

Exam I Solutions

1. An electron is released a short distance above the surface of the Earth. A second electron directly below it exerts an electrostatic force on the first electron just great enough to cancel the gravitational force on it. How far below the first electron is the second?

- ☒ 5.1 m
- ☐ 7.5 m
- ☐ 10.2 m
- ☐ 2.5 m

What we are really asking is, "at what distance below a proton would the upward force on an electron equal the electron's weight?"

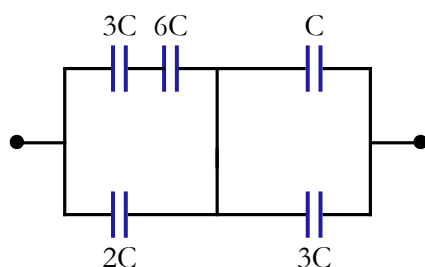
To determine that, all we need to do is write down the two forces and balance them. The electron has mass m_e and charge $-e$, the proton has charge e , and the distance between them we'll call d . Our origin will be at the proton's position:

$$\begin{aligned}
 m_e g - \frac{k_e e (-e)}{d^2} &= 0 \\
 d^2 &= \frac{k_e e^2}{m_e g} \\
 d &= \sqrt{\frac{k_e e^2}{m_e g}} = \sqrt{\frac{(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) (1.6 \times 10^{-19} \text{ C})^2}{(9.1 \times 10^{-31} \text{ kg}) (9.8 \text{ m/s}^2)}} \\
 d &\approx \sqrt{26 \frac{\text{N} \cdot \text{m} \cdot \text{s}^2}{\text{kg}}} = \sqrt{26 \frac{(\text{kg} \cdot \text{m/s}^2) \cdot \text{m} \cdot \text{s}^2}{\text{kg}}} \\
 \Rightarrow d &\approx 5.1 \text{ m}
 \end{aligned}$$

2. A point charge q is located at the center of a spherical shell of radius a that has a charge $-q$ uniformly distributed on its surface. Find the electric field for a point a distance $r > a$ from the center of the shell (*i.e.*, outside the sphere).

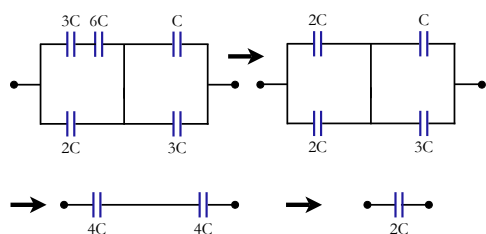
- ☐ $E = \frac{k_e q}{r^2}$
- ☐ $E = \frac{k_e q}{4\pi r^2}$
- ☒ $E = 0$
- ☐ $E = \frac{k_e q^2}{r^2}$

The simplest way to solve this one is with Gauss' law. First, Gauss' law told us that any spherically symmetric charge distribution behaves as a point charge. Second, Gauss' law tells us that the electric flux out of some surface depends only on the enclosed charge. If we draw a spherical surface of radius r and area A enclosing the shell *and* the point charge, centered on the center of the conducting sphere, the total enclosed charge is that of the shell plus that of the point charge: $q_{\text{encl}} = q + (-q) = 0$. If the enclosed charge is zero for any sphere drawn outside of and enclosing the spherical shell, then the electric field for all points outside the spherical shell.



3. Find the equivalent capacitance for the combination of capacitors shown at left.

- ☐ $4C/3$
- ☐ $3C$
- ☒ $2C$
- ☐ $C/6$



In order to find the equivalent capacitance, we want to find pure series or parallel sections of the circuit, and combine those one by one until we are left with only a single capacitance. First, we see that the $3C$ and $6C$ capacitors in the upper left are purely in series. Their equivalent capacitance is $1/C_{eq} = 1/3C + 1/6C$, or $C_{eq} = 2C$. After making that substitution (see figure at left), we now have two sets of capacitors in parallel, thanks to the middle connecting wire. In parallel, capacitors simply add together, and we can replace both halves of the circuit with $4C$ capacitors. That leaves us with two $4C$ capacitors in series, and an equivalent capacitance of $2C$.

