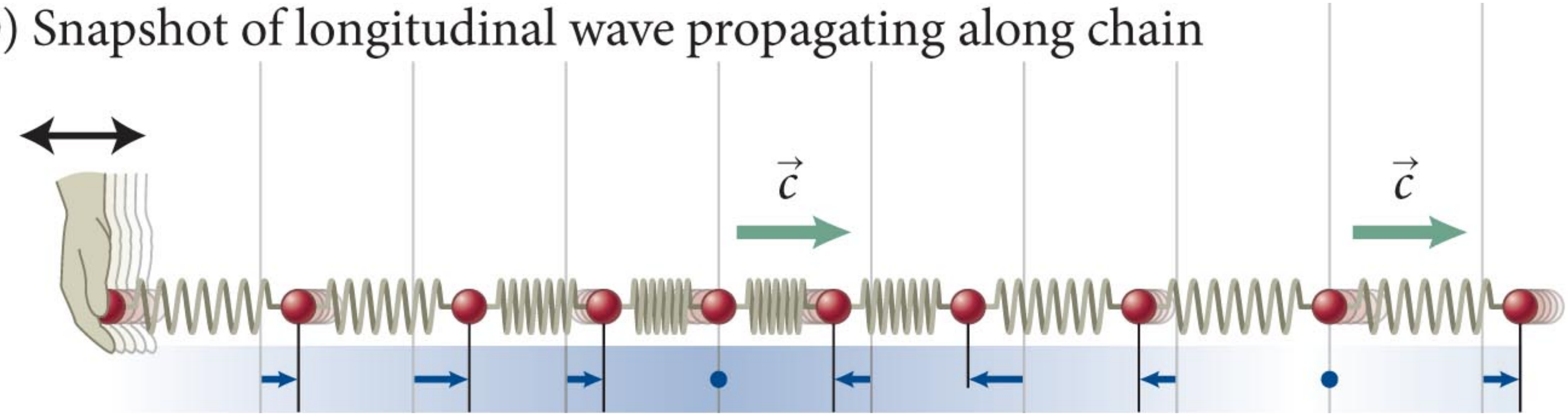
17.3 (a) Increase – the greater spring constant, the faster any disturbance is passed along. Just like increasing tension in a string (same mechanical model)!

(b) Decrease – greater mass slows down the transmission of the wave just like with beads on a string.

Checkpoint 17.4

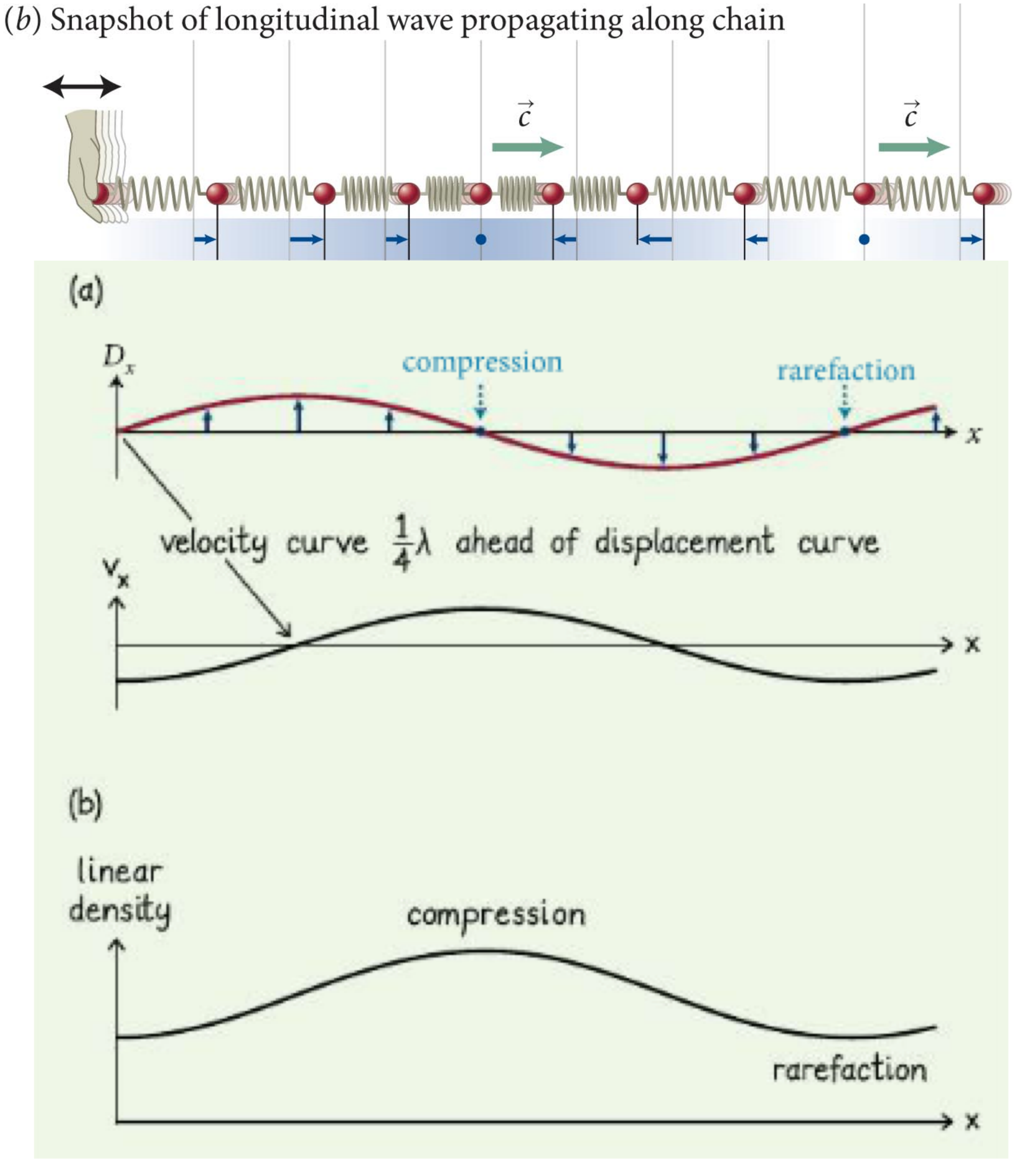
17.4 (*a*) Plot the velocity of the beads along the chain in Figure 17.9*b* as a function of their equilibrium position *x*. (*b*) Plot the linear density (number of beads per unit length) as a function of *x*.

(*b*) Snapshot of longitudinal wave propagating along chain



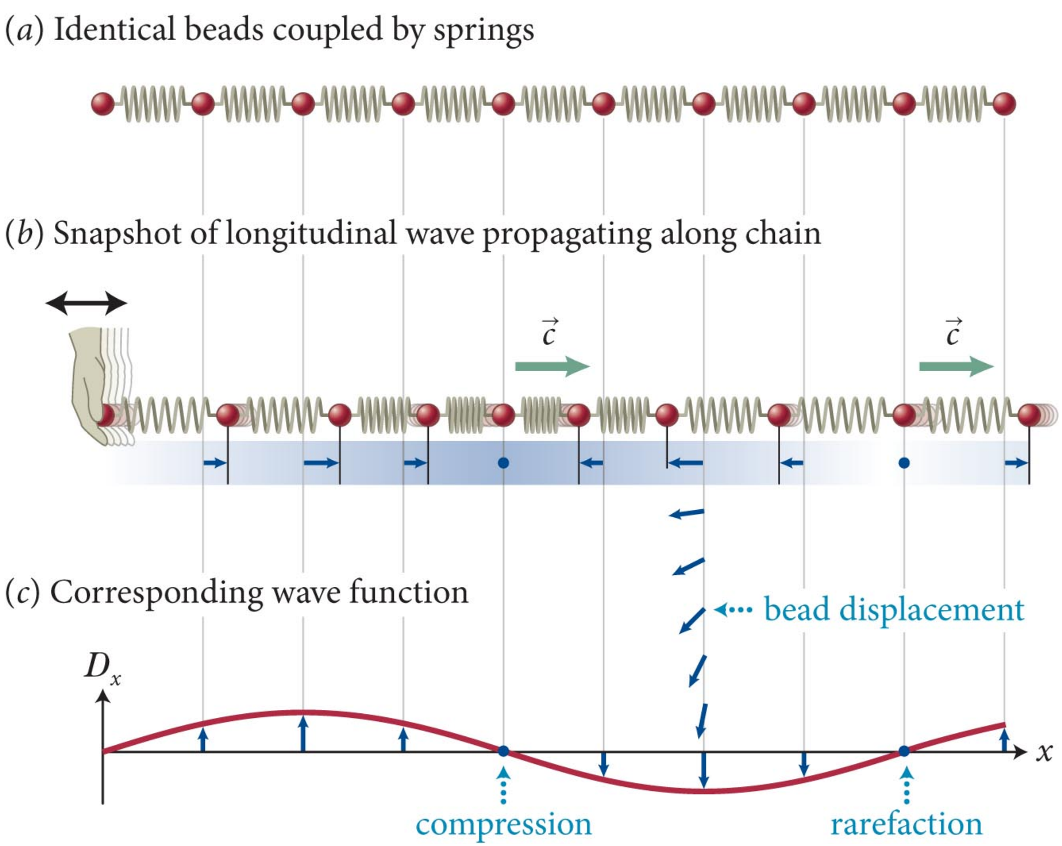
Checkpoint 17.4

$$v_x = \frac{dD_x}{dt}$$



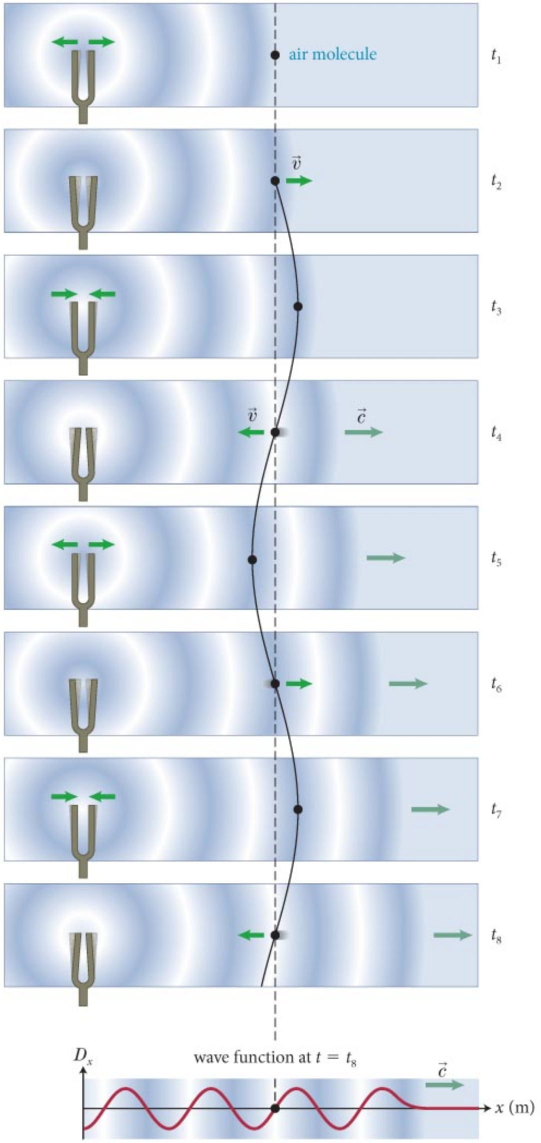
Section 17.2: Sound

- Longitudinal waves can also be represented by plotting the linear density of the medium as a function of position.
- **The compressions and rarefactions in longitudinal waves occur at the locations where the medium displacement is zero.**