4. If the end points of the capacitor arrangement above (the black circles) are connected to a 12.0 V source of potential difference, how much charge is stored if $C = 1\text{ }\mu\text{F}$?

- ☒ $24\text{ }\mu\text{C}$
- ☐ $10\text{ }\mu\text{C}$
- ☐ $3.6\text{ }\mu\text{C}$
- ☐ $82\text{ }\mu\text{C}$

The equivalent capacitance was $2C$, which in this case means $2\text{ }\mu\text{F}$. The total charge stored in any weird arrangement capacitors can always be found from the equivalent capacitance. That is the point of an equivalent capacitance - finding a single component of a certain value that will behave the same way as the whole arrangement to simplify things. Anyway: the total charge stored is $Q = C_{eq}\Delta V = (2\text{ }\mu\text{F})(12.0\text{ [V]}) = 24\text{ }\mu\text{[F} \cdot \text{V]} = 24\text{ }\mu\text{[C]}$.

5. Suppose three positively charged particles are constrained to move on a fixed circular track. If all the charges were equal, an equilibrium arrangement would obviously be a symmetrical one with the particles spaced 120° apart around the circle. Suppose two of the charges have equal charge q , and the equilibrium arrangement is such that these two charges are 140° apart rather than 120° . What is the *relative* magnitude and sign of the third charge?

- ☐ larger than either q_1 or q_2 and positive
- ☒ smaller than either q_1 or q_2 and positive
- ☐ larger than either q_1 or q_2 and negative
- ☐ smaller than either q_1 or q_2 and negative

First, all the charges must be of the same sign. With only three charges on a circular track, there is no way to keep opposite charges separated. The charges must be separated due to mutual repulsion.

If the angle between the two charges is 140° , it means that their mutual repulsion is *greater* than the repulsive force each feels due to the third charge. Thus, the third charge must be of the same sign, but smaller than either q_1 or q_2 . Remember this problem from your homework? In that case, the equilibrium angle was 90° , and the third charge was (approximately) three times larger than the other two.

6. If you place a negatively charged particle in an electric field, the charge will move

- ☐ from higher to lower electric potential and from lower to higher potential energy.
- ☐ from higher to lower electric potential and from higher to lower potential energy.
- ☐ from lower to higher electric potential and from lower to higher potential energy.
- ☒ from lower to higher electric potential and from higher to lower potential energy.

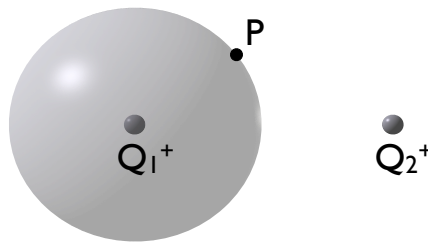
Negative charges will seek out regions of higher (more positive) electric potential - opposites attract. They will at the same time lower their potential energy, which is what any particle is going to try to do, left to its own devices. Remember, the negative charge means that seeking a higher potential V leads to a *lower* absolute potential energy qV .

7. A pyramid has a square base of side a , and four faces which are equilateral triangles. A charge Q is placed on the center of the base of the pyramid. What is the net flux of electric field emerging from one of the triangular faces of the pyramid?

- ☐ Uncertain: we must know whether Q is just above or below the base.
- ☐ 0
- ☒ $\frac{Q}{8\epsilon_o}$
- ☐ $\frac{Qa^2}{2\epsilon_o}$
- ☐ $\frac{Q}{2\epsilon_o}$

This is one of those cases where we have to rely on the symmetry of the situation in order to have any hope. First, we know that the *total* flux must just be Q/ϵ_o . If the charge Q is embedded in the base of our pyramid, it must be intercepting half of the total flux. Whether it is just above or just below the base is not important, it only matters that the charge Q is actually on the base. The field from the charge Q is then parallel to the base of the pyramid, and thus the base itself intercepts no field lines and has zero flux.

Thus, the pyramid intercepts half of the total flux, but does so only with its four triangular sides. Since the charge is exactly at the center of the pyramid, each of the other four sides of the pyramid must receive equal flux. Overall, the pyramid receives half of the total flux, and each triangular side must have one quarter of *that*, or one eighth of the total. The flux per triangular face must then be $Q/8\epsilon_o$.



8. In the figure above, a point charge $1Q^+$ is at the center of an imaginary spherical Gaussian surface and another point charge $2Q^+$ is outside of the Gaussian surface. Point P is on the surface of the sphere. Which one of the following statements is true?

- ☐ Both charges contribute to the net electric flux through the sphere but only $2Q^+$ contributes to the electric field at point P .
- ☒ Only $1Q^+$ contributes to the net electric flux through the sphere but both charges contribute to the electric field at point P .
- ☐ Both contribute to the net electric flux through the sphere but only $1Q^+$ contributes to the electric field at point P .
- ☐ Only $2Q^+$ contributes to the net electric flux through the sphere but both charges contribute to the electric field at point P .
- ☐ Only $2Q^+$ contributes to the net electric flux through the sphere and to the electric field at point P on the sphere.
- ☐ Only $1Q^+$ contributes to the net electric flux through the sphere and to the electric field at point P on the sphere.
- ☐ I don't know (this answer is worth 1/10 of full credit)

Only a charge inside a Gaussian surface can contribute to the net electric flux, so clearly only charge 1 contributes to the net flux on the surface shown. At point P , however, the electric field must still be a superposition of the field from charge 1 and charge 2 together. Remember, Gauss' law only says something about the net flux over a whole surface, it implies nothing about the field at point P alone. It tells us that the net flux over the *whole* spherical surface is only due to charge 1, not that the field at every *point* on the sphere is only due to charge 1. The effects of charge 2 simply cancel out when averaged over the whole sphere, since the same number of field lines enter the sphere as leave it. The fact that we are usually able to find the field at a particular point from Gauss' law only comes about through clever use of symmetry, and there are no obvious tricks to be played here.

9. A sphere the size of a basketball (radius 0.12 m) is charged to a potential of -500 V. About how many extra electrons are on it, per cm^2 of surface?

- ☐ 3.1×10^3
- ☒ 2.3×10^7
- ☐ 4.7×10^{10}
- ☐ 9.8×10^{21}

If the charge is spread evenly over the surface of a spherical object, like a basketball, then Gauss' law says we may treat the charge distribution as a point charge. Thus, we may consider the uniformly charged basketball equivalent to a single point charge at the center of the basketball. A men's regulation basketball has a diameter of 9.39 in, and thus a radius of 0.12 m. We know then that the potential at the surface of the basketball is -500 V, and this potential results from an effective point charge q at the center of the basketball, 0.12 m from the surface. Using the expression for the potential from a point charge:

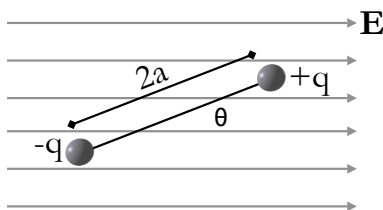
$$\begin{aligned} -500 \text{ V} &= \frac{k_e q}{r} = \frac{[9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2] q}{0.12 \text{ m}} \\ \Rightarrow q &= -6.65 \times 10^{-7} \text{ C} \end{aligned}$$

This is the effective charge on the basketball. Given that one electron has -1.6×10^{-19} C of charge, this must correspond to:

$$(\text{number of electrons}) = \frac{-6.65 \times 10^{-7} \text{ C}}{-1.6 \times 10^{-19} \text{ C/electron}} = 4.16 \times 10^{10} \text{ electrons}$$

If these electrons are spread out evenly over the surface, the electron density can be calculated from the surface area of the basketball (remembering that the surface area of a sphere is $4\pi r^2$, and we were asked to use cm^2):

$$(\text{density of electrons}) = \frac{4.16 \times 10^{10} \text{ electrons}}{4\pi (12 \text{ cm})^2} \approx 2.3 \times 10^7 \text{ electrons/cm}^2$$



10. An electric dipole in a uniform electric field E is displaced slightly from its equilibrium position, as shown at left, where θ is small. The separation of the charges is $2a$, and the moment of inertia of the dipole is I . Assuming the dipole is released from this position, which expression gives the correct variation of frequency with I , q , E , and a ?