Section 17.2: Sound

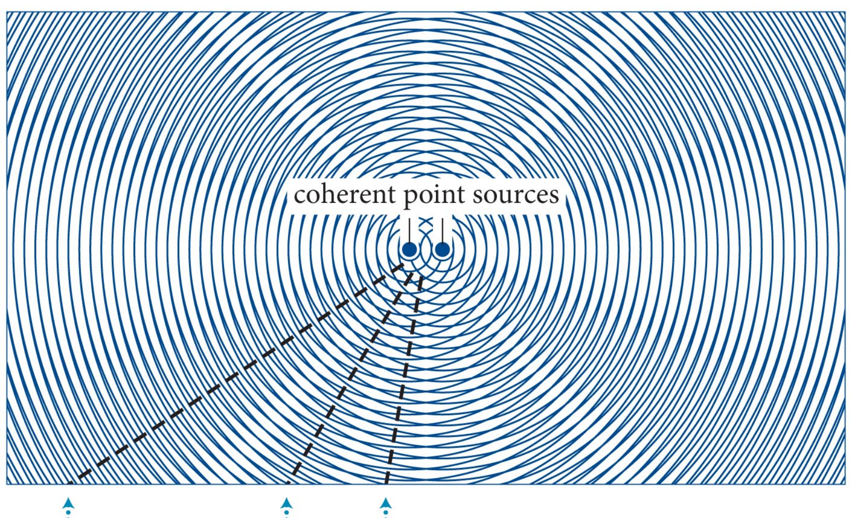
- The figure shows a sound wave generated by an oscillating tuning fork.
- At any fixed position: oscillates in time
- At any given time: spatial oscillation



Section Goals

You will learn to

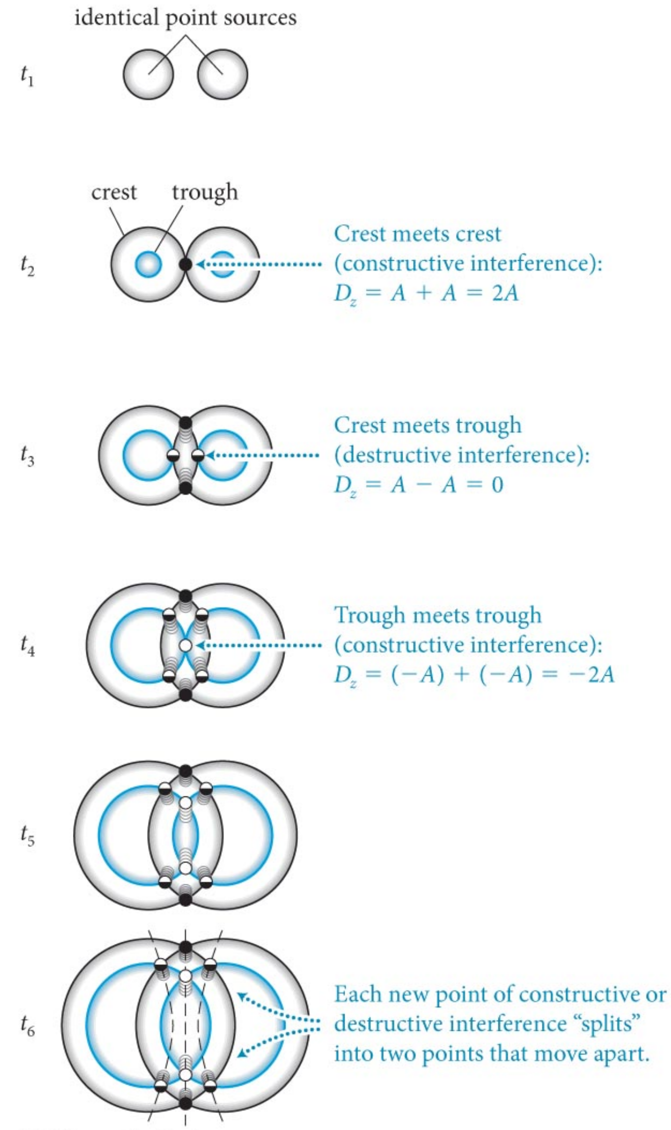
- Visualize the superposition of two or more two- or three-dimensional waves traveling through the same region of a medium at the same time.
- Define and represent visually the nodal and antinodal lines for interference in two dimensions.



nodal lines (lines along which waves interfere destructively)

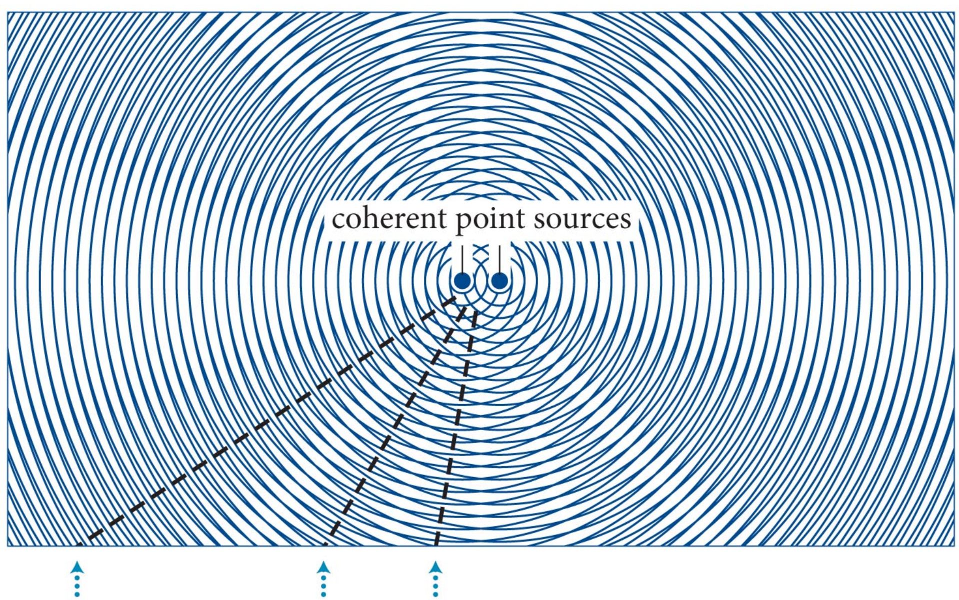
Section 17.3: Interference

- Let us now consider the superposition of overlapping waves in two and three dimensions.
- The figure shows the interference of two identical circular wave pulses as they spread out on the surface of a liquid.



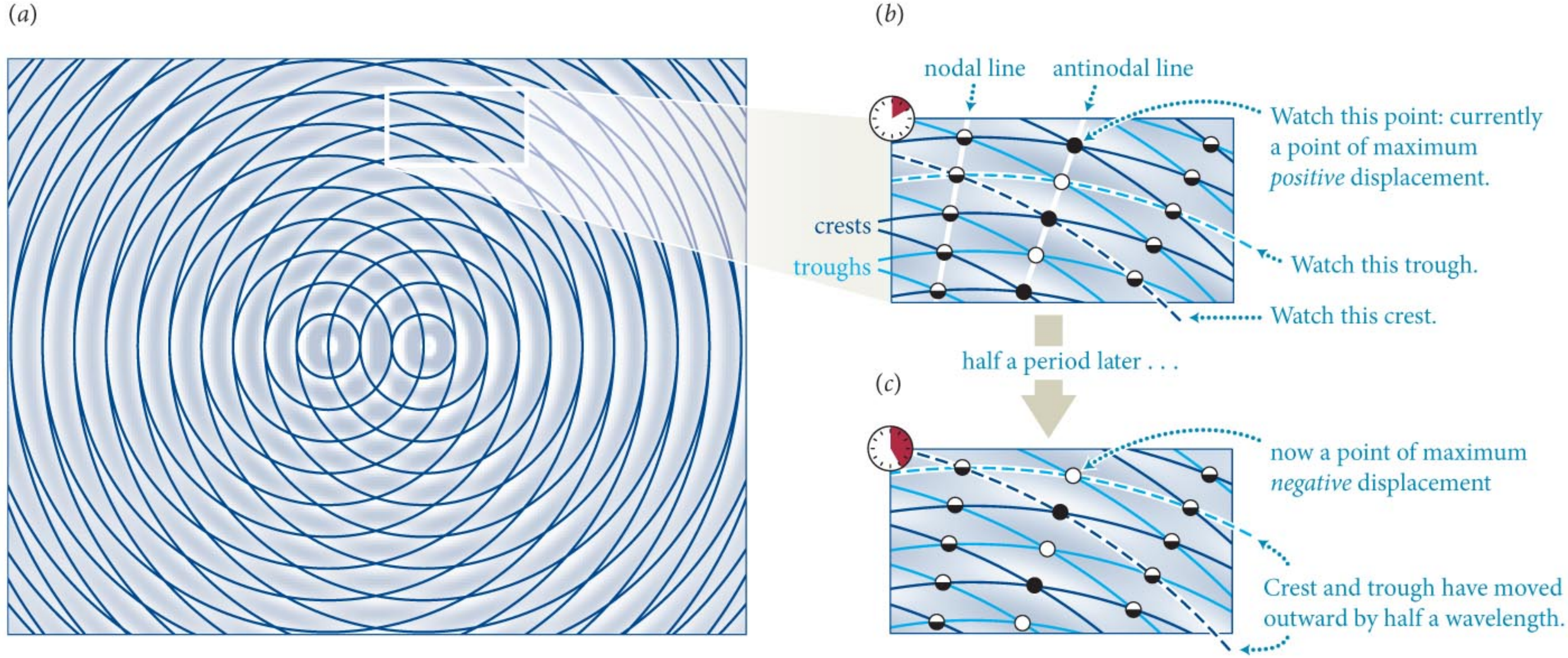
Section 17.3: Interference

- Sources that emit waves having a constant phase difference are called **coherent sources**.
- The pattern produces by overlapping circular wavefronts is called a *Moiré pattern*.
- Along **nodal lines** the two waves cancel each other and the vector sum of the displacement is always zero.



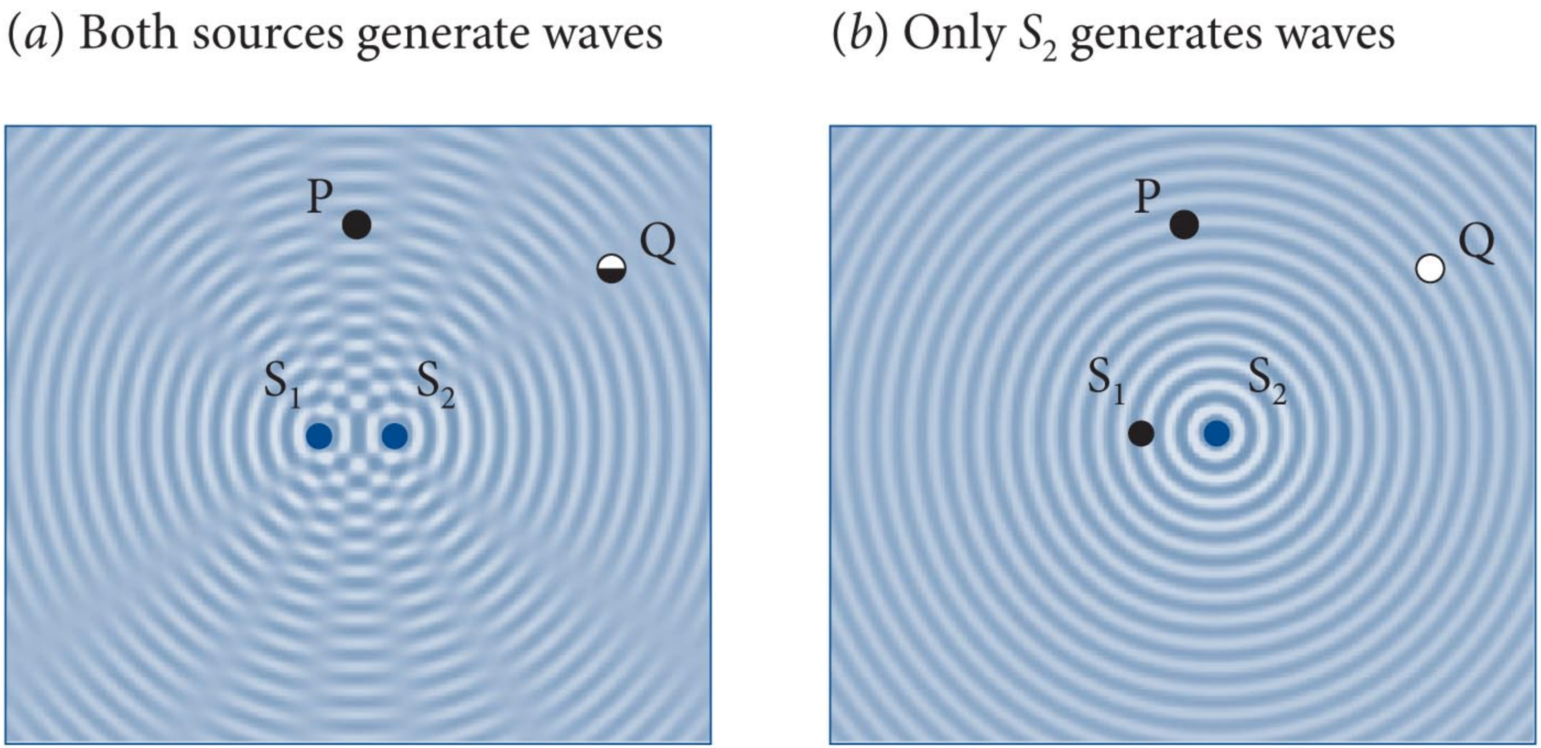
Section 17.3: Interference

- The figure shows a magnified view of the interference pattern seen on the previous slide.
- Along **antinodal** lines the displacement is a maximum.



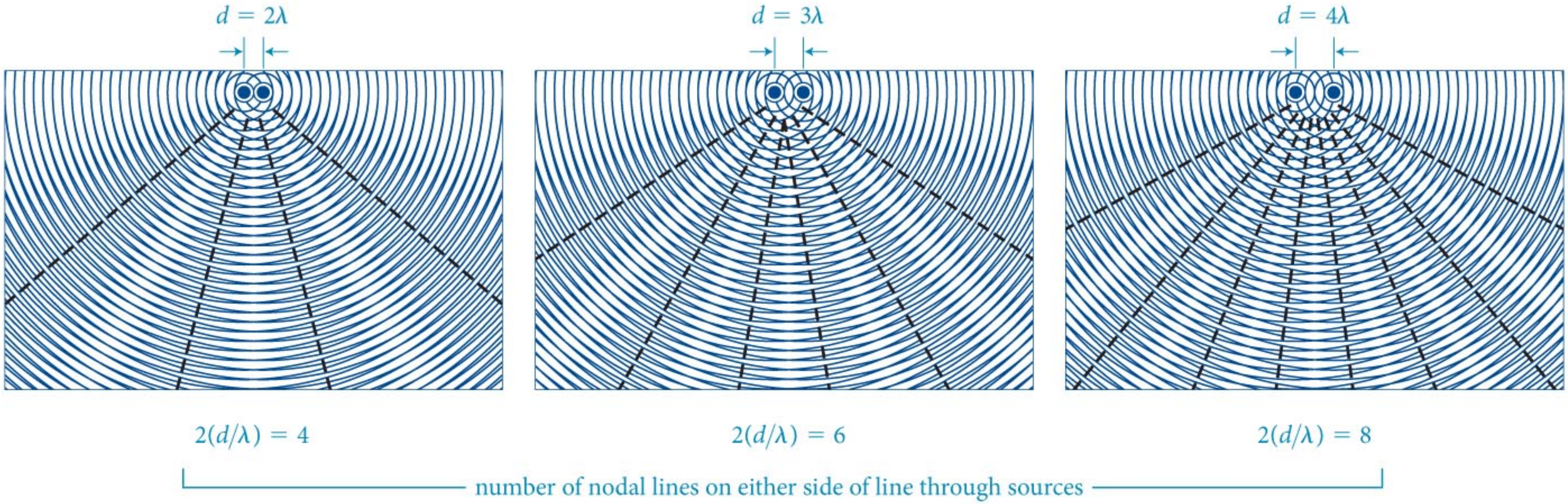
Section 17.3: Interference

- One consequence of nodal regions is illustrated in the figure.
  - **When the waves from two coherent sources interfere, the amplitude of the sum of these waves in certain directions is less than that of a single wave.**



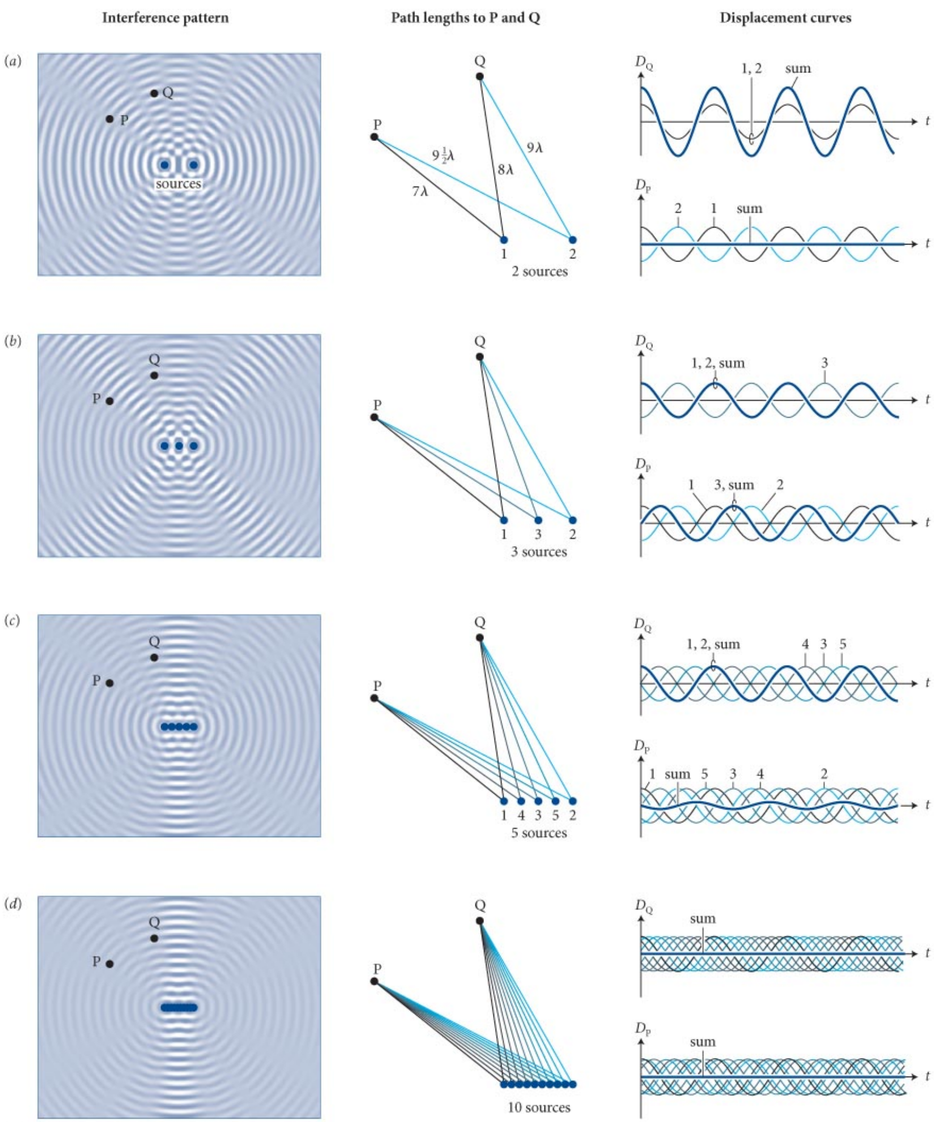
Section 17.3: Interference

- The effect that the separation between the two point sources have on the appearance of nodal lines is shown in the figure.
  - **If two coherent sources located a distance *d* apart emit identical waves of wavelength *λ*, then the number of nodal lines on either side of a straight line running through the centers of the sources is the greatest integer smaller than or equal to 2(*d*/*λ*).**



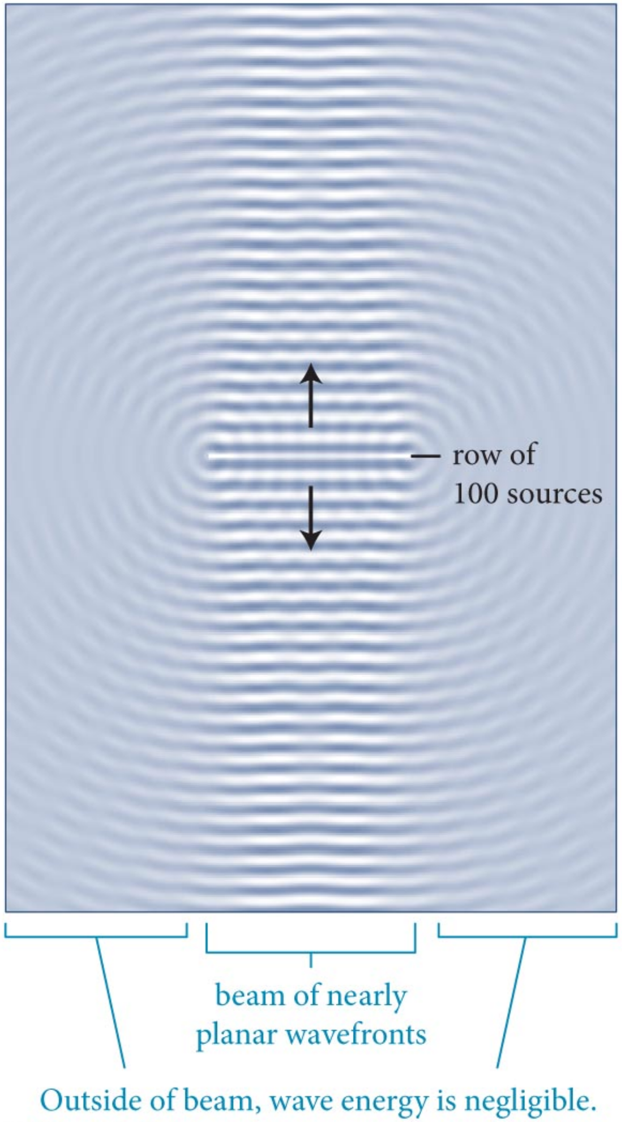
Section 17.3: Interference

- With more than two coherent sources?
- Do one pair first, then add a third source to the resultant of that pair. Repeat.
- Find path lengths from either source, divide by λ
- Difference is ½ **integer: destructive**
- Difference is **integer: constructive**



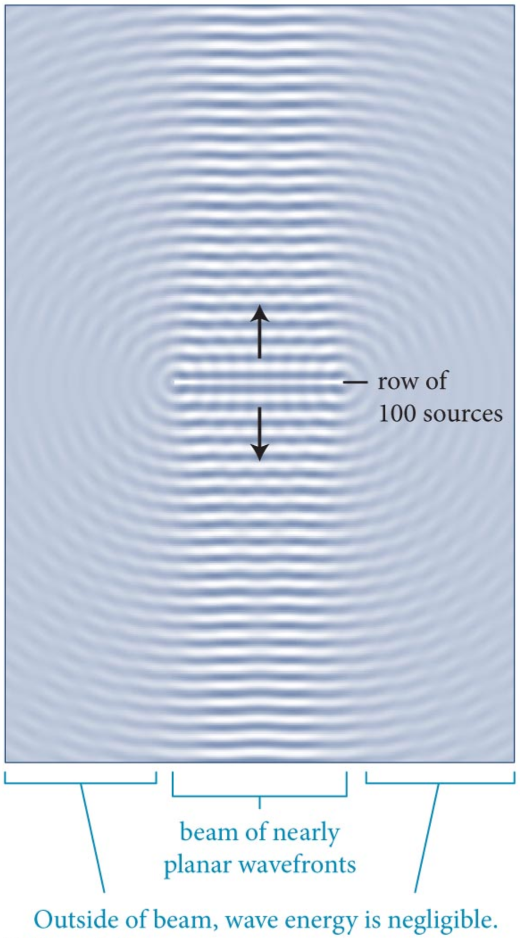
Section 17.3: Interference

- The figure shows what happens when 100 coherent sources are placed close to each other:
  - **When many coherent point sources are placed close together along a straight line, the waves nearly cancel out in all directions except the direction perpendicular to the axis of the sources.**



Checkpoint 17.11

17.11 How does the wave amplitude along the beam of wavefronts in Figure 17.20 change with distance from the row of sources?