- ☐ $f \propto \sqrt{\frac{qa}{EI}}$
- ☒ $f \propto \sqrt{\frac{qaE}{I}}$
- ☐ $f \propto \frac{qE}{Ia}$
- ☐ $f \propto \frac{EI}{q^2a}$

First, in a problem like this one does not need to necessarily solve for the frequency, but merely figure out its proper dependencies. We can think of the dipole in an electric field as analogous to a mass on a spring, and try to intuitively figure out what the answer should look like. First, the restoring force is provided by the electric field E acting on the charge q , and increases as both q and E . A stronger restoring force is like a stronger spring constant, and increasing the charge or electric field should increase the frequency of vibration. Thus, answers that are proportional to qE or \sqrt{qE} are physically correct, while the last one, going as $1/q$, or the first, going as $1/\sqrt{E}$, are not.

That only rules out one answer, but what about the other variables? The moment of inertia I is analogous to mass for rotational problems, and increasing the amount of mass connected to a spring should decrease the period of motion. Thus, physical answers should behave as $1/I$ or $1/\sqrt{I}$, also ruling out the the last answer. Finally, we have the dipole spacing $2a$. The larger we make the dipole spacing, the larger the torque will be for a given E and q . A larger torque also corresponds to a larger restoring force, which means that as a increases, so should the frequency of oscillation. This rules out the third and fourth answers, which leaves overall only the second choice as physically reasonable.

The Full Solution. Define the positive x axis to be in the direction of the electric field, and the positive y axis perpendicular to it in the upward direction. This means the z axis points out of the page for a right-handed coordinate system.

On the $+q$ charge, there will be force $F_+ = qE$ along \hat{x} , and on the $-q$ charge, a force $F_- = -qE$ (along $-\hat{x}$). Both of these forces will try to cause the dipole to rotate and orient itself along the electric field; that is, both will result in a clockwise torque about the center of the dipole. The sum of these torques must equal the dipole's moment of inertia I times the resulting angular acceleration α . Let the position of the $+q$ charge be defined by a vector \vec{r}_+ whose origin is at the dipole center, and similarly \vec{r}_- will give the position of the $-q$ charge. We also define a unit vector \hat{r} pointing from the $-q$ to the $+q$ charge. Finally, remember that a clockwise rotation defines a *negative* torque - this is the version of the right-hand rule for torques.ⁱ

ⁱWe have to pick the sign because $\vec{\tau}$, resulting from a cross-product, is a pseudovector.

$$\begin{aligned}\sum \vec{\tau} &= (\vec{r}_+ \times \vec{F}_+) + (\vec{r}_- \times \vec{F}_-) = qEa \hat{r} \times \hat{x} + (-qE)(-\hat{r} \times \hat{x}) \\ &= qEa(-\hat{z} \sin \theta) + qEa(-\hat{z} \sin \theta) = -2qaE \sin \theta \hat{z} = I\vec{\alpha} \\ \sum |\vec{\tau}| &= -2qaE \sin \theta = I|\vec{\alpha}|\end{aligned}$$

We could have avoided the vector baggage right off the bat, if we just chose the resultant torque to be negative based on the right-hand rule. In order to show simple harmonic motion, we need to show in this case that $\alpha = -\omega^2\theta$. Recalling the definition of α , we have:

$$I\alpha = I \frac{d^2\theta}{dt^2} = -2qaE \sin \theta$$

If the angle θ is small, we can approximate $\sin \theta$ by the first term in its Taylor expansion (a “first-order” approximation):

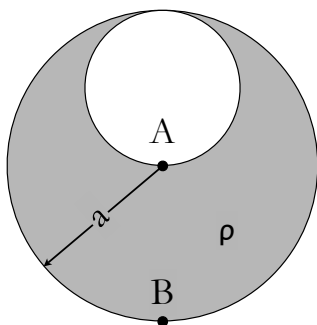
$$\begin{aligned}\text{Taylor expansion} \quad \sin \theta &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \theta^{2n+1} = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots \\ \text{small } \theta : \quad \sin \theta &\approx \theta\end{aligned}$$

Using this approximation,

$$I \frac{d^2\theta}{dt^2} \approx -2qaE\theta \quad \text{or} \quad \frac{d^2\theta}{dt^2} \approx -\frac{2qaE}{I}\theta$$

This is our beloved differential equation for simple harmonic motion, viz., $a = -\omega^2 x$, and thus for small θ ,

$$\omega = \sqrt{\frac{2qaE}{I}} \quad \text{or} \quad f = \frac{1}{2\pi} \sqrt{\frac{2qaE}{I}}$$



11. The sphere of radius a was filled with positive charge at uniform density ρ . Then a smaller sphere of radius $a/2$ was carved out, as shown in the figure, and left empty. Which expression *could* give the expression for the electric field *anywhere* inside the cavity? The \hat{y} direction is vertical, and r is measured from the center of the large sphere. *Hint: if it is true anywhere inside the cavity, pick an easy example point.*

- ☐ $\vec{E} = \frac{2k_e \pi \rho}{r} \hat{y}$
- ☐ $\vec{E} = \frac{2k_e \pi \rho a}{r^2} \hat{y}$
- ☒ $\vec{E} = \frac{2k_e \pi \rho a}{3} \hat{y}$
- ☐ $\vec{E} = \frac{2k_e \pi \rho r}{a} \hat{y}$

There are two very simple way to eliminate answers here: first, not all of the choices have the proper units. Only the third choice gives the correct units for E . Second, if the expression is supposed to give the field anywhere inside the cavity, we can pick one easy example point where we know how to calculate the field. At point A, the total field would be equivalent to that of a negatively charged sphere of radius $a/2$ and charge density ρ at a distance $a/2$ (see full solution below). A simple application of Gauss' law gives the third answer as the correct field.

Below we reproduce the full solution to the problem, taken from Homework 2.

By the principle of superposition, a carved out sphere of positive charge is equivalent to a *filled* sphere of positive charge plus a smaller sphere of radius $a/2$ having a charge density $-\rho$ (see figure below).

Thus, the original, nasty-seeming problem is just equivalent to finding the electric field due to two spheres. Further, we earlier derived from Gauss' law that any spherically symmetric charge distribution is equivalent to a point charge, so long as you are considering points outside the distribution. Taking that into account, we actually only have to worry about the field due to point charges - even easier!

At point A, we are at the exact center of the larger sphere, and by symmetry, its net contribution to the electric field should be zero. We are also on the edge of the smaller sphere, and since we are outside its volume, we may consider it as a point charge a distance $a/2$ away. The magnitude of the equivalent point charge is the same as the total charge on the smaller sphere, which we can find by multiplying its volume by the charge density:

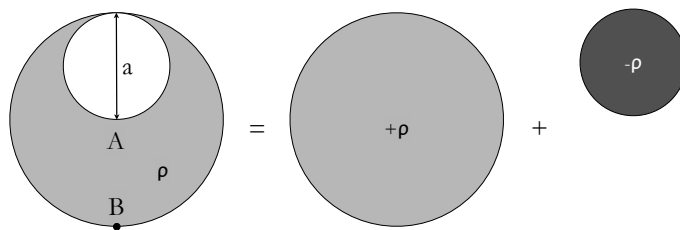


Figure 1: Problem 1: using superposition

$$Q_{\text{small}} = \rho V = \rho \left(\frac{4}{3} \pi r^3 \right) = \frac{4}{3} \pi \rho \left(\frac{a}{2} \right)^3 = \frac{4\pi \rho a^3}{24} = \frac{\pi \rho a^3}{6}$$

Now we can find the electric field at A easily, noting that the field from the smaller sphere (and equivalent point charge) must be pointing in the vertical direction, which we will call the \hat{y} direction. Formally, we should say that the appropriate unit vector points in the $-\hat{y}$ direction, from the charge's center to point A , but this is counteracted by the negative charge density:

$$\vec{E}_{\text{tot}} = \vec{E}_{\text{big}} + \vec{E}_{\text{small}} = 0 + \frac{k_e Q_{\text{small}}}{(a/2)^2} (-\hat{y}) = \frac{k_e \pi \rho a^3}{6 (a/2)^2} \hat{y} = \frac{2k_e \pi \rho a}{3} \hat{y}$$

This gives us the field at point A , which must agree with any expression for the field anywhere within the cavity. Thus, the third choice is the correct one.