It doesn’t very much!

Neighboring sources ‘shore each other up’

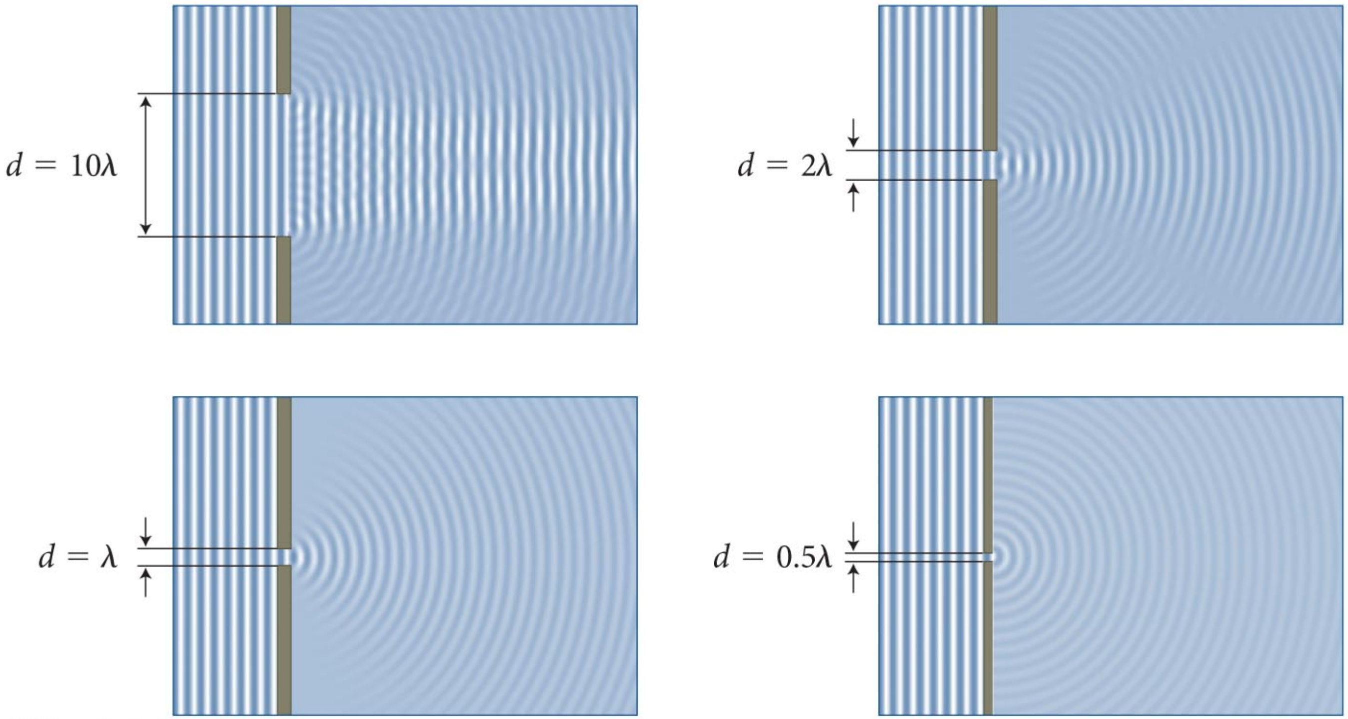
Utility? directional transmission!

Section 17.4: Diffraction

Section Goals

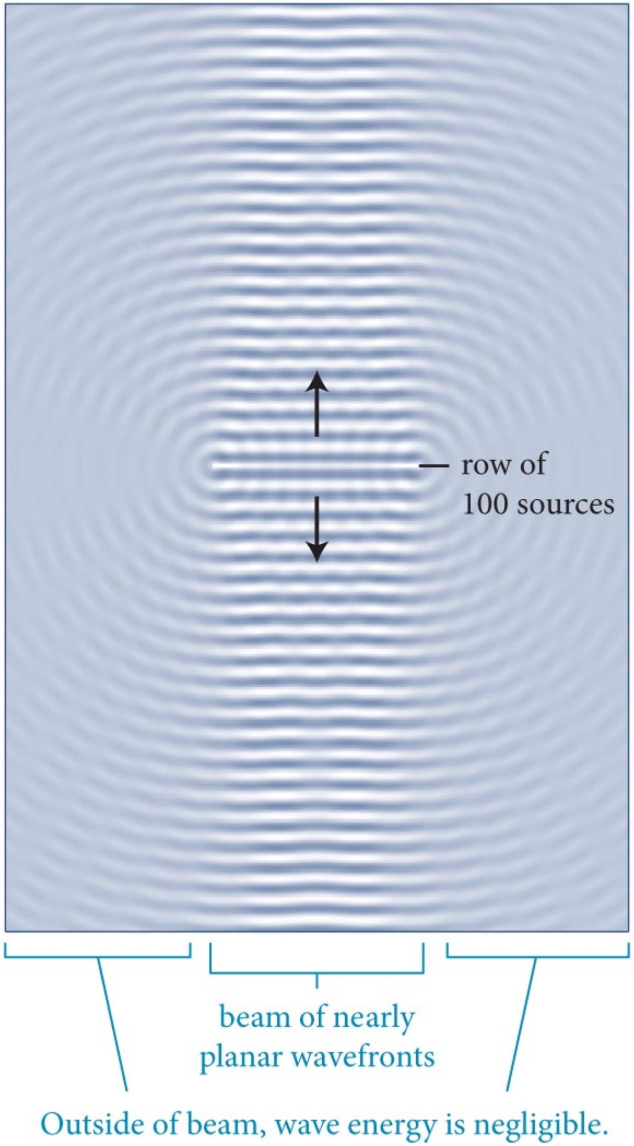
You will learn to

- Define the physical causes of **diffraction**.
- Represent diffraction **graphically**.



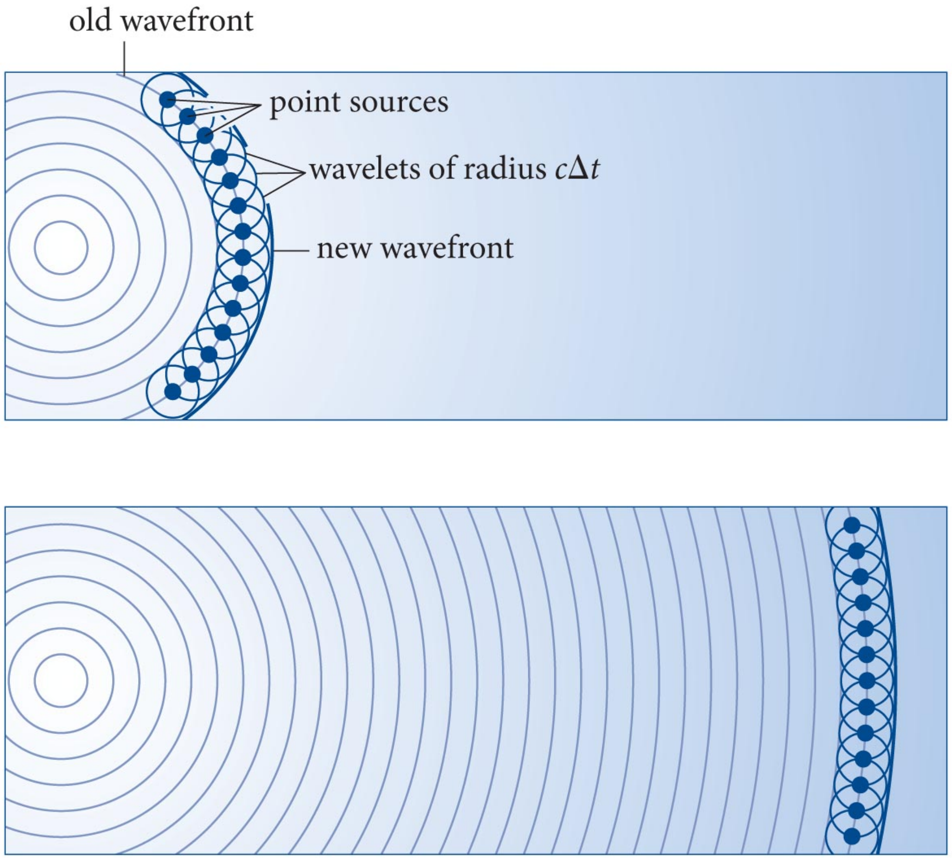
Section 17.4: Diffraction

- **Huygens' principle** states that any wavefront can be regarded as a collection of closely spaced, coherent point sources.
- All these point sources emit wavelets, and these forward-moving wavelets combine to form the next wavefront.