Any point inside the cavity? As a bonus, it turns out we can find the field at *any* point inside the cavity, it is not much harder at all. Take the origin to be the center of the large sphere. From this coordinate system, the center of the small sphere can be represented by a position vector $\frac{a}{2} \hat{y}$. The electric fields generated by each sphere is most easily represented with their respective radial vectors \vec{r} and \vec{r}' as shown in the figure below - \vec{r} gives the field from the larger sphere, and \vec{r}' from the smaller. However, we can see from the figure that $\vec{r} = \vec{r}' + \frac{a}{2} \hat{y}$. For any point within the hollow cavity, at an arbitrary point represented by \vec{r} , we can easily find the amount of charge enclosed from the larger and smaller spheres:

$$Q_{\text{big}} = \frac{4}{3} \pi r^2 \rho$$

$$Q_{\text{small}} = \frac{4}{3} \pi r'^2 \rho$$

From the enclosed charges, we can now readily find the total field, and we again find the third choice to be correct:

$$\begin{aligned} \vec{E}_{\text{tot}} &= \vec{E}_{\text{big}} + \vec{E}_{\text{small}} = \frac{k_e Q_{\text{big}}}{r^2} (-\hat{y}) + \frac{k_e Q_{\text{small}}}{r'^2} (-\hat{y}) \\ &= \frac{4k_e \pi \rho}{3} \vec{r} + \frac{4k_e \pi \rho}{3} \left(\vec{r} - \frac{a}{2} \hat{y} \right) = \frac{2k_e \pi \rho a}{3} \hat{y} \end{aligned}$$

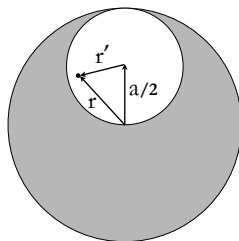


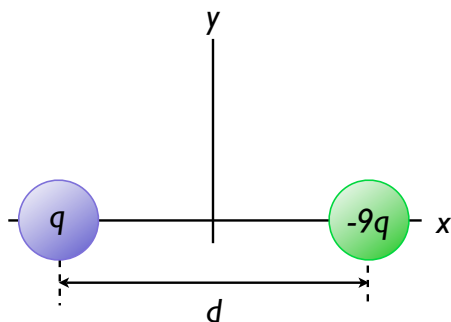
Figure 2: Problem 1: using superposition

12. A spherical balloon contains a positively charged object at its center. As the balloon is inflated to a greater volume while the charged object remains at the center, does the electric **flux** at the surface of the balloon:

- ☐ increase

- ☐ decrease
- ☒ remain the same

The electric flux at the surface of the balloon depends only on the enclosed charge, which does not change.



13. A positive charge of q and a negative charge of $-9q$ are placed a distance d apart, as shown at left.. Determine one point (other than infinity) at which the total electric field is zero.

- ☒ $d/2$ to the left of the positive charge
- ☐ $d/4$ to the right of the positive charge
- ☐ $d/2$ to the right of the negative charge
- ☐ $d/4$ to the left of the negative charge
- ☐ there is not such point

If there is a point where the electric field is zero, we know it must be to the left of the positive charge. For the field to cancel, we can't be closer to the larger charge. Since electric forces get larger as separation decreases, the only way the force due to the larger charge can be the same as that due to the smaller charge is if *we are farther away from the larger charge*. This leaves only the first choice. More explicitly ...

If we want there to be no net field, the electric fields from both the positive and negative charges must cancel. We want to find the position r such that both fields are equal in magnitude, knowing that they can be opposite in direction only either to the right of the negative charge, or to the left of the positive charge. As stated above, to the right of the positive charge is not a physically reasonable choice, thus we need only consider the region to the left of the positive charge. Both charges are on the x axis, so the problem is one-dimensional and does not require vectors.

Let E be the total field due to both charges, which we will simply call q_1 and q_2 to be perfectly general (so $q_1 \equiv q$, $q_2 \equiv 9q$).ⁱⁱ If we call the distance to the **left of the positive charge** x , then the total electric field at x is simply

$$E_{\text{tot}} = \frac{k_e q_1}{x^2} - \frac{k_e q_2}{(x + d)^2} = 0$$

The negative sign is crucial to account for the fact that the fields act in opposite directions, since we did not explicitly say that q_2 is negative. We could have said q_2 is negative explicitly, and then not used the minus sign above, but the result is the same. We have now a simple equation to solve. Canceling the common factor of k_e :

$$\begin{aligned} \frac{q_1}{x^2} &= \frac{q_2}{(x + d)^2} \\ q_2 x^2 &= q_1 (x^2 + 2xd + d^2) \\ (q_1 - q_2) x^2 + 2dq_1 x + q_1 d^2 &= 0 \\ \Rightarrow x &= \frac{-2dq_1 \pm \sqrt{4q_1^2 d^2 - 4q_1 d^2 (q_1 - q_2)}}{2(q_1 - q_2)} \\ x &= d \left[\frac{-q_1 \pm \sqrt{q_1^2 - q_1^2 + q_1 q_2}}{q_1 - q_2} \right] \\ x &= d \left[\frac{-q_1 \pm \sqrt{q_1 q_2}}{q_1 - q_2} \right] \end{aligned}$$

Now we just need to plug in our specific values for q_1 and q_2 :

ⁱⁱIt is not considered a general solution to the problem if we have arbitrary integers like 9 floating around.

$$\begin{aligned}
 x &= d \left[\frac{-q_1 \pm \sqrt{q_1 q_2}}{q_1 - q_2} \right] \\
 &= d \left[\frac{-q \pm \sqrt{9q^2}}{-8q} \right] \\
 &= d \left[\frac{-1 \pm 3}{-8} \right] \\
 &= d \left[\frac{1}{2}, -\frac{1}{4} \right]
 \end{aligned}$$

The negative solution would correspond to a position to the right of the positive charge, between the two charges, which is unphysical - our equations do not take into account that the field from the positive charge changes sign for $x < 0$. When we set up this problem, we explicitly restricted the validity of our solution to the region $x > 0$, since we already knew that between the two charges the fields would act in the same direction and could not possibly cancel. Thus, the positive root is the correct solution, or $d/2$ to the left of the positive charge.

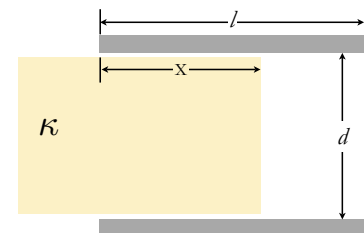
14. If the current carried by a conductor is doubled, what happens to the electron drift velocity?

- ☐ it is halved
- ☐ it remains the same
- ☒ it is doubled
- ☐ it quadruples
- ☐ it decreases by four times

Within our simple electron gas model, the current is related to drift velocity by $I = nqv_d A$. If the current doubles for a given conductor, for which n , q , and A are fixed (this must be true if it is the same conductor) then v_d must double.

15. A capacitor is constructed from two square plates of sides l and separation d . A dielectric is inserted a distance x into the capacitor, as shown at right. For what value of x would the capacitor store the most energy when connected to a voltage source?

- ☐ $x = l/2$
- ☐ $x = 0$
- ☒ $x = l$
- ☐ it does not matter



The simple solution is to remember that dielectrics allow capacitors to store more charge at a given potential difference, and thus store more energy. The maximal energy stored will be when the dielectric is fully inserted. The full solution to this problem from homework 4 yields the same conclusion.

16. Referring to the figure above, be if the inserted section were a *conductor* instead of a dielectric, in what direction would the force on it be?

- ☐ up
- ☐ to the right
- ☒ to the left
- ☐ down
- ☐ there is no net force

Again, see the solution to homework 4. The capacitor stores no energy with a metal plate inserted. This means that the system will try to push out the metal plate to maximize its stored energy, and so the force must be to the left.