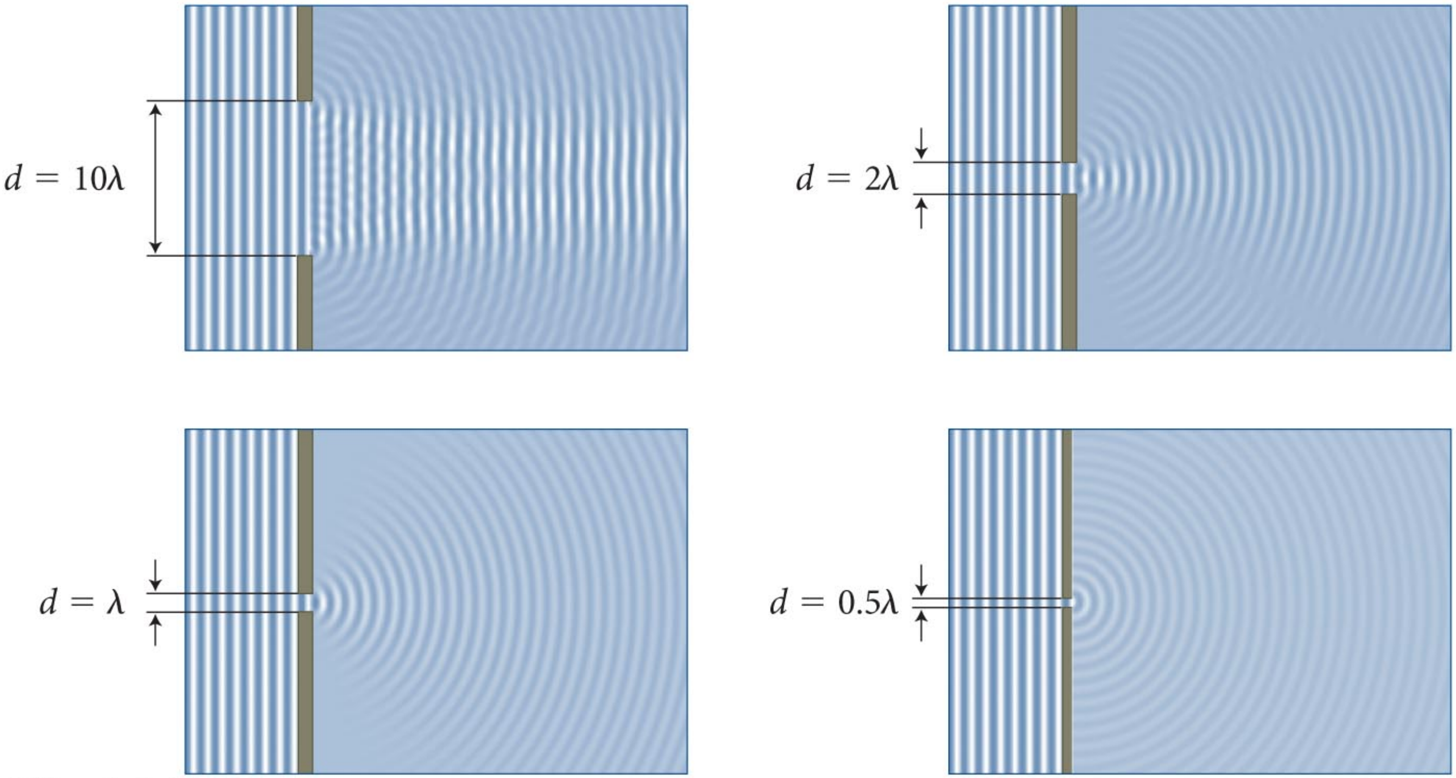
Section 17.4: Diffraction

- The figure shows planar wavefronts incident on gaps of varying size.
- **Obstacles or apertures whose width is smaller than the wavelength of an incident wave give rise to considerable spreading of that wave.**
- The spreading is called **diffraction**.



Checkpoint 17.12

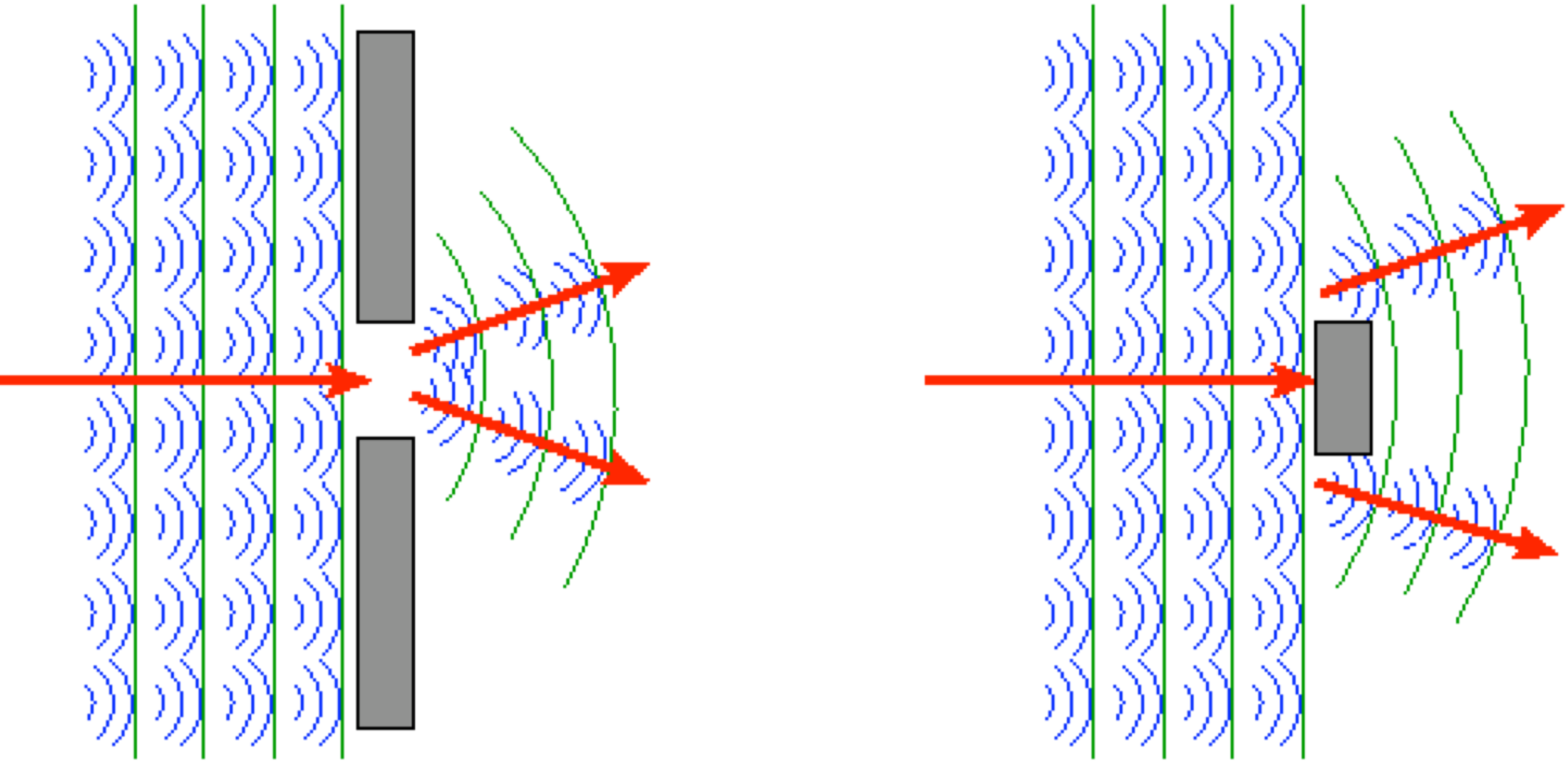
17.12 Suppose the barriers in Figure 17.22 were held at an angle to the incident wavefronts. Sketch the transmitted wavefronts for the case where the width of the gap is much smaller than the wavelength of the incident waves.



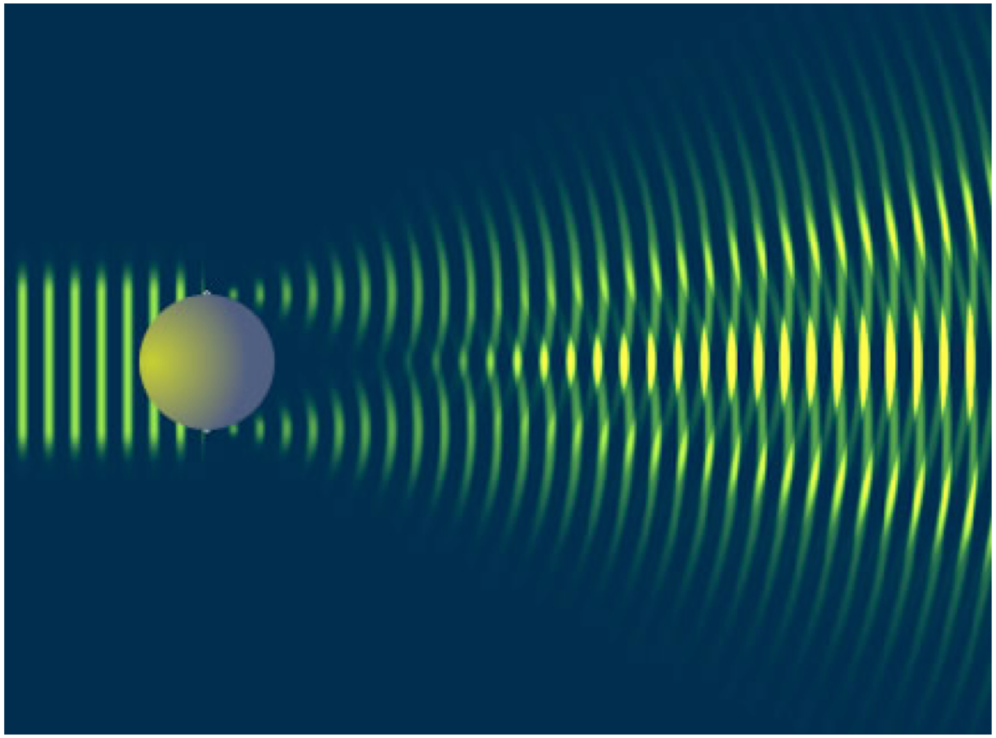
Checkpoint 17.12

17.12 Incident angle doesn't make a difference: the gap causes the same diffraction regardless. Only relies on incident waves causing the gap to become a point source.