17. An electron (of charge $-e$ and mass m_e) enters a region of uniform electric field $\vec{E} = 800 \hat{x}$ [N/C] with velocity $\vec{v}_i = 1.5 \times 10^5 \hat{y}$ [m/s]. What is magnitude the acceleration $|\vec{a}|$ of the electron due to the electric field?

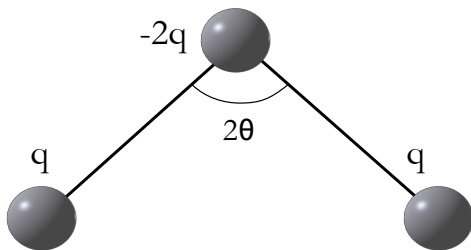
- ☐ $-3.5 \times 10^{13} \left[\text{m/s}^2 \right]$
- ☐ $4.6 \times 10^8 \left[\text{m/s}^2 \right]$
- ☒ $-1.4 \times 10^{14} \left[\text{m/s}^2 \right]$
- ☐ $6.8 \times 10^{12} \left[\text{m/s}^2 \right]$

The velocity is an unnecessary quantity here. The presence of an electric field gives rise to an acceleration through the electric force:

$$\vec{F}_e = q\vec{E} = m\vec{a}$$

Since \vec{E} is in the \hat{x} direction, the acceleration \vec{a} due to that field will be in the \hat{x} direction as well, we can drop the vector notation. Using the quantities given, and rearranging the above:

$$a = \frac{qE}{m} = \frac{-eE}{m_e} = \frac{(-1.6 \times 10^{-19} [\text{C}]) (800 [\text{N/C}])}{9.11 \times 10^{-31} [\text{kg}]} = -1.4 \times 10^{14} [\text{N/kg}] = -1.4 \times 10^{14} \text{ m/s}^2$$



18. In the figure at left, three point charges are connected by unbreakable strings of length d . What is the equilibrium angle 2θ ? Note that $\frac{d}{dx} \frac{1}{\sin x} = -\frac{\cos x}{\sin^2 x}$.

- ☐ 90°
- ☒ 180°
- ☐ 135°
- ☐ 90°

If the arrangement of charges were not perfectly linear, the q charges would repel each other toward a linear configuration. The only plausible configuration that leads to no net forces is the linear one. The math proving this is messy, but as in homework 2, you can write down the total energy of the system and minimize it with respect to θ .

19. In a region of uniform electric field \vec{E} , a charged particle experiences an acceleration \vec{a} . If a second particle with **four** times the charge and **twice** the mass of the first particle enters that same region, it will experience an acceleration

- ☐ $\frac{1}{4} \vec{a}$
- ☐ $\frac{1}{2} \vec{a}$
- ☐ \vec{a}
- ☒ $2\vec{a}$
- ☐ $4\vec{a}$

If the charges experience no other forces, the electric force must give the acceleration: $F_e = qE = ma$, which leads to $a = qE/m$. If the charge increases by four times, the acceleration increases by four times; if the mass doubles, the acceleration is halved. Overall, this leads to a doubling of the acceleration.

20. Two particles are separated by a distance of 3.0 m; each exerts an electric force of 1.0 N on the other. If one particle carries 10 times as much electric charge as the other, what is the magnitude of the smaller charge?

- ☐ 10 pC
- ☒ 10 μC
- ☐ 10 nC

□ 10 kC

Let one charge be q and the other $10q$. We have then:

$$\begin{aligned} F &= 1 \text{ [N]} = \frac{k_e q (10q)}{(3 \text{ [m]})^2} \\ 9 \text{ [N} \cdot \text{m}^2] &= 10k_e q^2 \\ \Rightarrow q &= \sqrt{\frac{9 \text{ [N} \cdot \text{m}^2]}{10k_e}} = \sqrt{\frac{9 \text{ [N} \cdot \text{m}^2]}{9 \times 10^{10} \text{ [N} \cdot \text{m}^2/\text{C}^2]}} = 10^{-5} \text{ [C]} = 10 \mu\text{[C]} \end{aligned}$$