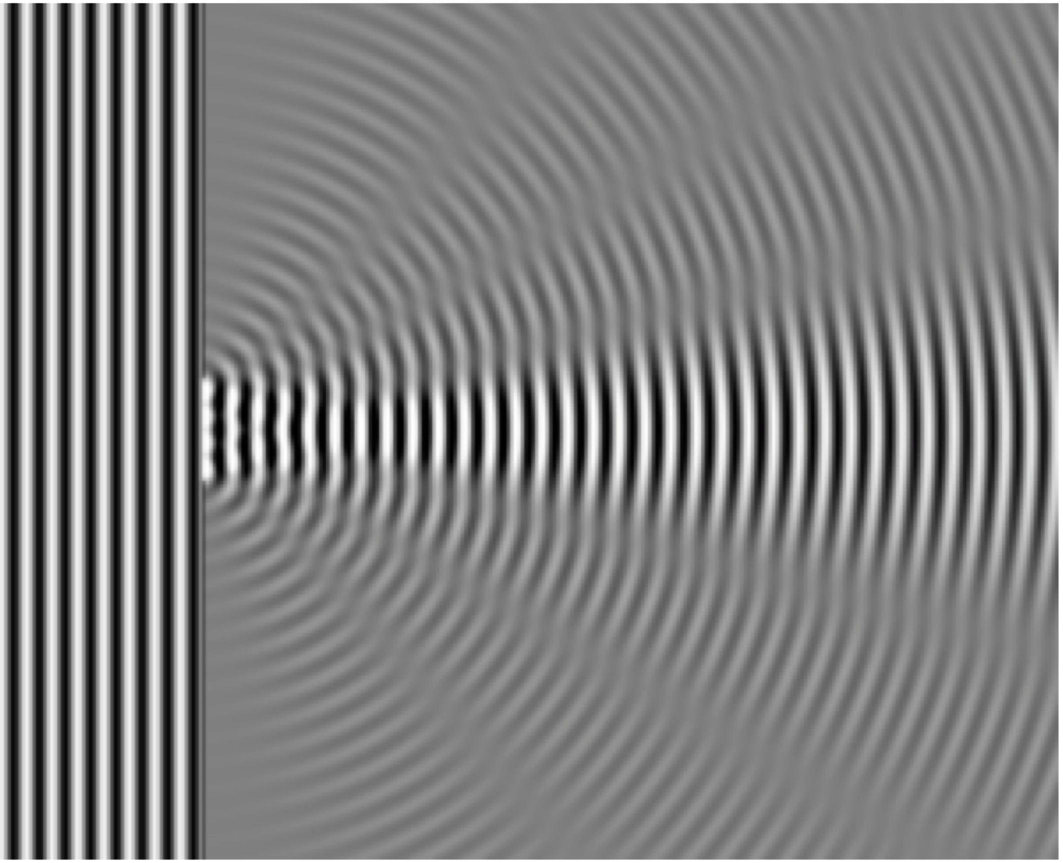
Examples

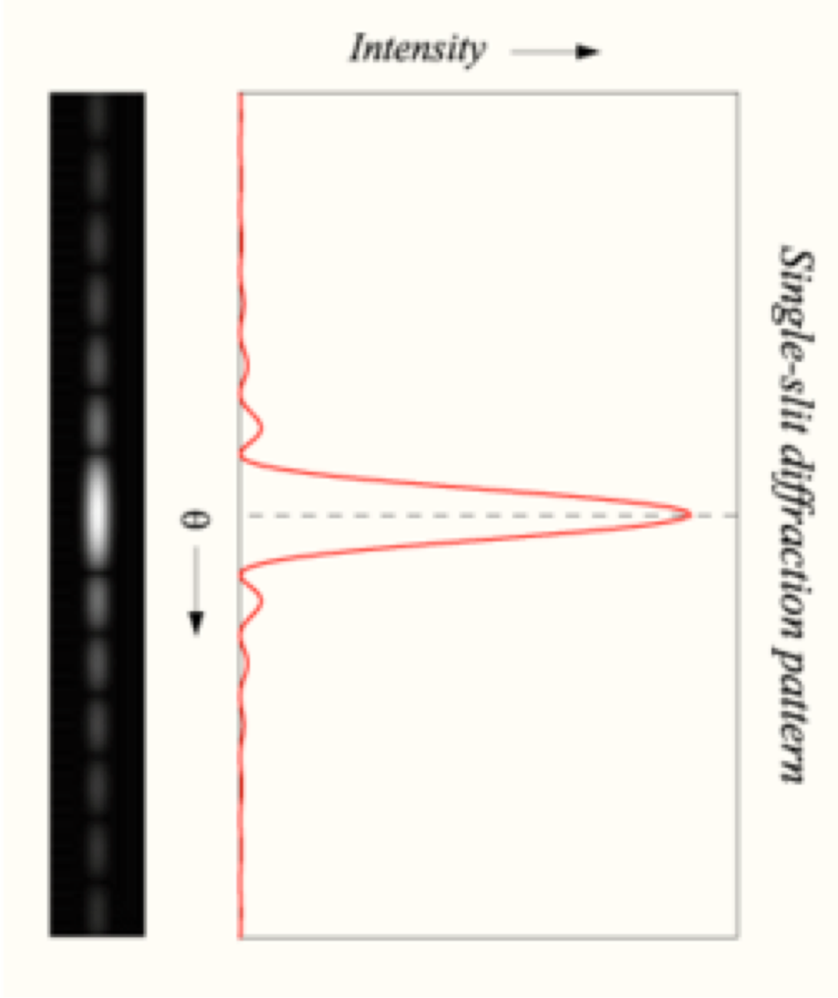


Examples

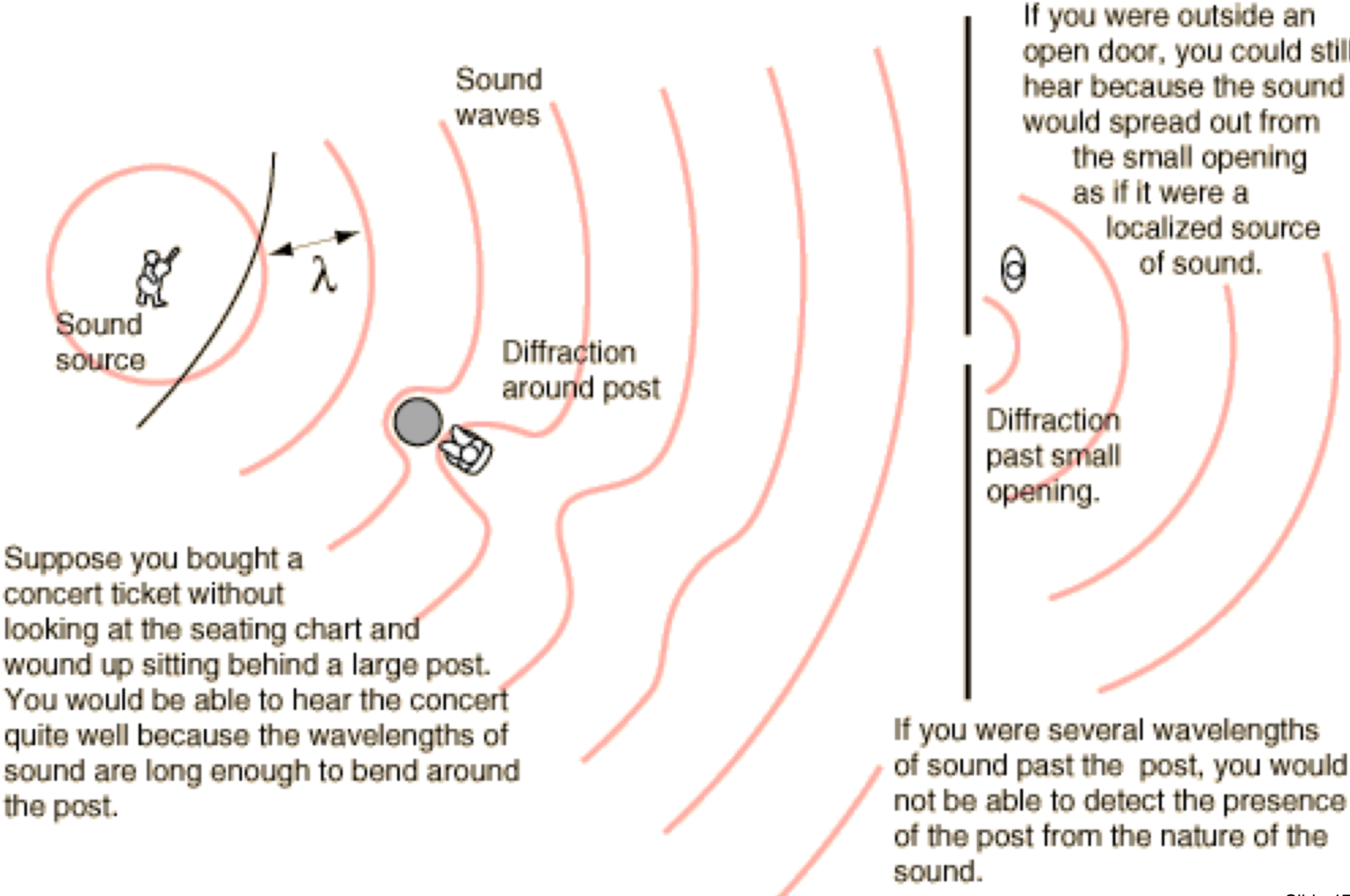


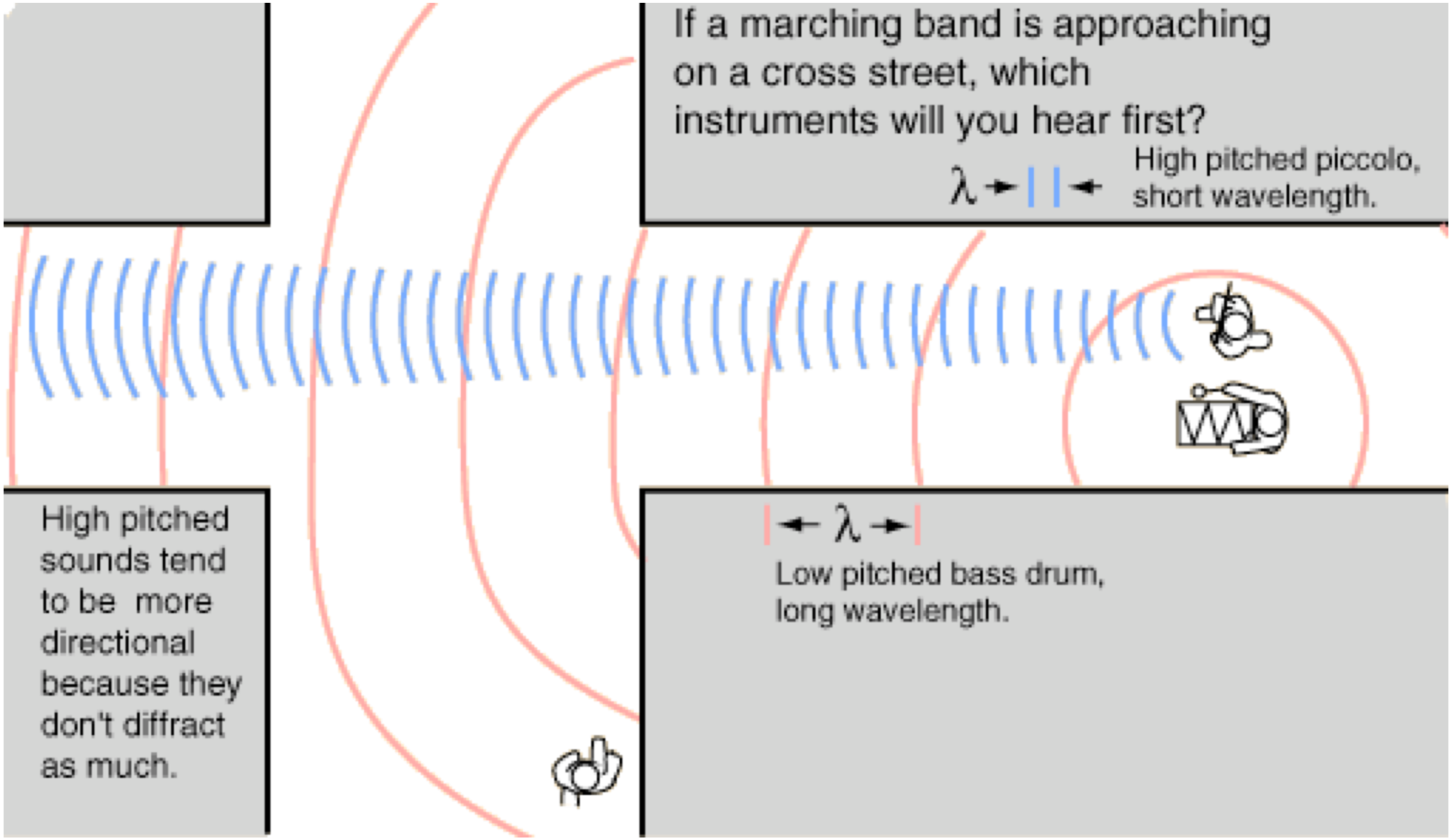
Examples



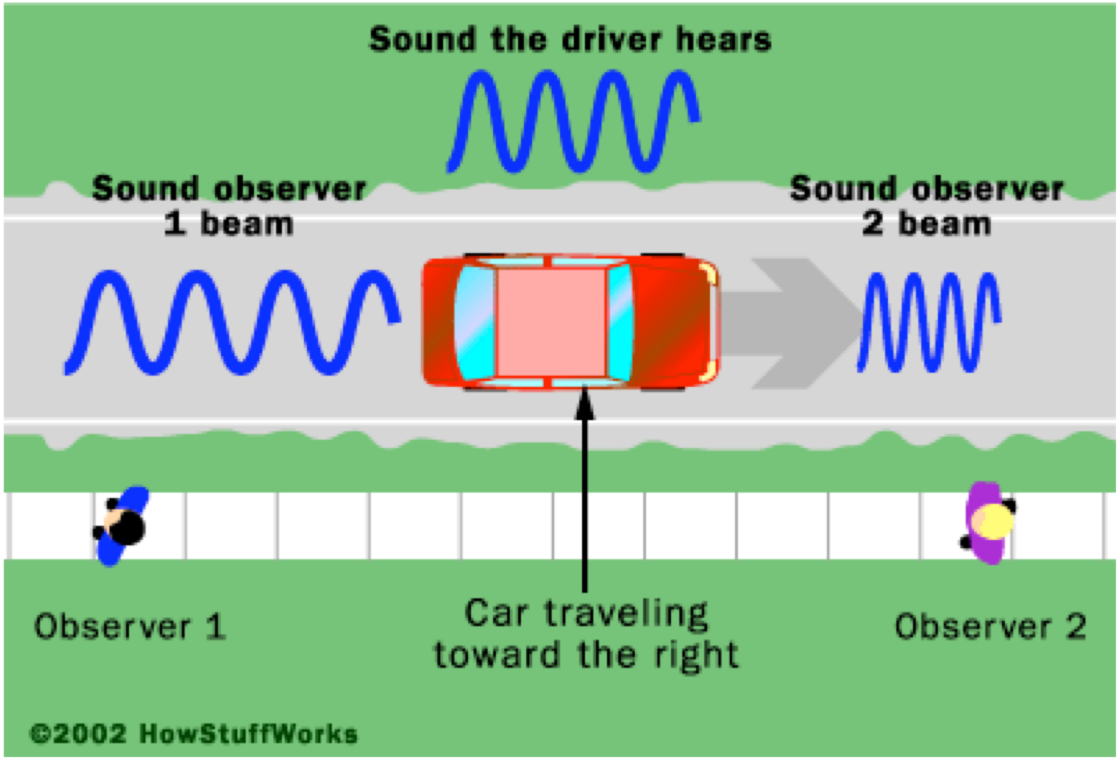


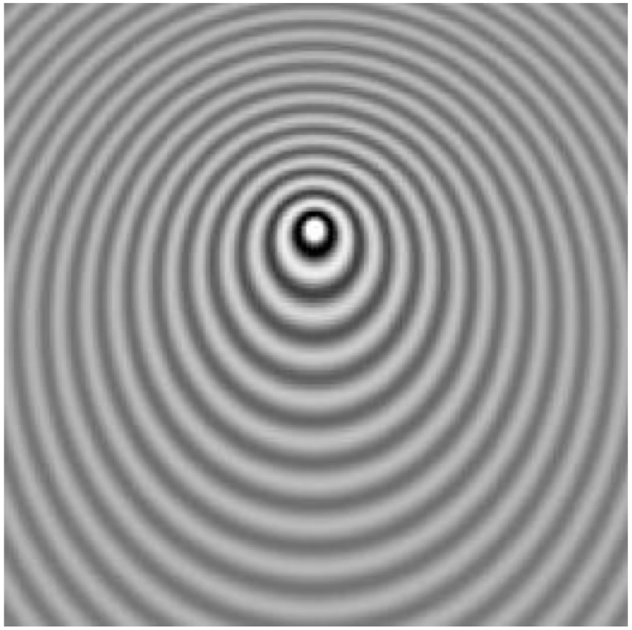
This happens with sound too!




If a marching band is approaching on a cross street, which instruments will you hear first?
λ
High pitched piccolo, short wavelength.
λ
Low pitched bass drum, long wavelength.
High pitched sounds tend to be more directional because they don't diffract as much.

Doppler Effect:
moving relative to waves

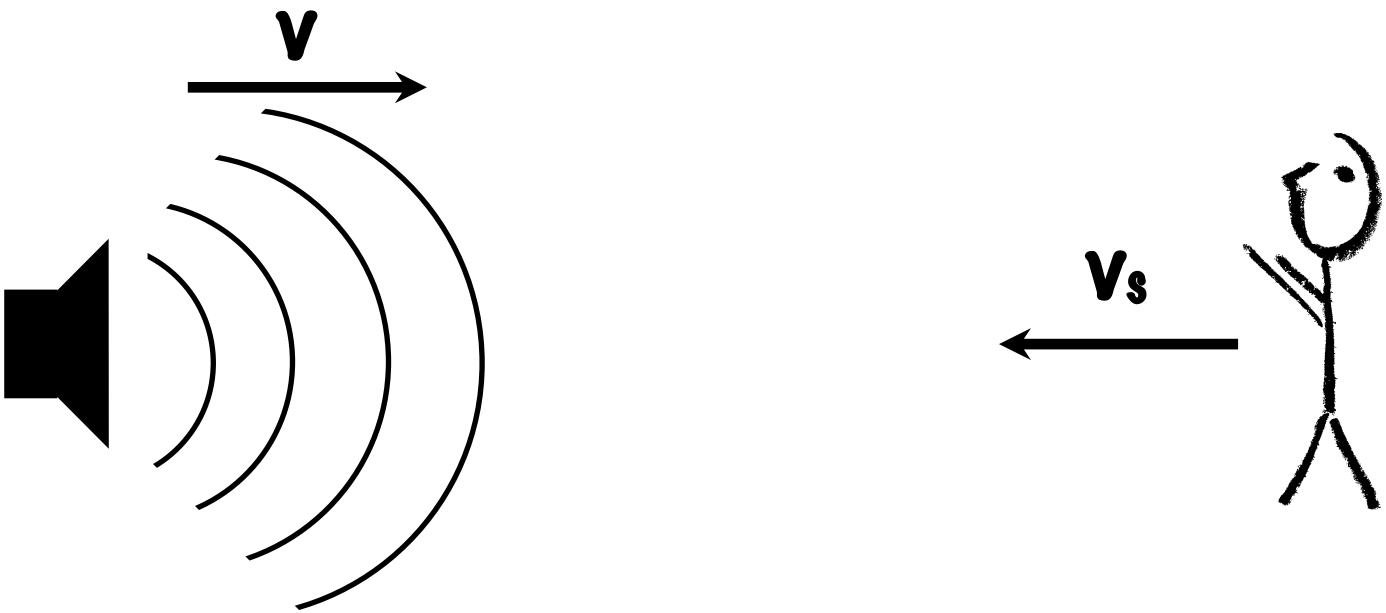




in one period T, you move closer to the source by $v_s T$

the waves appear squashed together by $v_s T$

the apparent frequency (1/T) is still velocity / wavelength



$$f' = \frac{v}{\lambda - v_s T} = \frac{v}{v/f - v_s T} = \frac{v}{v/f - v_s/f} = \left(\frac{v}{v - v_s}\right) f$$

Approaching the source:

pitch (freq) seems higher

$$f' = \left(\frac{v}{v - v_s} \right) f$$

Moving away from source:

pitch (freq) seems lower

$$f' = \left(\frac{v}{v + v_s} \right) f$$

Only has to do with RELATIVE motion!

e.g., ambulance - driver hears no change

similarly: doesn't matter who is moving

happens for light too - receding galaxies have "red shift" (lower freq)

Chapter 17: Self-Quiz #5

Because sound waves diffract around an open doorway, you can hear sounds coming from outside the doorway. You cannot, however, see objects outside the doorway unless you are directly in line with them. What does this observation imply about the wavelength of light?

Chapter 17: Self-Quiz #5

Answer

Because light does not diffract as it travels through the doorway, this observation implies that the wavelength of the light must be smaller than the width of the doorway.

Given that visible light has wavelengths between 4×10^{-7} m and 7×10^{-7} m and most doorways are about 1 m wide and 2 m tall, this is indeed the case.

Quantitative Tools

Section 17.5: Intensity

Section Goals

You will learn to

- Define the **intensity** of a wave.
- Calculate the intensity of a wave using the **decibel scale**.

Section 17.5: Intensity

- For waves in three dimensions, **intensity** I is defined as

$$I \equiv \frac{P}{A}$$

  - P is the power delivered by the wave over an area A.
  - SI units: W/m^2
- If the power delivered by a point source is P_s, the intensity at a distance r from the source is

$$I = \frac{P_s}{A_{sphere}} = \frac{P_s}{4\pi r^2} \quad \text{(uniformly radiating point source)}$$

- For two-dimensional surface waves, the intensity is

$$I_{surf} \equiv \frac{P}{L}$$

  - SI units: W/m

Section 17.5: Intensity

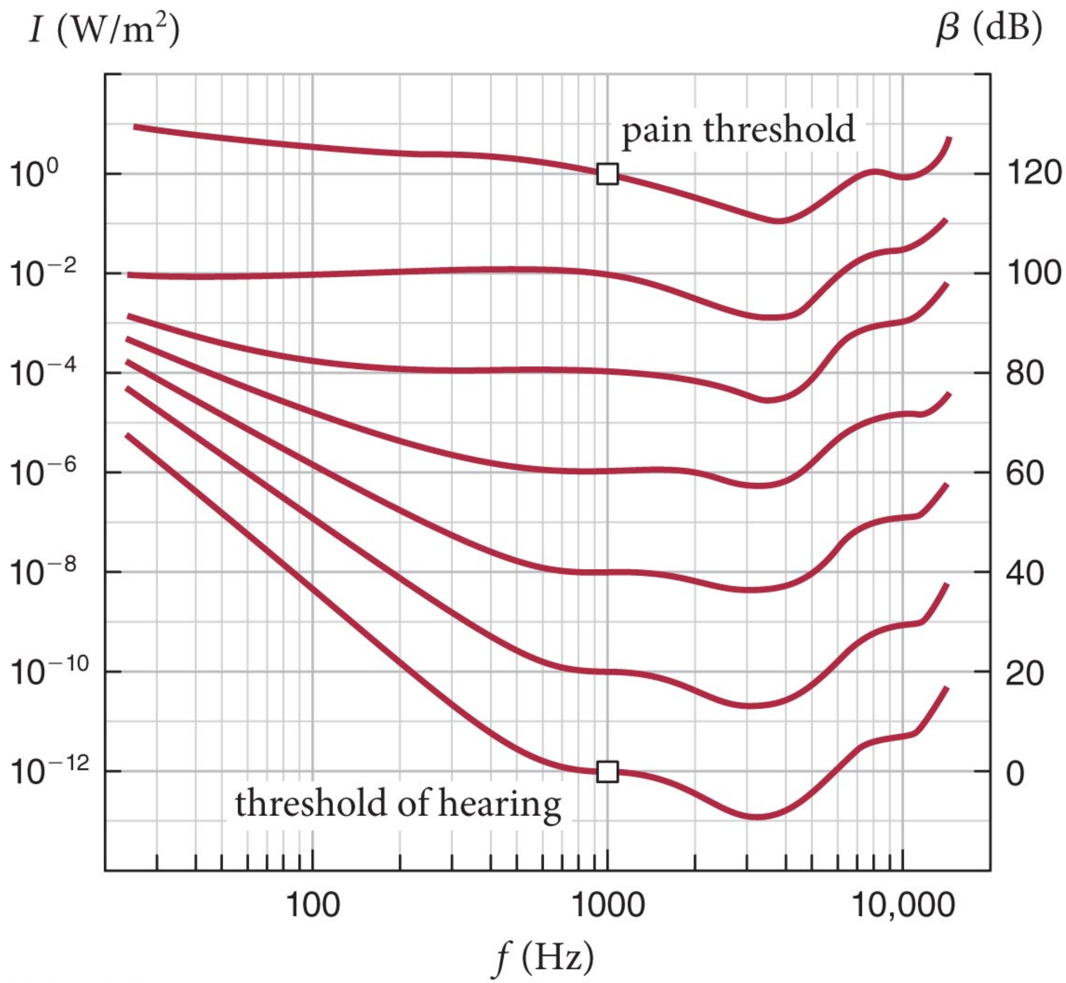
- The human ear can handle an extremely wide range of intensities, from the *threshold of hearing* $I_{th} \approx 1 \times 10^{-12}$ W/m^2 to the *threshold of pain* at ≈ 1.0 W/m^2.
- To deal with this vast range of intensities, it's convenient to use a logarithmic scale and it's logical to place the zero of the scale at the threshold of hearing.
- To do so, we define the **intensity level** β, expressed in **decibels** (dB), as

$$\beta \equiv (10 \text{ dB}) \log\left(\frac{I}{I_{th}}\right)$$

where $I_{th} = 1 \times 10^{-12}$ W/m^2.

Section 17.5: Intensity

Average auditory response of the human ear. Most sensitive at 3kHz. (lower magnitude means more sensitive. 3kHz is very annoying.)



Section 17.5: Intensity

Table 17.1 Approximate intensity levels

Source	distance (m)	β (dB)	Description
Jet engine	50	140	pain
Pneumatic hammer	10	110	
Shout	1.5	100	very loud
Car horn	10	90	
Hair dryer	0.2	80	loud
Automobile interior		70	
Conversation	1	60	moderate
Office background		50	
Library background		40	
Suburban bedroom		30	quiet
Whisper	1	20	
Normal breathing	5	10	barely audible

Exercise 17.5 Doubling the intensity

A clarinet can produce about 70 dB of sound. By how much does the intensity level increase if a second clarinet is played at the same time?

Exercise 17.5 Doubling the intensity (cont.)

SOLUTION If the intensity of the sound produced by one clarinet is I_c, the intensity level of one clarinet is

$$\beta_1 = (10 \text{ dB}) \log\left(\frac{I_c}{I_{th}}\right) = 70 \text{ dB}.$$

Exercise 17.5 Doubling the intensity (cont.)

SOLUTION The second clarinet doubles the intensity, so the intensity level becomes

$$\beta_2 = (10\text{ dB})\log\left(\frac{2I_\text{c}}{I_\text{th}}\right) = (10\text{ dB})\left[\log 2 + \log\left(\frac{I_\text{c}}{I_\text{th}}\right)\right]$$

$$= (10\text{ dB})\log 2 + \beta_1,$$

where I have used the logarithmic relationship log $AB = \log A + \log B$. Because $\log 2 \approx 0.3$, the intensity level increases to $\beta_2 \approx (10\text{ dB})(0.3) + 70\text{ dB} = 73\text{ dB}$. So, even though the intensity doubles, the intensity *level* increases by only 3 dB. ✔

Checkpoint 17.13

17.13 In Exercise 17.5, how many clarinets must play at the same time in order to increase the intensity level from 70 dB to 80 dB?

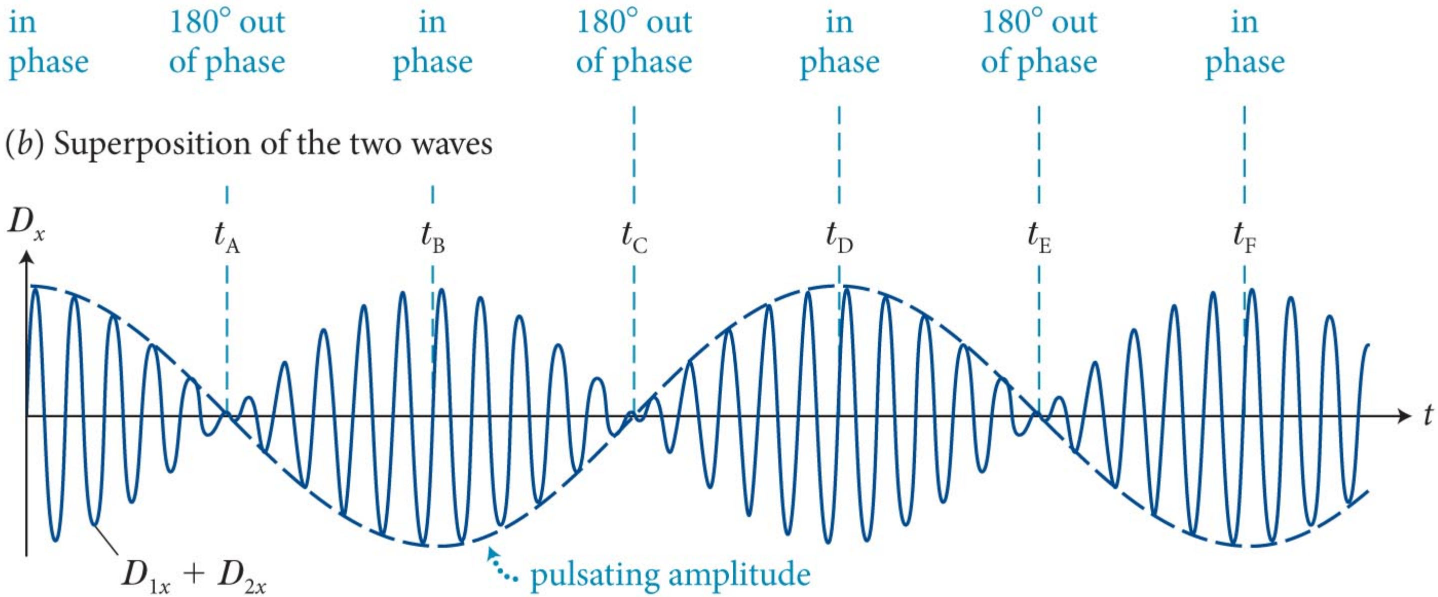
10 dB means a factor of 10 increase in intensity (log scale!), so we need 10 clarinets playing at the same time.

Section 17.6: Beats

Section Goals

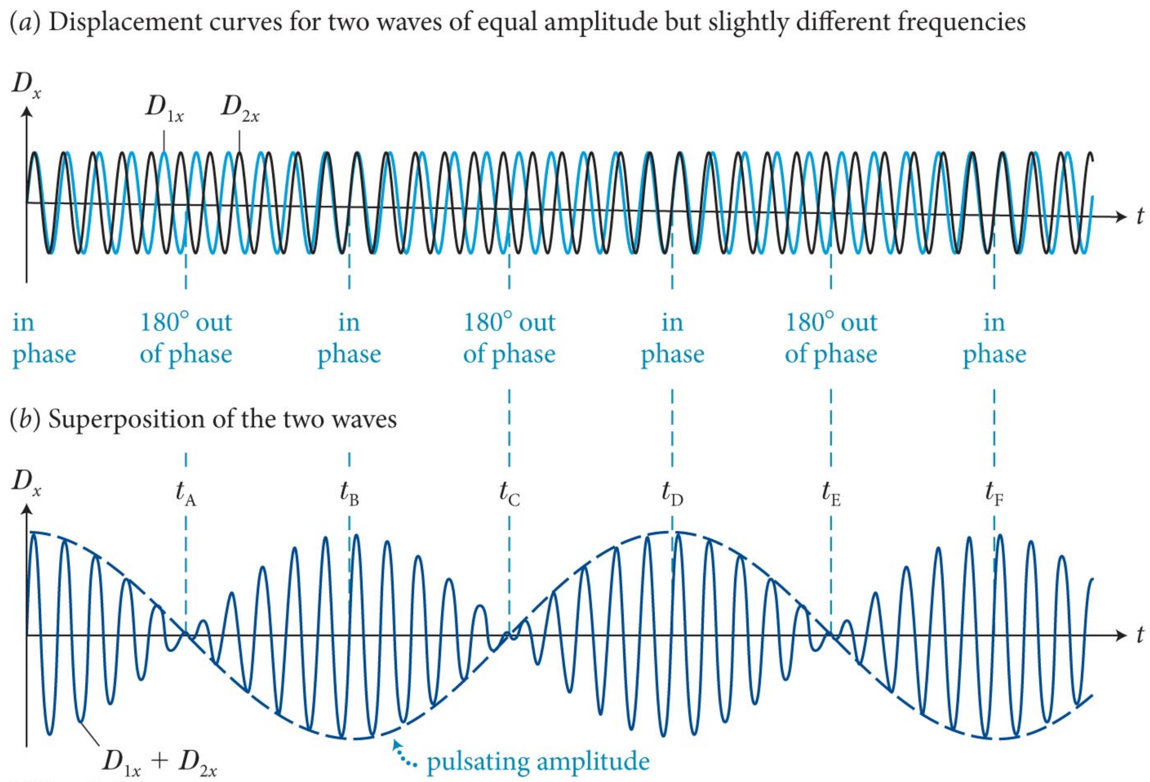
You will learn to

- Establish the concept of **beats**, which arises from the overlap of equal amplitude waves with slightly **different frequency**.
- Derive the mathematical formula that relates the frequency of the beats to the frequencies of the overlapping waves.



Section 17.6: Beats

- Part (*a*) shows the displacement curves for two waves of equal amplitude *A*, but slightly different frequencies.
- The superposition of the two waves result in a wave of oscillating amplitude as shown in part (*b*).
- This effect is called **beating**.



Section 17.6: Beats

- The displacement caused by the two individual waves at some fixed point is given by

$$D_{1x} = A\sin(2\pi f_1 t)$$

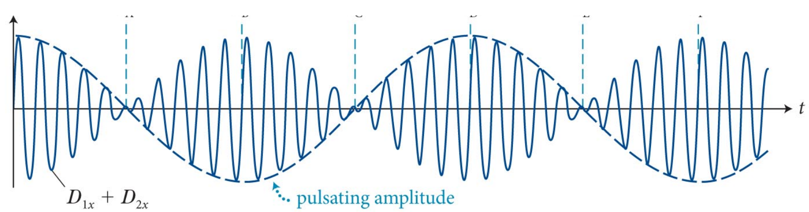
$$D_{2x} = A\sin(2\pi f_2 t)$$

- The superposition of the two waves gives us

$$D_x = D_{1x} + D_{2x} = A(\sin 2\pi f_1 t + \sin 2\pi f_2 t)$$

- Using trigonometric identities, we can simplify the equation to

$$D_x = 2A\cos\tfrac{1}{2}\left[2\pi(f_1 - f_2)t\right]\sin\tfrac{1}{2}\left[2\pi(f_1 + f_2)t\right]$$



Section 17.6: Beats

- Using $\Delta f = |f_1 - f_2|$ and $f_{av} = ½ (f_1 + f_2)$, we can write

$$D_x = 2A \cos\left[2\pi(\tfrac{1}{2}\Delta f)t\right]\sin(2\pi f_{av} t)$$

- We can see that the resulting wave has a frequency of f_{av}.
- The frequency of the amplitude variation is $½\Delta f$.
- However, since two beats occur in each cycle of this amplitude variation, the **beat frequency** is twice that

$$f_{beat} \equiv |f_1 - f_2|$$

- This is how you know you're out of tune. Faster beating means farther apart.

Exercise 17.7 Tuning a piano

Your middle-C tuning fork oscillates at 261.6 Hz. When you play the middle-C key on your piano together with the tuning fork, you hear 15 beats in 10 s. What are the possible frequencies emitted by this key?

Exercise 17.7 Tuning a piano

SOLUTION The beat frequency—the number of beats per second—is equal to the difference between the two frequencies (Eq. 17.8).

I am given the frequency of the tuning fork, $f_t = 261.6$ Hz, and the beat frequency, $f_B = (15 \text{ beats})/(10 \text{ s}) = 1.5$ Hz.

I do not know, however, whether the frequency f_p of the struck middle-C piano key is higher or lower than that of the tuning fork.

Exercise 17.7 Tuning a piano

SOLUTION If it is higher, I have $f_B = f_p - f_t$.

If it is lower, then $f_B = f_t - f_p$.

So

$$f_p = f_t \pm f_B = 261.6 \text{ Hz} \pm 1.5 \text{ Hz}$$

and the possible frequencies emitted by the out-of-tune middle-C key are 260.1 Hz and 263.1 Hz. ✔

Section 17.6
Question 6

One way to tune a piano is to strike a tuning fork (which emits only one specific frequency), then immediately strike the piano key for the frequency being sounded by the fork, and listen for beats. In making an adjustment, a piano tuner working this way causes the beat frequency to increase slightly. Is she going in the right direction with that adjustment?

1. Yes
2. No

Section 17.6
Question 6

One way to tune a piano is to strike a tuning fork (which emits only one specific frequency), then immediately strike the piano key for the frequency being sounded by the fork, and listen for beats. In making an adjustment, a piano tuner working this way causes the beat frequency to increase slightly. Is she going in the right direction with that adjustment?

1. Yes
2. ✔ No – faster beating means larger difference in freq.